Driving the Design of the STEM-Prep Pathway

What's Meaningful to Students and Essential to Success

Dana Center **Mathematics** PATHWAYS

Updated January 2017

The creative principle [of science] resides in mathematics.
—Albert Einstein
(1933, June). On the Method of Theoretical Physics.
The Herbert Spencer Lecture conducted from Oxford.

What is the STEM-Prep pathway?

This pathway is a sequence of courses designed to prepare developmental mathematics–level students interested in STEM careers to enter—and succeed in—calculus and other technical courses that require strong algebraic skills and a mastery of functions. The new courses under development to support the STEM-Prep pathway include *Reasoning with Functions I* and *Reasoning with Functions II*.

Foundations of Mathematical Reasoning
https://dcmathpathways.org/resources/dcmp-foundations-mathematical-reasoning-table-contents-and-curriculum-overview

Frameworks for Mathematics and Collegiate Learning
https://dcmathpathways.org/resources/dcmp-frameworks-mathematics-and-collegiate-learning-course-overview-and-learning-outcomes

Students begin the STEM-Prep pathway after completing the Dana Center Mathematics Pathways (DCMP) developmental course *Foundations of Mathematical Reasoning* and its co-requisite *Frameworks for Mathematics and Collegiate Learning*.

With three semesters of coursework, students can complete this accelerated pathway to calculus:

- Semester 1: *Foundations* and *Frameworks* (which students take concurrently when they traditionally would be entering a beginning algebra course)
- Semester 2: *Reasoning with Functions I*
- Semester 3: *Reasoning with Functions II*

What's the STEM-Prep content design team doing?

To identify the most appropriate content for the STEM-Prep pathway, we invited prominent leaders in mathematics education research and instruction to participate in a content design team.

This team convened in spring and summer 2014 through a series of phone conferences and face-to-face meetings facilitated by Frank Savina (STEM-Prep pathway coordinator, Dana Center) and Stuart Boersma (STEM-Prep pathway lead author, Central Washington University, Ellensburg, Washington).

The content design team began this process[1] by drawing on its members' experience in the classroom, knowledge of mathematics education research, and understanding of how undergraduate institutions approach calculus to identify the aspects of calculus that tend to be most demanding and difficult for students.

STEM-Prep Pathway Content Design Team

David M. Bressoud, DeWitt Wallace Professor of Mathematics, *Macalester College, St. Paul, Minnesota*

Helen Burn, Professor of Mathematics, *Highline Community College, Des Moines, Washington*

Marilyn P. Carlson, Professor of Mathematics, *Arizona State University, Phoenix, Arizona*

Eric Hsu, Professor of Mathematics, *San Francisco State University*

Michael Oehrtman, Associate Professor of Mathematical Sciences, *University of Northern Colorado, Greeley, Colorado*

While it is well known that weak algebraic skills can cause students difficulty in calculus, the content design team concluded that success in calculus was even more strongly dependent on a deep conceptual understanding of function processes and covariational reasoning.

Some Difficult Concepts in Calculus

- *Function notation*—including distinguishing input from output and variables from parameters; using different symbols to represent equal processes
- *Exponential and logarithmic growth*—including the fact that each is a function in its own right and also an inverse of the other
- *Communicating about change and rates of change*—including understanding what it means for negative quantities to increase or decrease; interpreting graphs of f, f', f''; understanding limiting behavior; identifying rate as a quantity in and of itself; examining change over entire intervals
- *Riemann sums*—including making connections between the use of technology and summation notation; using and conceptualizing approximation rules
- *Function as process*—including thinking of a function as a complete activity; understanding functions are processes that can be reversed and composed
- *Covariation*—including understanding dynamic geometric relationships; communicating change with correct language; understanding the chain rule and how compositions of functions transmit change
- *Limits and approximations*—including understanding the definite integral as an accumulator

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[1] The content design team's process is presented in more detail in "They Will Need It for Calculus" available at https://dcmathpathways.org/resources/they-will-need-it-calculus-structure-and-content-stem-prep-pathway.

What's meaningful to students and what's essential to success?

How do we incorporate meaningful problem solving in mathematics coursework, and what masteries and understandings are essential to student success in calculus?

The content design team used these two driving questions to focus its thinking on the most crucial content for the STEM-Prep pathway. This focus, grounded in research,[2] entails streamlining the traditional precalculus syllabus.

While the team recognized the many content areas in traditional precalculus courses that are clearly *helpful* in preparing students for calculus, team members focused on identifying those content areas that were both immediately *meaningful to the students* and *essential to student success*.

The content—and the pedagogical approach—for the STEM-Prep courses *Reasoning with Functions I* and *II* were thus designed to better prepare students to master difficult concepts in calculus by giving them targeted skills and conceptual understanding to succeed.

The final content and sequencing of this course material was informed by current research in mathematics education and recommendations from the Mathematical Association of America's outreach work to partner disciplines.[3]

For example, the content design team determined that some algebraic skills, such as polynomial long division, were not immediately meaningful for the precalculus student. Further, many calculus instructors revisit this topic.

The team determined that it could be more efficient—and helpful to students and faculty—to adopt a just-in-time teaching approach (in the calculus course) for this particular topic.

As the team continued to examine essential content more deeply, four overarching principles emerged. To succeed in calculus and beyond, it is crucial that students first achieve . . .

- *Deep understanding of the function process:* A strong conceptual understanding of the process view (rather than the action view) of function gives students a critical mathematical foundation to support future learning in STEM fields. By stressing the process view of a function, the STEM-Prep curriculum will prepare students to analyze function outputs on entire intervals of inputs, reason about inverting functions by reversing a process, and make stronger connections between the graph of a function and the function's relationship to generalized inputs and outputs. Students will understand that a function is independent of a formula and that to conceptualize fully the notion of function composition, a process view of functions is essential.

- *Proficiency in covariational reasoning:* The ability to analyze two quantities simultaneously, how those quantities change, and how they covary enables students to better understand the unique and dynamic problem situations addressed by calculus and related disciplines. The STEM-Prep curriculum will provide students many opportunities to explore dynamic function relationships and help students

[2] See the Resources section at end of this document.

[3] See References and Suggested Reading in the Resources section at the end of this document.

more easily conceptualize the notions of an average rate of change and the transition between an average rate of change and an instantaneous rate of change.

- *Fluency in communication with functions and function notation:* Students will communicate orally and in writing as they analyze function behavior in multiple representations. The STEM-Prep curriculum will engage students at the notational level by having them directly examine the need for function notation and by requiring them to interpret to and from function notation. Such communication skills are essential to developing students' mathematical knowledge and logical reasoning skills.

- *Facility with meaningful approaches to algebraic reasoning*: Students will engage with the curriculum content as they develop their algebra and problem solving skills within authentic STEM contexts. Students will create, explore, and interpret mathematical models and use algebra as a way of extracting additional information from a model or mathematical problem. This approach to algebra will give students an immediate appreciation of the usefulness of algebra and algebraic reasoning.

The content design team strongly believes that an active pedagogy seriously and intentionally built on these principles will help students succeed in calculus.

What's next?

Currently, the STEM-Prep pathway is being implemented at seven colleges and universities across the country. Four institutions are collaborating with the Dana Center to test the efficacy of the pathway, providing feedback for revision of *Reasoning with Functions I* and *Reasoning with Functions II*. Summative evaluation of the pathway will be completed by July 2018.

To receive updates on events and releases of materials through the monthly Dana Center Higher Ed In Brief, email us at **dcmathpathways@austin.utexas.edu**.

Resources

Learning Outcomes

Reasoning with Functions I: Student Learning Outcomes (Draft for Implementation Spring 2016), developed by the content design team and Dana Center staff. Available at https://dcmathpathways.org/resources/dcmp-reasoning-functions-i-table-contents-and-curriculum-overview.

Reasoning with Functions II: Student Learning Outcomes (Draft for Implementation Fall 2016), developed by the content design team and Dana Center staff. Available at https://dcmathpathways.org/resources/dcmp-reasoning-functions-ii-table-contents-and-curriculum-overview.

Detailed descriptions of course content can be found in the Brief Course Outlines available at the DCMP Resource Site, http://www.dcmathpathways.org.

References and Suggested Reading

Susan L. Ganter, & William Barker (Eds.). (2004). *The Curriculum Foundations Project: Voices of the Partner Disciplines: Reports from a series of disciplinary workshops organized by the Curriculum Renewal Across the First Two Years (CRAFTY) subcommittee of the Committee for the Undergraduate Program in Mathematics (CUPM)*. Washington, DC: Mathematical Association of America.

David M. Bressoud, Marilyn P. Carlson, Vilma Mesa, & Chris Rasmussen. (2013). The calculus student: Insights from the Mathematical Association of America national study. *International Journal of Mathematics Education in Science and Technology, 44*(5), 685–698.

Michael Oehrtman, Marilyn Carlson, & Patrick W. Thompson. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. In Marilyn P. Carlson and Chris Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education*. *MAA Notes, 73,* 27–41. Washington, DC: Mathematical Association of America.

About this resource

Authors and reviewers

Stuart Boersma, Central Washington University, Ellensburg, Washington

Rebecca Hartzler, Seattle Central Community College, Seattle, Washington

Francisco J. Savina, course program specialist, mathematics, higher education, Charles A. Dana Center at the University of Texas at Austin

Editing and production

Ophella Dano, editing, Dana Center

Peggy Evans, proofreading, Dana Center

Rachel Jenkins, editing and layout, Dana Center

About the development of this document

The work on STEM Prep began in earnest in January 2014. This document is one of several resources that will document the development of the STEM-Prep pathway, for which the Dana Center is developing two college-level transferable mathematics courses—*Reasoning with Functions I* and *Reasoning with Functions II*. This pathway will move students from developmental mathematics through college-level precalculus with learning outcomes specific to STEM preparation.

As the STEM-Prep development process continues, we plan to issue revisions and additional documents to reflect on and clarify the approaches and strategies that the DCMP is employing to support students in learning rigorous mathematics content and as they progress to and through a program of study that leads to a certificate or degree.

About the Dana Center

The Dana Center develops and scales mathematics and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations.

For more information about the DCMP, see **www.dcmathpathways.org**.