Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra

Edited by
Susan L. Ganter
and
William E. Haver

MAA
MATHEMATICAL ASSOCIATION OF AMERICA

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Edited by

Susan L. Ganter
Clemson University

and

William E. Haver
Virginia Commonwealth University

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CUPM Subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY) Members 2005–2010

Name	**Affiliation**
Norma Agras	*Miami Dade College*
William C. Bauldry	*Appalachian State University*
Andrew G. Bennett	*Kansas State University*
Stuart Boersma	*Central Washington University*
Cheri Boyd	*Nazareth College of Rochester*
Morton Brown	*University of Michigan*
Jenna Carpenter	*Louisiana Tech University*
Ray Collings	*Georgia Perimeter College*
Minerva Cordero	*University of Texas - Arlington*
Bruce Crauder	*Oklahoma State University*
Suzanne Dorée	*Augsburg College*
Barbara Edwards	*Oregon State University*
Laurette Foster	*Prairie View A&M University*
Susan Ganter	*Clemson University*
Brian Gill	*Seattle Pacific University*
Sheldon P. Gordon	*Farmingdale State University*
Bill Haver (Chair)	*Virginia Commonwealth University*
Jon Johnson	*Elmhurst College*
Robin Lock	*St. Lawrence University*
David Manderscheid	*University of Nebraska - Lincoln*
Donald Small	*United States Military Academy*
Bruce Yoshiwara	*Los Angeles Pierce College*
Katherine Yoshiwara	*Los Angeles Pierce College*

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Preface

This volume contains recommendations from representatives of partner disciplines obtained during phase two of the Curriculum Foundations Project: Voices of the Partner Disciplines. The reports from the first round of Curriculum Foundation workshops and the CUPM *Curriculum Guide 2004* recommendations concerning general education and introductory college courses led CRAFTY (Committee on Renewal and the First Two Years) to the decision to focus first on College Algebra. This volume also contains reports on this multi-level effort to begin to respond to the recommendations of the partner disciplines by renewing college algebra.

In the late 1990s, the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA) began discussing the preparation of its next Curriculum Guide, a document published once each decade since CUPM's formation in 1953. The purpose of the Curriculum Guide is to assist college mathematics departments in the on-going development and improvement of their undergraduate programs. Historically, this document had focused on the traditional mathematics major, with little attention to alternative courses and programs and virtually no mention of mathematics courses for non-mathematics majors. However, it was clear that this important document could no longer ignore the wealth of new programs, courses, and materials resulting from the reform movement in the undergraduate mathematics community. In particular, the dramatic changes being implemented in introductory college mathematics courses such as precalculus, calculus, and differential equations needed to be studied and directly addressed in the recommendations of the Curriculum Guide.

As a result, in 1999 CUPM initiated a major analysis of the undergraduate mathematics curriculum. As the subcommittee of CUPM concerned with the first two years of the college mathematics program, CRAFTY has had a major role in analyzing and formulating recommendations concerning the foundational years in mathematics instruction. Moreover, given the impact of mathematics instruction on the sciences and quantitative social sciences—especially instruction during the first two years—there was a need for significant input from these partner disciplines. Therefore, CRAFTY was charged with gathering this necessary information for the "mathematics intensive" disciplines (e.g., physics, chemistry, and engineering). In 1999–2001, CRAFTY conducted a series of disciplinary workshops known as the Curriculum Foundations Project.

Each Curriculum Foundations workshop consisted of 20–35 participants, the majority chosen from the discipline under consideration, the remainder chosen from mathematics. Each workshop, lasting 2–3 days, was a discussion among the representatives from the partner discipline, with mathematicians present to listen and serve as resources. The result of each workshop was a report, written and reviewed by partner discipline representatives and directed to the mathematics community. The Curriculum Foundations Steering Committee supplied a common set of questions to guide each workshop's discussions. These common questions provided uniformity and the ability to compare findings across workshops. The 17 mathematics intensive workshops culminated with a curriculum conference to analyze and synthesize the workshop findings. These findings and the 17 disciplinary reports were published by MAA in the document *Curriculum Foundations Project: Voices of the Partner Disciplines* [Ganter and Barker, 2004] and contributed significantly to the content of the most recent Curriculum Guide [MAA, 2004].

Based on these findings—as well as input from numerous focus groups, surveys and research findings— the CUPM *Curriculum Guide 2004* made the following bold recommendation:

> All students meeting general education or introductory requirements in the mathematical sciences should be enrolled in courses designed to

- Engage students in a meaningful and positive intellectual experience;
- Increase quantitative and logical reasoning abilities needed for informed citizenship and in the workplace;
- Strengthen quantitative and mathematical abilities that will be useful to students in other disciplines;
- Improve every student's ability to communicate quantitative ideas orally and in writing;
- Encourage students to take at least one additional course in the mathematical sciences.

In addition the Guide states, "Unfortunately, there is often a serious mismatch between the original rationale for a college algebra requirement and the actual needs of the students who take the course. A critically important task for mathematical sciences departments at institutions with college algebra requirements is to clarify the rationale for the requirements, determine the needs of the students who take college algebra, and ensure that the department's courses are aligned with these findings." [MAA, 2004]

In response to this bold and challenging recommendation from CUPM, CRAFTY set a goal of having every mathematics department review its general education and introductory offerings. CRAFTY has worked toward this goal by: offering a series of panel discussions at regional and national meetings; sponsoring special sessions highlighting innovative courses offered at many colleges and universities; sponsoring PREP workshops to provide professional development for faculty offering introductory courses; conducting, with support from the Harvard Calculus Consortium and in conjunction with MAA's Committee on Mathematics Across the Disciplines (MAD), a second series of disciplinary workshops focusing on the social sciences and humanities; developing a detailed set of *College Algebra Guidelines* that were ultimately endorsed by CUPM; and conducting, with support from the Division of Undergraduate Education of the National Science Foundation, a program to support faculty teams from eleven institutions to change their college algebra courses so that they more closely aligned with the MAA *College Algebra Guidelines*.

This volume includes information on many of these efforts. It begins with reports from participants in the five disciplinary Curriculum Foundation II Workshops (Agriculture, Arts, Economics, Meteorology and Social Science) and a summary of the recommendations of the overall Curriculum Foundations project. It continues with the *College Algebra Guidelines*, reports from the NSF supported college algebra project, and includes papers describing the results of efforts led by four different members of CRAFTY to improve college algebra, three at their home institutions and one with a consortium of HBCUs. Finally a set of recommendations for departments that are considering revitalizing college algebra are outlined. Revitalizing our introductory courses as proposed in the CUPM Curriculum Guide is not an easy task. The papers in this volume do not gloss over the difficulties, but instead are honest descriptions of efforts at a number of institutions and feature the ups and down of curricular development. We believe that the volume will be a valuable tool for departments taking on this challenge.

The members of CRAFTY, the authors of the reports presented here, and the editors of this volume recognize that this work is only a first step on the long road of renewing and reinvigorating College Algebra and other introductory collegiate mathematics courses.

However, it is our hope that these initial efforts will inspire others in the mathematics community to rethink the nature of these courses—both locally and globally—so that our students can benefit from a mathematics curriculum that is both rigorous and relevant.

Susan L. Ganter
William E. Haver
July 2011

References

Ganter, S.L. and W. Barker (2004), *Curriculum Foundations Project: Voices of the partner disciplines*, Mathematical Association of America, Washington, DC.

Mathematical Association of America (2004), *CUPM Curriculum Guide 2004*, Mathematical Association of America, Washington, DC.

CURRICULUM FOUNDATIONS II

This section begins with reports from the five Curriculum Foundation II Workshops:

Agriculture, hosted by Kansas State University;

Arts, hosted by Eastern Michigan University and the University of South Florida;

Economics, hosted by Farmingdale State College;

Meteorology hosted by Valparaiso University and

Social Science, hosted by Virginia Commonwealth University.

The section concludes with a summary of the findings of these Curriculum Foundations II Workshops, a comparison with the findings of the Curriculum Foundations I Workshops and a summary of CRAFTY's response to these findings.

Agriculture

CRAFTY Curriculum Foundations II Project
Kansas State University, March 27–28, 2008

Report Editor: Andrew Bennett and Don Boggs
Workshop Organizers: Don Boggs and Andrew Bennett

Summary

Agricultural instruction addresses business, science, economics, communications and natural resources. With such a diverse set of topics, there come diverse needs for mathematics, and the requirements for different disciplines range from basic algebra through the calculus sequence. However, one point is clear: the mathematical needs of agriculture students are increasing. Fortunately, the general mathematical needs of agriculture students have a strong common core focusing on mathematical understanding. Most students are good at mathematical manipulations, but often have a negative emotional reaction to mathematics and do not understand the meaning of the numbers they have computed, even for relatively simple mathematical ideas. Mathematics classes need to adjust their pedagogy to help students see mathematics in context so that students can develop the understanding needed to effectively use mathematics as a tool. The suggestions from workshop participants range from simply introducing more problems in an agricultural context to potentially significant redesigns that allow students to personalize the material to their own interests. Improving communications between agriculture and mathematics faculty is important to developing such targeted assignments. In addition, more radical changes in pedagogy may be possible in light of new technologies, and may be necessary to help a new generation of students master the additional mathematics they need.

Narrative

Introduction and Background

Agriculture is a very broad area of academic study, which varies somewhat according to location. This is natural since the types of crops that grow best or animals that can best be raised will be different in different environments. The workshop at Kansas State University drew largely from colleges of agriculture in the Midwest; however, the conclusions are nationally relevant. Participants came from many different disciplines with very different program requirements. For example, departments that focus on production agriculture may require just algebra, while agribusiness requires business calculus, and students in soil science may find themselves using differential equations. Furthermore, different schools have different admissions requirements, which lead to different expectations of their students. Therefore, the workshop participants often divided into groups based on the mathematical requirements for individual programs. Somewhat surprisingly, discussions in both groups resulted in similar responses. The major issues in all cases were about students' abilities to apply what they learn, rather than individual mathematical topics.

Understanding and Content

Prior to the workshop, participants were surveyed about the importance of different mathematical topics for their students. The list of topics was initially generated by discussions between the mathematicians and agriculture faculty on the organizing committee. One issue discovered through this process is that topics might be familiar to both groups but using different names, so that while the content of "proportional reasoning" was very important to faculty in agriculture, they did not recognize it by that name. Dealing with such issues may be important in helping students transfer their learning from mathematics to agriculture. Most important overall were the concepts of percent, decimals, graphs, averages (mean/median), fractions, linear equations, and systems of simultaneous equations. Note that a topic might receive a moderate to low score overall but be very important for specific subgroups of agriculture students. Many of the topics at the top of the list in importance are at an intermediate algebra level, and therefore often are assumed to be understood by incoming students who are placed in college algebra and above. However, this assumption does not seem to be universally valid; i.e., mathematics departments need to identify and remediate specific weaknesses with basic skills even for students who have placed into higher level courses.

While the survey of participants addressed specific topics, most of the workshop discussions focused on conceptual understanding and computational skills. It was universally agreed that students need to have both, but the sense of the agricultural faculty (and not disputed by the mathematicians) was that students typically develop reasonable computational skills but weak conceptual understanding. Students need to understand that mathematics is not just abstract, but has real applications. In the words of one participant, "they need to know what the numbers mean." Another participant noted that his students were fine at dealing with $y = mx + b$, but couldn't relate that to the formula Weaned Weight = days × average daily growth + birth weight. Students are generally good at following procedures, but are weak at problem solving. They often lack the ability to take a formula with three variables and plug in two values to solve for a third, and need more experience with solving multi-step word problems. Additionally, students need to:

- have solid notions of percent, including percent as "parts per hundred" and percentage increases
- be able to translate words into formulas and then use order of operations and algebraic manipulations to compute specific values
- understand what each part of a formula is telling them
- be able to carry out and comprehend unit conversions, including changes from percent to parts per million or per pound to per hundredweight
- understand the time-value of money that is important in many business situations
- understand data and basic concepts
- use regression and related statistics
- experience non-normal and non-linear situations
- understand basic number sense; e.g., that 3.5 is midway between 2.5 and 4.5 or that there are not 200 inches in 2 feet
- understand and interpret graphs, especially bar graphs, line graphs, and scatter plots.

Technology

Technology was not a major concern of participants. It was felt that pencil, paper, and simple calculator should be generally available to students, but there was not a strong sentiment for or against graphing calculators or other tools. In general, spreadsheets are very useful in agriculture and it would be beneficial for students to have experience with setting them up and not just filling numbers into a predefined page. It also would be beneficial for students to have experience with some of the advanced features of spreadsheets, such as using the Solver to solve min/max problems or to hit a target value. Spreadsheets also can be used for necessary statistics; specialty statistical software is not needed.

Instructional Techniques

The main issue with instruction is the need to make mathematics more relevant and "less scary." There were many suggestions for reaching this goal, but most were ideas for things to try rather than proven recipes. A common suggestion was to embed the mathematical instruction into content familiar to the students and utilize multiple contexts. In addition, students often learn better when they see mathematicians value what the students value. Of course, there are practical issues to developing such instruction. For example, few mathematics faculty are sufficiently familiar with agriculture to be able to use agricultural examples in instruction. Improving communications between faculty in agriculture and mathematics will be important in addressing this need. A number of participants noted that they had no communications with the mathematics faculty at their home campuses. Finding ways to make it easier for mathematics faculty to find appropriate problems in an agricultural context, perhaps via a website, would be beneficial.

Of course, the large mathematics courses that cover the topics agriculture students need draw on students from many different disciplines. There was discussion of the possibility of allowing students to personalize their course in various ways. Perhaps students could attend common lectures but opt for different problem sections (recitations) focused on a variety of different majors, with agriculture as one of the choices. Or perhaps problem sets could be devised using a menu approach in which students pick one problem from each column, where the columns offer problems addressing the same mathematical content in different contexts. Appropriate assessment, placement, and remediation should be addressed for all students; even students who place into more advanced courses often have specific weaknesses in the prerequisite material that should be addressed in some targeted fashion. Perhaps electronic materials could be used to help students "self-remediate" in specific areas of weakness, without repeating an entire course. And, other innovations in pedagogy, such as a year-long algebra/calculus sequence with continuous review, may be a better way for students to learn. While these approaches would require significant modifications to existing courses, experimenting with such ideas drew interest from several participants.

In addition to the more speculative ideas listed above, some more common issues in pedagogy were also discussed. Group work found favor with some but not all participants, with everyone agreeing that extra care in implementation is required. As such, the choice to use groups should be left to the individual instructor. General technical communication skills are important and should be included in all technical courses; i.e., students should learn to be precise, concise, and clear in explaining their work.

Appendix: List of mathematics topics considered important for agriculture students

Mean Rating of Importance (5=very Important, 1=Unimportant)	Topic
4.76	Percentage
4.62	Decimals
4.62	Graphs
4.62	Averages (mean/median)
4.55	Fraction
4.52	Proportion
4.41	Linear Equations
4.17	Area and Volume
4.03	Regression (best fit lines)
3.86	Marginal Revenue/Cost
3.86	Probability
3.76	Deviation
3.76	Hypothesis Testing
3.48	Optimization (min/max problems)
3.45	Quadratic Equations
3.38	Coordinate Geometry (distance formulas, etc.)
3.32	Polynomial Functions and Equations
3.25	Exponential Functions
3.24	Logarithms
3.24	Systems of Linear Equations
3.2	Rational Functions
3.14	Inequalities
3.14	Differentiation (including applications)
3.04	Integration (including applications to area and volume)
2.97	Properties of Functions (continuity, increasing/decreasing, etc.)
2.93	Solving Triangles (e.g., find the third side given two sides and the included angle)
2.86	Matrices
2.68	Trigonometric Functions (periodic functions)

Resources

Math Concepts for Food Engineering, by R.W. Hartel, T. A. Howell Jr., and D. B. Hyslop, Technomic Publishing Co, Lancaster PA, 1997

Math for Soil Scientists, by Mark Coyne and James Thompson, Thomson/Delmar Learning, Clifton Park, NY, 2006

Mathematical Application in Agriculture, by Nina Mitchell, Thomson/Delmar Learning, Clifton Park, NY 2004.

Workshop Participants

Agriculture

Don Boggs, Kansas State University
Kevin Donnelly, Kansas State University
Hulya Dogan, Kansas State University
Dave Mengel, Kansas State University
Arlo Biere, Kansas State University
Jennifer Bormann, Kansas State University
Fred Fairchild, Kansas State University
Chris Raines, Kansas State University
Sajid Alavi, Kansas State University
Melinda Wilkerson, KSU College of Veterinary Medicine
Doug Trumble, Ag 1 Source
Dale Strickler, Winfield Solutions/Croplan Genetics
Stan Henning, Iowa State University
Paul Gallagher, Iowa State University
Dennis Buckmaster, Purdue University
Mickey Latour, Purdue University
Frank Dooley, Purdue University
Rick Waldren, University of Nebraska
Dennis Burson, University of Nebraska
Joe Schatzer, Oklahoma State University
Doug Malo, South Dakota State University
Aimee Wertz, South Dakota State University
Erin Debler, Frontier Farm Credit
Kristy Umscheid, Farmers State Bank, Westmorland, KS

Mathematics

Andy Bennett, Kansas State University
Gabriel Nagy, Kansas State University
Rekha Natarajan, Kansas State University
Gordon Woodward, University of Nebraska
Tevian Dray, Oregon State University
Tim Warkentin, Cloud County Community College

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The Arts

CRAFTY Curriculum Foundations II Project
University of South Florida, November 1–4, 2007

Report Editors: John DeHoog and Chris Hyndman
Workshop Organizers: Joanne Caniglia, Hartmut Hoft, and Elaine Richards

Summary

When artists, art educators, mathematics educators, and mathematicians gather together to discuss mathematics and art, and the connections between the disciplines, many insights emerge. The Arts workshop focused on the points of intersection and the amazing connections within and between mathematics and art. Participants provided a series of mathematical content topics and technological tools that will be required of art majors and although the list is not exhaustive, it provides recommendations for college algebra and finite mathematics courses. Workshop discussions also highlighted advising structures that would assist arts students, such as sequencing courses.

In addition to the skills listed, arts students need to explore—to solve problems and "play" with materials and space with creativity, much like Pólya's analysis of problem solving. As artists and mathematicians discussed their art and the mathematics that sometimes emerged (often without intent), beautiful symmetries and patterns were found. The participants discovered that many of these patterns invite mathematical analysis—but require none—and need only be seen, as music need only be heard. Such forms and dynamics of nature are the things that inspire artists and mathematicians.

Narrative

Introduction and Background

This paper will not only describe the mathematical content, pedagogical dispositions, and technological prerequisites necessary for the development of artists but also will make known the interconnections between mathematics and the arts. As languages, both mathematics and art give expression to the beauty of abstraction and to the complexity of reality. It is in their collaboration and connection that we see the creativity of interdisciplinary study.

With this focus as background, University of South Florida, Osaka City University Advanced Mathematical Institute, Eastern Michigan University, and John Sims Projects presented a unique conference that interwove the discipline of mathematics and mathematical art through a schedule of lectures, workshops, and exhibitions. From these events, artists, sculptures, art educators, dancers, and mathematicians discussed questions designed to assist mathematics educators in determining the mathematical, technological, and pedagogical knowledge necessary for the development of art, dance, and theater majors. For the purpose of this report, the terms "artist," "sculptors," and "dancers" refer to individuals without mathematical specialization, and the terms "art educator," "mathematician," and "mathematics educator" refer to individuals engaged in college teaching.

Although the conference focused on the visual arts, a small portion of the conference focused on the relationship between dance and mathematics. These connections also will be recorded in this paper.

On November 1–4, 2007, the University of South Florida hosted the Knotting Mathematics Conference in Low Dimensional Topology and Mathematical Art. Participants included artists who expressed their work with mathematical themes and mathematicians who use artistry as a means of making the abstract concrete. Presentation teams of artists and mathematicians on the program articulated this interplay between mathematics and art.

It was through numerous round table discussions, interviews, team presentations (one artist and one mathematician), exhibits, and informal conversations that the patterns emerged on the mathematical topics and dispositions that are necessary in the development of artists. A two-hour panel discussion culminated these discussions. Below are the comments from participants that reflect the workshop discussions and the views expressed by most of the workshop participants.

This report represents a consensus of the artists, art educators, mathematicians, and mathematics educators' comments. Following initial drafts, artists and mathematicians were asked to review the materials. The content within each section reflects the consensus of ideas.

Understanding and Content

Artists discussed mathematical topics that are prerequisites to drawing, painting, and 2- and 3-D design. These topics with prerequisite understandings include:

Prerequisite Understandings to Drawing, Painting and Design	
Percentages	• percentage of increase/decrease (mixing of paints) • scaling operations represented by changes in percent of the *xy*-axis (*z* in 3-D)
Measurement	• conversion between units within the same system (metric and customary) • sense of space and orientation • understanding of surface area, volume, perimeter, and length
Use of Tools	• ability to use a ruler, protractor, and compass • ability to estimate answers obtained on calculators
Patterns	• recognition, transference, extension, and creation of patterns
Ratio and Proportion	• ratio of the perimeters of two similar polygons varies proportionally to the lengths of their corresponding sides; the ratio of their areas varies proportionally to the squares of the lengths of their corresponding sides • ratio of the areas varies proportionally to the squares of the lengths of their corresponding sides
Scale	• shapes can be dilated to similar figures with proportional corresponding sides and congruent angles
Perspective Drawing	• orientation of an object does not affect other properties
Transformation	• transformations (translations, rotations, and reflections) preserve distance
Symmetry	• figures can possess line and rotational symmetry
Two- and Three-Dimensional Figures	• shapes or combinations of shapes can be arranged without overlapping to completely cover the plane • there are a finite number of shapes (or combination of shapes) that can be arranged to completely cover the plane • solid figures can be viewed from different perspectives • there are multiple ways to classify most solids

Mathematics Courses Required of Art Majors

Although every art educator and artist acknowledged the importance and relevance of mathematics in the development of artists, they do not believe that the mathematics courses required as part of general education at most institutions addresses the essential mathematical skills necessary for art. At the institutions where the general education mathematics course content is considered to be adequate, art educators expressed concern that most undergraduate art majors do not take these courses prior to drawing, painting, or 2- and 3-D design. Art majors often wait until the end of their program to study mathematics. For example, when students are required in art classes to mix paints using a particular ratio, art educators often find that students have not yet studied the necessary skills of scale, ratio, and proportion.

Additionally, the following mathematical concepts often are needed by art majors:

- sets
- logic
- modeling
- algebra
- probability
- statistics
- consumer mathematics
- matrices
- mathematical systems
- geometry
- topology
- mathematics of finance

Art educators and artists at the conference acknowledged that many art students are afraid to study mathematics, and recommended integration of mathematics and art *throughout* the curriculum. Discussions not only centered on content that was similar, but also on the process of interpreting, knowing, and solving problems.

All participants agreed that a constructivist perspective is essential. Students need to see art and mathematics as processes in which they take an active part. Mathematics and art students alike are not merely recipients of other's work; rather they are creators and problem solvers.

Technology

Because art educators, mathematics educators, and artists use different technologies, this section will summarize technologies used by each of these groups.

Mathematics and art educators used Geometers Sketchpad (Key Curriculum Press) as a dynamic construction and exploration tool that adds a powerful component to the study of mathematics. Students can use this software program to build and investigate objects, figures, diagrams, and graphs. With Sketchpad, students acquire a tangible, visual way to explore and understand abstract concepts. Concepts that initially may be difficult for students to understand become very clear when they see visual representations on the screen and interact with them using Sketchpad.

To make physical models of diverse polyhedra and three-dimensional fractal structures, artists take advantage of "solid freeform fabrication" (SFF) technologies (see http://www.georgehart.com/rp/). Also known as rapid-prototyping fabrication processes, these technologies convert a design, expressed in a suitable language, into an accurate, compact model of a structure by building it up layer by layer.

Ivars Peterson provided an overview of artists' use of technology in his keynote address, while conference participants shared the tools and technology they used to create their art throughout the conference. A sample of these technologies follow:

SFF technologies: These are used to create a wide variety of mathematical mini-sculptures. For example, Artist Bathsheba Grossman has produced pocket-sized, fused-metal models of such geometric objects as the 120-cell and the gyroid (see http://www.bathsheba.com/math/). Such models, when made out of wax, or plastic, also allow her to use the ancient lost-wax method to cast imaginative, math-inspired, metal sculptures (see http://www.bathsheba.com/

gallery/bronze/). She has been able to create her mini-sculptures by doing 3D printing directly in metal (see http://www.bathsheba.com/sculpt). SFF fabrication costs are expensive, but they are decreasing as the technology develops. Some universities already have access to the technology.

Pedagoguery Software: Polyhedra can be created with this software at a reasonable cost. Students can create structures out of wood or cardboard, use origami techniques to create paper variants, and other 3-D materials. Stronger than analogous plastic or paper models, these solids help students to visualize figures (see http://www.peda.com/models/).

Art educators emphasized the use and knowledge of simple tools for construction. Compasses, rulers, grids, and mechanical devices often are used in the mathematics classroom as well as for the creation of art. Without the power of mathematical relationships and processes, these tools have little creative power. Yet undergraduate art majors often do not possess the measurement skills necessary to use them.

Instructional Techniques

Mathematicians noted that most problem-solving frameworks in U.S. mathematics textbooks use Pólya's problem solving stages. However, it is important to note that Pólya's "stages" are more flexible than the "steps" often described in today's mathematics textbooks. These stages often are simplified in textbooks as understanding the problem, making a plan, carrying out the plan, and looking back. This simplification gives the impression that problem solving is a linear process

The National Council of Teachers of Mathematics and the Mathematics Association of America recommend that problem solving and critical thinking are of primary importance. "How to think" is a theme that underlies genuine inquiry and problem solving in mathematics. However, care must be taken so that efforts to teach students "how to think" in mathematics problem solving are not translated into teaching "what to think" or "what to do." This is, in particular, a byproduct of an emphasis on procedural knowledge about problem solving as seen in the frameworks of U.S. mathematics textbooks and the very limited routine problems/exercises included in lessons. Artists and art educators indicated that the charts and models of problem solving found in mathematics textbooks are inconsistent with genuine problem solving.

Clearly, the linearity of the models used in U.S. mathematics textbooks does not promote the spirit of Pólya's creativity and his goal of teaching students to think critically. Throughout the workshop, mathematics and art educators discussed the following deficiencies in the current methods of teaching problem solving:

1. depict problem solving as a linear process;
2. present problem solving as a series of steps;
3. imply that solving mathematics problems is a procedure to be memorized, practiced, and habituated; and,
4. lead to an emphasis on "getting the answer."

This linear formulation is not consistent with problem solving activities in art or mathematics. Artists and art educators suggested that a framework is needed that emphasizes the dynamic and cyclic nature of genuine problem solving. A student may begin with a problem and engage in thought and activity to understand it, a phase that art educators term as "play" or "open exploration." Specifically, it is important to emphasize the process, not the end product. Students then will attempt to make a plan, and in the process may discover a need to understand the problem better. Or when a plan has been formed, the student may attempt to implement it and be unable to do so. The curriculum should then emphasize the process of creating a new plan, or going back to develop a new understanding of the problem, or posing a new (possibly related) problem on which to work. This approach would therefore be described in terms of intention, problem definition, exploration, planning, production, and integration.

Artists often commented:

- The solution is not going to come in ten minutes.
- Design is a process, not a product.
- Artists look outside of their discipline for a process to generate art.

Assignments that Makes the Conceptual Visual

"I ask my students to record their activities in a 24-hour day. They bring their chart to class and I ask them to translate the data into visual form and hierarchies. That is what I mean by looking to other disciplines to gather ideas to create art. For example, statistics can be a big help in making the concepts come to life."

Another example of artists using mathematics to create designs occurs in the work of John Sims. Taking the numbers found in pi, he created a design with a numerical spiral effect.

Instructional Interconnections

Art exhibits at the Museum of Science and Industry in Tampa and the Center Gallery and Oliver Gallery on the University of South Florida's Tampa campus became the backdrop of many discussions on the interconnections between art and mathematics. As artists described their work, both content and process interconnections between mathematics and art emerged. For example mathartist John Sims elaborated on the development of his art and how he connects mathematics in his teaching through his rendition of "Pi," a work that utilizes an unusual pattern of pi while being inspired by mathematics and art. The following interconnections emerged through this and other discussions among the workshop participants.

Content-Based Interconnections

Patterns pervade nature. Mathematics often is described as the science of patterns. Indeed, pattern perception or pattern recognition is important to the survival of every organism. The ability to perceive the most subtle of patterns is key to navigating and understanding the world. Each "orderly arrangement of things" allow individuals to perceive something different about the world and recognize beauty. Workshop participant Brent Collins expressed these ideas eloquently: "Being able to perceive the world in patterns of aesthetic congruence is an incentive to live, no doubt given to us over the course of our evolution because it does have survival value. It is why artists work, suffering through the complexities of their work for the serene transparency to be found on the other side."

Transformations. Another content theme that seemed to permeate many of the art pieces in the galleries, the artists' presentations, and John Conway's lively Nagel address was that of transformation. All materials and objects—actual and represented—bear some relationship to the concepts of time, change, and motion. Reflections, rotations, translations, and glides all connect interesting artistic patterns to mathematics.

Symmetry. Some of the artists and mathematicians shared a common background in design, engineering, and architecture. These individuals spoke of the artists' ability to create order from chaos. The ability to make order from chaos is one of the special talents of the artist and the designer. Good design is often a matter of perceiving potential patterns within disorder or visual complexity. The principle of symmetry was mentioned most often as a principle that can be applied to almost any type of visual information.

Process-Based Interconnections

Constraint. A theme articulated by many of the artists and art educators was the role of mathematics as a constraint to art and inspiration. In the words of workshop participant Charles O. Perry, "The basic difference in the discipline of architecture and sculpture is that I can't force a solution in sculpture, where in architecture, one can arrive at an apparent 'rational' solution through continual work." For Perry, the appropriateness of the form is the final goal or criteria.

Rather than confining art or requiring art to conform to a narrow set of rules, an understanding of essential mathematical constraints frees artists to use their full intuition and creativity within the constraints, even to push the boundaries of those constraints. Constraints need not be negative—they can show the often limitless realm of the possible.

Inspiration. The process of creating art and the process of creating mathematics often inspire each other. "When I set off to be an artist, I would avoid the arbitrary, esteem the orders of God in Nature, make things that were beautiful, try to make things that appeared to have no author, things you thought you had seen before; entwined with mathematics, geometry, topography, spinning, interlocking, always saying thank you God."—workshop participant Charles O. Perry

Mathematics and Dance

It may seem that mathematics and dance are unrelated. The former deals with very rational and logical processes, the latter explores physical and emotional expression. However, presentations and discussions with Karl Schaeffer and the work of Erik Stern demonstrate the many connections. When dancers produce choreography for a new dance or when mathematicians solve non-routine problems, they do much the same thing: creatively exploring patterns in space and time with an eye toward aesthetic potential.

According to dancers, the mathematical topics that are helpful (but not required) include geometry and spatial awareness. For the general dance student interested in technique and choreography, mathematics courses with some physics are useful. Some algebra is helpful within a dance curriculum where the concepts of acceleration, velocity, and other forces on the body in motion are necessary. Obviously mathematical concepts are a rich field for exploring choreographic possibilities and certainly could complement and enhance conceptual approaches to the choreographic craft.

Resources

http://www.mathdance.org/ The dance company, founded in 1987, tours nationally and is co-directed by Karl Schaffer and Erik Stern. Company members include Gregg Lizenbery, Scott Kim, and Chris Jones.

http://www.cs.berkeley.edu/~sequin/SCULPTS/collins.html Website of sculptures by Brent Collins and Carlos Sequin.

http://www.georgehart.com A collection of web pages authored by George Hart.

http://knotart.cas.usf.edu/ Homepage of the Knotting Mathematics and Art Conference

Kalajdzievski, S. (in Press). *Math and art: An introduction to visual mathematics*. CRC Press, 2008.

Workshop Participants

The Arts

Brent Collins, Artist
John DeHoog, Art Department, Eastern Michigan University
Charles Frohman, Artist, Department of Mathematics, University of Iowa
Chaim Goodman-Strauss, Artist, Department of Mathematics, University of Arkansas
George W. Hart, Geometric Sculptor, Department of Mathematics, Stony Brook University
Chris Hyndman, Art Deparment, Eastern Michigan University
Charles O. Perry, Artist
David Popaliskey, Dance, University of Santa Clara
Tony Robbin, Artist
Carlo Sequin, Artist, Engineering, and Mathematics, University of California, Berkeley
Karl Schaffer, Dance Choreographer, Mathematics, De Anza College
John Sims, Mathartist

Mathematics

Thomas Banchoff, Department of Mathematics, Brown University
Joanne Caniglia, Department of Mathematics Education, Kent State University
John H. Conway, Department of Mathematics, Princeton University
Hartmut Hoft, Department of Computer Science, Eastern Michigan University
Natasa Jonoska, Department of Mathematics, University of South Florida

Sasho Kalajdzievski, Department of Mathematics, University of Manitoba
Ivars Peterson, Director of Publications, Mathematical Association of America
Elaine Richards, Developmental Mathematics, Eastern Michigan University

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Economics

CRAFTY Curriculum Foundations II Project
Farmingdale State College, April 30–May 2, 2008

Report Editors: Bala Veeramacheneni, Natsuko Iwasaki, and Richard Vogel
Workshop Organizers: Sheldon Gordon and Richard Vogel

Summary

Mathematics is an integral part of economics, and understanding basic mathematical concepts is important in terms of expressing and communicating ideas in economics. The mathematics curriculum historically has been organized to support the sciences and engineering, though some attention recently has turned to the life sciences and business. However, since the mathematical requirements in economics are ubiquitous and increasing in sophistication, there is a growing need for a revision of the nature and scope of the mathematics taught to support both the large numbers of students in introductory economics courses and those who major in economics. Economics students today are expected to communicate their ideas and solutions in different formats, understand the relationship between variables, build analytical models and test them empirically, and analyze quantitative data to make inferences and draw policy conclusions. They should be able to apply theory to real world problem solving and evaluate alternative solution proposals.

On April 30–May 2, 2008, a group of economists and mathematicians gathered at Farmingdale State College for a workshop to discuss these and other curricular issues relative to the education of economics majors. The economists welcomed the opportunity to work with the mathematics community on possible solutions for some of these issues to its attention. In addition, they expressed strong interest in continuing such collaborations between the disciplines and strongly encouraged mathematicians to reach out to their local colleagues in economics.

Narrative

Introduction and Background

Mathematics and economics are complementary disciplines. Indeed, there has been a long symbiotic relationship between the two disciplines. Economic questions have motivated the creation of some very significant mathematics and conversely mathematics has illuminated the understanding of economic systems. Mathematics allows economists to form meaningful, testable propositions about wide ranging and complex subjects that could not be adequately expressed informally. Further, the language of mathematics allows economists to make clear, specific, positive claims about controversial or contentious subjects that would be difficult otherwise.

Some one million students take economics courses each year, about 95% of all economics enrollments are in the introductory Principles courses. These general courses are the prerequisites for all intermediate and advanced courses and are also used for recruiting virtually all economics majors. Thus, although the intent of the Curriculum Foundations workshop was to focus on the mathematics needs of economics majors, it quickly become clear that it was not possible to separate that from the mathematical needs of the students taking introductory economics courses. Both audiences are therefore addressed in this report.

Mathematics in the Economics Curriculum

Almost all economics courses require some level of mathematical ability and knowledge. Economics instructors recognize that students bring different levels of mathematical skills and knowledge into economics courses, and many introductory and intermediate textbooks include mathematical appendices to familiarize students with algebraic, geometric, and mathematical concepts and methods that they likely have been exposed to in the past.

Every student enrolling in an introductory economics course such as Microeconomics and Macroeconomics Principles is expected to have a knowledge of basic arithmetic, algebra, and geometry—the slope of a line, the area of a triangle or rectangle, and the relationship between variables. They should be able to build and interpret basic graphs, to interpret and solve simple systems of simultaneous linear equations, and to understand concepts such as growth rates and present discounted values.

Majors in economics will take courses such as Intermediate Microeconomics, Intermediate Macroeconomics, and more advanced economics courses and electives, most of which require some background in core mathematical courses such as calculus, linear algebra, and statistics. Undergraduate mathematics courses for economics majors should cover topics useful in economics applications including, for example, constrained optimization techniques in multivariable calculus, derivatives and partial derivatives, matrices and quadratic forms of linear algebra, basic statistics and probability, probability distribution functions, hypothesis testing, and linear and multiple regression analysis. Economics majors in intermediate and advanced courses in economics should be exposed to how these tools and methods are applied in different contexts. Basic economic and mathematical principles introduced in beginning courses are reinforced and refined in intermediate theory courses and later extended in advanced elective courses. In addition, a strong background in mathematics is an asset for students going on to graduate studies in economics as well as business or public policy. Most economics related positions in government, business, and finance require strong quantitative skills.

The nature, scope, and goals of an economics major are quite different from other programs of study and certainly different from a major in mathematics. However, the mathematics that should be taught to economics majors is important. Economics students are expected to:

1. understand the relationship between variables,
2. build analytical models and test them empirically,
3. analyze quantitative data to make inferences and policy conclusions,
4. apply theory to real world problem solving, and
5. understand that every problem has alternative solutions.

Understanding and Content

At many institutions, students may take the introductory economics courses in their first or second college semester, and thus a number of these students may have had only limited exposure to mathematics beyond the high school level. It also is very likely that many of these students will be concurrently enrolled in mathematics courses such as a general first-year survey or pre-calculus/calculus sequence, or even an introductory course in statistics. Much of the discussions during the workshop focused on some of the mathematical weaknesses that the students may have, and their ability, or inability, to apply the mathematical principles and concepts that they have already learned or are currently learning from their mathematics courses to the economics classroom. Understanding of simple arithmetic ideas—such as working with fractions, decimals, percentages, and ratios—is often weak. Many students have great difficulty with mathematical notation (such as the use of variables other than x and y) and the meaning of parameters—particularly in the context of their effects on the graph of a linear function. Many students have had little experience with tables of data, and often have trouble creating or interpreting graphs. Also, while most of the participants did not expect that students in an introductory course should have completed a calculus course, many did express the desire for their students to understand some of the concepts from calculus as applied to the graphs of functions, including the ideas of increasing versus decreasing, and increasing or decreasing at either an increasing or a decreasing rate (concavity).

On the other hand, students taking intermediate economics courses are expected to have had some calculus, including the use of summation notation, the meaning of the derivative as it relates to the behavior (the rate of change) of a function, partial derivatives (again as it relates to the behavior of a function), and constrained optimization from the point of view of Lagrange multipliers. A major emphasis in economics is on the effects on a function of changes in the parameters, and this is particularly important in functions of several variables. The issue is rarely one of calculating the partial derivatives, but rather recognizing whether the partial derivative is positive or negative, enabling students to interpret or predict the economic behavior modeled by the function as each of the parameters changes.

In addition, students in the intermediate courses need to have attained some manipulative skills, particularly the ability to solve systems of two equations in two unknowns and fairly simple systems of three equations in three unknowns using the substitution method. They should also be familiar with some of the basic arithmetic operations on matrices. Moreover, it is extremely important that they are familiar with some statistical ideas, particularly the ability to calculate and interpret the mean, variance, and standard deviation from a given data set. Some branches of economics require considerably more in the way of statistical preparation, including at least one fairly substantial statistics course that covers a significant amount of probability (including conditional and joint probability, expected values, and Bayes Theorem), a wide variety of distribution functions (including the binomial, Poisson, normal, chi-square, *t*- and *F*- distributions), estimation and confidence intervals, hypothesis testing (for means and proportions, as well as their differences), and linear and multiple linear regression.

Technology

Many economics graduates become financial analysts, marketing analysts, and economic consultants. Among the different tasks required to work in these areas are analyzing data and preparing economic reports. Therefore, the ability to work with data is critical. Since real-world data can be massive and complex, technology is essential in completing tasks required in a business environment. For example, spreadsheet-based software such as Excel is useful in plotting and charting data, conducting parametric analysis, and analyzing the impacts of changes in parameters. Students also should be familiar with basic technologies, such as graphing calculators and internet search engines.

However, knowledge about technology alone (i.e., how to use Excel to perform computations) is insufficient. Students also must know how to model problems, which requires conceptual understanding of mathematics and its applications. Then, they must know what type of technology to use and how. The ability to work with data can be acquired only when an effective problem-solving skill is combined with a command of technology to execute the necessary computations.

Instructional Techniques

There was general agreement among the workshop participants that to be successful in economics courses and the economics major, students need to develop and acquire a number of skill sets. In particular, the discussions focused on the following skills that economics students could be exposed to in their mathematics courses that would help them develop necessary application skills.

1. **Mathematical Notations** The variables that students may encounter in the mathematics classroom are very frequently x and y. This practice fails to encourage students to think of other variable representations. For example, in economics, q represents quantity and p represents price. Mathematics courses should utilize many symbols as appropriate to the context of applied problems.

2. **Applied Skills** Economics students must have facility with numbers and dimensions. They should know to ask whether the unit, sign, and scale of an answer are reasonable for a given context. They should be able to interpret the meaning of answers based on tabular and graphical presentation of numbers. Conversely, they should be able to express how people and economic systems behave using equations based upon a given scenario (what the economists call a “story”).

   To develop such application skills, mathematics courses could show students possible applications of mathematical concepts to economic issues and encourage students to think of the meaning of equations and solutions in a given economic context. For example, a method of calculating a growth rate for an exponential process can easily be applied to economic variables, such as a growth rate of the Gross Domestic Product and an inflation rate. Students can then interpret the long-term economic growth rate for a country such as the United States. Applications can help make mathematics more concrete.

3. **Problem Solving Skills** Economics students should know to ask whether a given problem is solvable. They should be able to choose from potential alternative solution methods, construct an equation or a set of equations as a mathematical model based upon the given data and current business environment, and solve the equations they have developed using technology. In summary, effective problem solving may involve the following steps:

   (1) identify the mathematical aspects of a problem;

   (2) create a mathematical model to represent the problem, listing possible methods for solving the problems;

   (3) decide on the most suitable method to solve the problem;

   (4) solve the problem;

   (5) decide if the solution is reasonable and, if there are multiple solutions, decide which is most reasonable;

   (6) apply the solution to the mathematical model to solve the problem; and,

   (7) communicate the solution and its interpretation/significance verbally or in writing.

   In order to foster the problem solving skills needed for economics, mathematics instructors could incorporate practical application exercises. For example, instructors may have students work with real data. Instructors also may ask students to construct an equation based on a given scenario.

Workshop Participants

Economics

James Booker, Professor and Head, Department of Economics, Siena College

Bill Bosshardt, Associate Professor and Director, Center for Economic Education, Florida Atlantic University

John Brock, Professor, Director, Center for Economic Education, University of Colorado

William Dawes, Professor and Chair, Department of Economics, State University of New York at Stony Brook

Eleanor Fapohunda, Professor of Economics and Associate Dean, School of Arts and Sciences, State University of New York at Farmingdale

Rae Jean B. Goodman, Director of Teaching and Learning and Professor of Economics, U.S. Naval Academy

Natsuko Iwasaki, Assistant Professor, Department of History, Economics, and Politics, State University of New York at Farmingdale

Mary Lesser, Professor of Economics and Director of Eastern Economics Association, Iona College

Jane Lopus, Professor, Department of Economics, California State University at East Bay

Michael Murray, Charles Franklin Phillips Professor, Bates College

James Payne, Professor and Chair, Department of Economics, Illinois State University

Dennis Placone, Associate Professor, Department of Economics, Clemson University

Robert Rycroft, Professor, Department of Economics, University of Mary Washington, President, Omicron Delta Epsilon

Michael Salemi, Professor, Department of Economics, University of North Carolina at Chapel Hill

Roberta Schroder, Assistant Professor and Chair, Department of Economics & Finance, Nassau Community College

Rhonda Sharpe, Assistant Professor, Department of Economics, University of Vermont

John Siegfried, Secretary Treasurer, American Economic Association, Department of Economics, Vanderbilt University

Bala Veeramacheneni, Assistant Professor, Department of History, Economics, and Politics, State University of New York at Farmingdale

Richard Vogel, Professor and Chair, Department of History, Economics, and Politics, State University of New York at Farmingdale

Michael Watts, Professor and Director, Center for Economic Education, Purdue University

Mathematics

Deborah Hughes Hallett, Professor, Department of Mathematics, University of Arizona
Alan Tucker, Distinguished Teaching Professor, Department of Applied Mathematics, State University of New York at Stony Brook
Warren Page, Professor Emeritus, Department of Mathematics, New York City College of Technology, City University of New York
Sheldon Gordon, Distinguished Teaching Professor, Department of Mathematics, State University of New York at Farmingdale
Jack Winn, Distinguished Teaching Professor and Chair, Department of Mathematics, State University of New York at Farmingdale

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Appendix A

For Introductory Courses

- Basic Arithmetic and Algebraic Skills (e.g. Fractions, Decimals, Percentage Changes/ Ratios)
- Natural logs for linearizing data
- Slope of a line/ slope at a point
- Linear/ Exponential data/ graph
- Plot numbers/ interpret graphs (see Appendix B: Examples 1, 6, 7, and 8)
- Total/ Average/ Marginal Concepts (see Appendix B: Example 3)
- Using multiple variables/ parameters
- Equations and algebra – relationship between variables (dependent and independent variables)
- Variable names other than x and y (e.g., p and q), parameters can be stated numbers or alphabetical letters
- Practical limits on the values of variables (i.e., domain and range in context)
- Graphing and Interpreting Linear Equations
- Effect of changing parameters in linear equations
- Calculating area of relatively simple geometric figures—rectangles, triangles, etc (see Appendix B: Example 2)
- Two Linear Simultaneous Equations (solve by substitution method)
- Increasing at increasing/decreasing rates (see Appendix B: Example 5)
- Compound (growth) interest rate
- Net present value/ Future value/ Discount (see Appendix B: Example 4)

For Higher Level Economics Courses:

- Solving systems of two or three (simple) linear equations (by the substitution method) (see Appendix B: Example 10)
- Constrained Optimization (e.g. method of Lagrange multipliers) (see Appendix B: Example 10)
- Arithmetic Operations of Matrices (e.g. addition, subtraction, multiplication, determinant, inverse)
- Calculation and Interpretation of the Mean/ Variance/ Standard Deviation (from data)
- Expansion and Manipulation of Summation Notation (see Appendix B: Example 10)
- Derivatives
- Simple Examples of Partial Derivatives with interpretation (see Appendix B: Example 9)
- Iterative systems
- Word problems

Statistics (see Appendix B: Example 11)

- Conditional Probability, Joint Probability
- Expected value
- Distribution (Binominal, Poisson, Normal, Chi-square, t- and F-distributions)
- Confidence interval
- Hypothesis testing (one- and two-sided) for means and proportions, differences in means and differences in proportions.
- Estimation
- Linear Simple and Multiple Regressions
- Type I and type II errors
- P-values
- R^2
- Bayes Theorem
- Cumulative Distribution Function, Probability Density Function
- Central Limit Theorem

Appendix B

For Introductory Courses

Example 1. The demand curve illustrates the price consumers in a market are willing and able to pay for a good for a given quantity of goods. As the quantity of goods in a market increases, the price consumers are willing to pay decreases.

Total revenue for goods sold in a market can be calculated by multiplying the price of the good times the quantity of goods sold. For example, if the quantity of goods in the market is 300, then the price consumers are willing to pay is \$14. Total revenue equals \$4,200. Geometrically, total revenue can be represented by the area of a rectangle. In this case, the rectangle has a width of \$14 and a length of 300 units. The area of the rectangle is: width × length = \$14 per unit × 300 units = \$4,200.

1. Can you find the price and quantity combination that would give the highest revenue for this market?
2. What is the maximum total revenue for this price and quantity?

Consumers benefit in a market by paying less than they are willing to pay for goods they buy. Consumer surplus represents the gains to a consumer of consuming a good. Consumer surplus is represented by the area between the demand curve (what consumers are willing to pay) and the price the consumers actually pay. For example, suppose the price of a good is $14 and consumers buy 300 units. The line $P = 14$ represents the price consumers pay. The demand curve shows what the consumers would have been willing to pay. The area between these lines represents consumer surplus. In this case, consumer surplus is the area of a triangle with a base of 300 and a height of $6. The area of the triangle is: ½ base × height = ½ × 300 units × $6 per unit = $900.

3. Find the price and quantity combination that would give the highest consumer surplus for this market?

Example 2. Total/ Average/ Marginal Concepts Suppose a business firm's total cost (TC) of production is as depicted in the table below. Calculate the average total cost (ATC) and the marginal cost (MC) of production and indicate in the appropriate places of the table.

Units Produced (Q)	TC	ATC	MC
0	30		
1	100		
2	150		
3	210		
4	300		
5	405		
6	516		

Example 3. Net present value/ Future value/ Discount

1. Suppose the rate of interest at which you can borrow and lend is 8% per annum. What is the present value of a discount bond which pays $3000.00 at the end of three years (and, by definition, nothing until then)? Explain why it makes sense that the present value is less than $3000.00? What happens to the present value of this bond if the interest rate falls to 7.5% per annum? Explain why your answer makes sense.
2. Suppose that you are considering subscribing to *The Economist*. The cost of a one-year subscription is $90.00 payable in advance. The cost of a two-year subscription is $170.00 also payable in advance. In either case, you don't have the cash now so you plan to pay for your subscription by charging it to your credit card. Are you better off subscribing for two years or subscribing for one with the intention of renewing your one-year subscription next year? Explain. How would your answer change if you are paying for the subscription with money you currently hold in a saving account?
3. You have received a gift of $1000 from Uncle Buck and wish to save it to use as the down payment on a car that you intend to buy in three years. Which of the following financial strategies is the best?
   a. Buy for $1000 a U.S. Saving Bond (discount bond) that matures in three years and pays $1200 at maturity.
   b. Buy a three-year U.S. Treasury Bond with a face value of $1000, a coupon rate of 6.5% and a current market price of $1000.
   c. Buy a five-year U.S. Treasury Bond with a face value of $1000, a coupon rate of 6.7% and a current market price of $1000.
4. It is January, 2007, and you have decided to plan your life. You plan to have one child who will enroll at UNC in January, 2032. You estimate that you will need $100,000.00 in January, 2032, to finance the expense of your child's college education package. You expect to earn, on average, 8 percent on any funds that you save during the next 25 years.
   a. UNC offers you the following prepayment option. Pay $20,000.00 now and UNC will cover the cost of your child's education package in 2032 provided (s)he is admitted. If (s)he is not admitted, UNC will refund your money including compound interest computed at 8 percent. You do not have $20,000 but your parents

will lend it to you. Should you take advantage of the prepayment option? Explain your answer completely and show any calculations in complete detail. There will be a substantial penalty for an incomplete answer.

b. Suppose that you decide to turn down the prepayment option for some reason. You decide instead to save a constant amount at the end of each year to finance your child's college education package. How much must you save each year? Explain your answer completely and show any calculations in complete detail. There will be a substantial penalty for an incomplete answer.

Example 4. Increasing at increasing/decreasing rates. The table shows the short-run production function for a production process that uses homogeneous capital (the fixed factor) and homogeneous labor (the variable factor) to produce a homogeneous output.

Labor	Total Output	Marginal Product of Labor
0	0	
1	80	
2	200	
3	260	
4	300	
5	330	
6	350	
7	362	

Graph the production function. Locate the region where output is increasing at an increasing rate. Locate the region where output is increasing at a decreasing rate. Compute the marginal product of labor. What is the relationship between the marginal product of labor and whether total output is increasing at an increasing or decreasing rate?

Example 5. Production Possibilities and the Law of Increasing Costs The table below shows the production possibilities for an economy that produces only paper goods and containers.

Paper Goods (units/day)	Containers (units/day)	Opportunity Cost (units)
1000	0	----
800	400	
600	750	
400	1000	
200	1150	
0	1200	

a) Complete the column labeled "Opportunity Cost" of producing one additional container in terms of the loss of one unit of paper goods.

b) Graph the production possibilities curve for this economy. Be sure to label the axes:

c) Explain why a linear production possibilities curve does NOT display increasing cost.

d) Does the production possibilities table shown above display the Law of Increasing Cost? Explain.

Example 6. Marginal Revenue/ Marginal Cost analysis; Profit-Maximizing Rule under Perfect Competition
Alpha Company has the following set of characteristics at its current output level: Current Output = 5,000 units; Market Price = $1.00/ unit; Fixed Costs = $2,000; Variable Costs = $2,500; Marginal Costs = $1.25 and rising

a) Is Alpha producing at the profit maximizing output level? Support your answer with a complete quantitative analysis.

b) If Alpha is NOT maximizing profits at the current level, what action should the firm take? That is, should the firm increase, decrease or shut down production? Explain.

Example 7. Interpreting linear equations Suppose that the simple consumption function in the U.S. economy is $C = 100 + 0.75\ Y^D$, where C is consumption expenditure and Y^D is disposable income. (Note that the D here is not an exponent; the standard usage in economics for disposable income is to write the D as a superscript.)

a) In mathematical terms, what does the coefficient on disposable income (0.75) represent?

b) In economic terms, what does the coefficient on disposable income (0.75) represent?

For Higher Level Economics Courses

Example 8. Simple examples of partial derivatives Suppose a business firm's production function for a product is represented by $Q = 0.1LK + 3L^2K - 0.1L^3K$.

a) Calculate the marginal product of labor (i.e., the partial derivative of Q with respect to L).

b) Suppose that Tom derives utility from consuming CDs (x) and DVDs (y) as given by the utility function, $U = x^a y^b$, where $0 < a, b < 1$. Calculate the marginal utility of good x (i.e., the partial derivative of U with respect to good x).

Example 9. Constrained optimization (Lagrangian) Suppose that a person's utility function between two goods, x and y, is $U = xy^2$, the income is 100, the price of x is \$2 and the price of y is \$3. Find the utility maximizing consumption bundle using the Lagrangian.

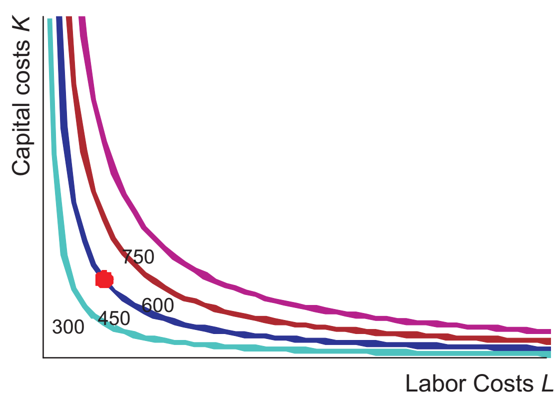
Example 10. The accompanying figure shows the contour diagram for a Cobb-Douglas model. At the indicated point,

a. Is the Production an increasing or decreasing function of Labor costs L?

b. Is the Production an increasing or decreasing function of Capital costs K?

c. Is the Production increasing at an increasing rate, increasing at a decreasing rate, decreasing at an increasing rate, or decreasing at a decreasing rate as a function of Labor costs L?

d. Is the Production increasing at an increasing rate, increasing at a decreasing rate, decreasing at an increasing rate, or decreasing at a decreasing rate as a function of Capital costs K?



Meteorology

CRAFTY Curriculum Foundations II Project
Valparaiso University, February 22–23, 2008

Report Editors: Craig Clark and Bill Marion
Workshop Organizers: Bill Marion and Bart Wolf

Summary

The meteorologists stated that meteorology students need to complete Calculus I, II, III and Differential Equations—preferably in the first two years of study—in order to be prepared for the quantitative meteorology courses which begin in the third year. Unfortunately, about half of the students who register in their first year for a meteorology program do not have the prerequisite knowledge to be successful in Calculus I. These students must therefore take at least one precalculus course. Mathematics is a struggle for some of these students, so there is a significant dropout rate from meteorology in the first year. No conclusions were reached as to how to ameliorate this problem, except to say that the underprepared students' problem-solving skills somehow must be addressed in precalculus courses.

There was consensus among the participants that conceptual understanding and problem-solving skills are equally important skills for success in upper-level meteorology courses. Since most calculus textbooks do not include many direct applications to problems in meteorology, it was observed that good physics applications could be used as a substitute. Vectors also are a very important topic for meteorology students, and a strong understanding of vectors is necessary in quantitative meteorology courses. In addition, the ability to interpret functions, limits, derivatives and definite integrals, when the data are presented as discrete data points and via graphs—not just by a well-defined function rule—is essential.

Workshop participants stressed that meteorology students need to be familiar with various software packages (Excel was specifically mentioned) and need experience with computer programming, particularly in FORTRAN.

Most important, the workshop participants emphasized that meteorology students' experiences in the first two years of collegiate mathematics should instill the ability to make sense of quantitative data in such a way that they have the confidence to use a variety of mathematical concepts and tools in a variety of settings.

Narrative

Introduction and Background

On February 22–23, 2008, fourteen meteorology faculty representing eleven colleges and universities met at Valparaiso University to discuss the mathematical needs of meteorology students. Four mathematics faculty also were present to facilitate the conversation, address questions about curricular issues and mathematics content, and listen to the conversation. Prior to their arrival, participants took part in an electronic discussion about expectations for the workshop and some of the issues to be discussed. It quickly became clear that a major concern of the meteorologists was the mathematical preparedness of first-year meteorology majors. Because the study of weather attracts a diverse group of students, they come to the major with wide disparity in their quantitative abilities. Hence, many meteorology majors struggle in the first two calculus courses and, as a result, drop out of the meteorology program. Thus, it was important to address this issue at the workshop.

During the workshop, the participants talked about the importance of mathematics in the meteorology curriculum, the quantitative nature of the discipline, and the common difficulties that many students face in mathematics courses and calculus-based meteorology courses. The discussion was valuable and far ranging, revealing similar pedagogical issues in every meteorology program represented. This report is organized around three primary categories that guided the conversation: understanding and content, student populations, and technology and instruction.

Understanding and Content

Mathematics Requirements

A quick survey of the mathematics courses required by the eleven meteorology programs revealed near-universal agreement that meteorology majors must complete Calculus I, II, III, and Differential Equations. It should be noted that a number of these students also take upper-level mathematics courses to complete a minor or major in mathematics. In addition, computer programming (most often in FORTRAN) and statistics courses are required at some schools, while they are elective courses at other institutions. Programs vary in the number of meteorology courses taught in the first two years, but at least one largely-qualitative survey course is typical. While some first- and second-year courses have a calculus prerequisite, the mathematics-intensive courses such as the atmospheric dynamic meteorology sequence and atmospheric thermodynamics are typically taught in the third year.

Relative Importance of Conceptual and Computational Skills

One of the mathematics participants pointed out that mathematics can be viewed as an abstract liberal art as well as a practical tool for solving problems. Both paradigms drive mathematical pedagogy in calculus courses, with a mix of theory and practical problem solving. The meteorologists responded that to be successful in the upper-level quantitative meteorology courses their students should have a good understanding of the basic concepts and applications introduced in calculus. For example, in addition to computing the derivative of a function, students must understand that the derivative is a function that represents the rate of change of the dependent variable with respect to the independent variable.

In addition, students need to see a variety of problems in which this concept is applied—including, for example, when a function is represented as a set of discrete data points. While textbook examples directly related to the study of weather are scarce, there are many problems from related fields (such as physics) that could be assigned. The meteorologists can then teach applications of mathematics to specific meteorology problems in upper-level courses. Practical problem-solving skills are critical nonetheless; for example, simplifying an expression using the rules of algebra and trigonometry or assessing the appropriateness of a numerical solution.

Importance of Calculus Topics

Within the three-course calculus sequence, the anecdotal evidence is that meteorology students have the greatest difficulty in the second course. The poorest students may not make it to Calculus II, but a significant number of students get adequate grades in Calculus I and subsequently struggle to pass Calculus II. One of the mathematics participants commented that the second semester course often is a mix of somewhat unrelated topics, while another mathematics participant suggested that some of the topics, such as sequences and series, might be seen as more abstract. Thus, these challenges make it more difficult for students to assimilate and master the material. In addition, several meteorologists pointed out that it is not unusual for the meteorology students to be taking a calculus-based physics course concurrently, resulting in a difficult course load.

From a list of calculus topics typically covered in a three-semester sequence, the meteorologists were asked to rank their importance and to suggest the order in which they should be covered. Most felt that meteorology students should be exposed to almost all topics listed. As for the order in which the topics are presented, the primary concern was that standard vector content needs to be covered early (if not multiple times), since an understanding of vectors is very important for the study of meteorology. Other than this, the mathematics topics in the first two years should be presented in the best pedagogical sequence for teaching mathematics, since the quantitative meteorology courses typically are not taken until the third year.

Student Populations

First- and Second-Year Students

While incoming students should know that meteorology is a mathematics-intensive major, many are not prepared for college-level calculus. The mathematics background of first-year students varies substantially; internal surveys at some institutions indicate that between 25% and 50% of the incoming students take Calculus I, with a much smaller number of beginning students starting in Calculus II. The remaining students begin the mathematics sequence in a pre-calculus course. In two newer meteorology programs represented, the transition from a primarily qualitative meteorology minor to a B.S. degree has caused growing pains for students not accustomed to calculus-based coursework.

In addition, students often have difficulty conceptualizing topics such as derivatives and integrals, even after seeing them in mathematics, physics, and meteorology courses. One theme from the discussions was that many students try to memorize and compartmentalize their learning, with various ideas from mathematics and meteorology stored in isolated bits of information. Students learn derivatives in calculus, but experience cognitive dissonance when asked to estimate a derivative from temperature data on a grid. This is especially true of students with poor mathematics grades, but not uncommon even among students with reasonable calculus grades. One suggestion is that students might master the concepts better if they started the meteorology sequence concurrently with the calculus sequence. Many students would benefit from seeing the concepts and applications at the same time, but it could be logistically challenging given the varying mathematics placement of incoming students.

Mathematics departments at the eleven represented institutions each offer help sessions and tutoring programs for students. Several of the meteorology programs also have tutoring or mentoring systems, in which first-year students are partnered with advanced meteorology majors. Unfortunately, some struggling students won't go to help sessions, even when prompted by faculty. It also was noted that students can benefit from working in peer groups, but many choose to study alone.

Students considering Graduate School

The primary student groups within undergraduate meteorology programs are students planning on 1) a career in operational meteorology, 2) a career in broadcasting, or 3) graduate study. Students considering graduate school are encouraged to take additional mathematics courses in areas such as linear algebra, numerical methods, Fourier analysis, statistics, and partial differential equations (PDE). One participant noted that it isn't crucial for students to have completed all these courses before entering graduate school, but they should complete them during their first year of graduate study. For example, a PDE course is not required for entry into a graduate meteorology program at most institutions; however, it can be a requirement for the core graduate courses, implying it must be completed before enrolling in these graduate courses.

Graduate school-bound students also need programming skills, most commonly in FORTRAN. Some meteorology programs teach such a course, while others have their students take programming courses in other departments. There was general agreement that knowledge and comfort with one programming language should transfer to other languages, but it may be difficult for some students. Many other computer tools also are commonly used in graduate school research or classes, including Unix/Linux, graphical packages, and other programming languages (especially IDL).

Students from Under-represented Groups

A recent success story within meteorology programs has been the increase in enrollment of women in undergraduate programs, in some programs approaching 50%. Unfortunately, the same cannot be said for traditionally disadvantaged minority populations. The participants discussed several possible factors to explain the continued low enrollment of disadvantaged groups, including less robust K–12 preparation for college within urban centers and geographic biases produced by the fact that meteorology programs are located primarily in rural regions.

Technology and Instruction

There was a short discussion about the technology background of incoming students and the use of technology in the classroom. Incoming college students have widely varying experiences using computers; while many of them have used multiple software packages, some need help with the most basic applications in Excel. There was a consensus that meteorology students need to be comfortable with a variety of software tools and be able to learn how to use additional tools on their own, since different tools are useful for different problems. It was suggested that both mathematics and meteorology faculty continue to find creative ways to incorporate technology into their courses on a daily basis.

Conclusion

The meteorology participants emphasized most that students need to learn to make sense of quantitative data in a way that provides the confidence to apply a variety of mathematical tools and concepts in a variety of settings. Since meteorology is such a mathematics-intensive discipline, the meteorology faculty welcomed the opportunity to participate in a national dialogue with mathematicians about the quantitative needs of their undergraduate majors. They recommended that such discussions continue, even if only at the local level. All agreed that, as part of any revision of the MAA's CUPM Curriculum Guide, there should be a reemphasis on the importance of mathematics faculty in every college or university initiating a conversation with faculty in the partner disciplines. Significant benefits can accrue for both the mathematicians and the faculty in other disciplines. Such discussions already are underway at Valparaiso University, with several departments working to restructure second semester calculus and to revise the first computer science course. The goal for these changes is to more broadly meet the needs of science, mathematics, engineering, and meteorology majors.

Workshop Participants

Meteorology

Michael Baldwin, Department of Earth and Atmospheric Sciences, Purdue University
Teresa Bals-Elshols, Department of Geography and Meteorology, Valparaiso University
Martin Baxter, Department of Geology, Central Michigan University
William Capehart, Department of Atmospheric Sciences, South Dakota School of Mines
Craig Clark, Deparment of Geography and Meteorology, Valparaiso University
Paul Croft, Department of Geology and Meteorology, Kean University
Brian Etherton, Department of Geography and Earth Sciences, University of North Carolina, Charlotte
William Gallus, Department of Geological and Atmospheric Sciences, Iowa State University
Anthony Hansen, Department of Earth and Atmospheric Sciences, Saint Cloud State University
Redina Herman, Department of Geography, Western Illinois University
Eric Hoffman, Department of Chemical, Earth, Atmospheric, and Physical Sciences, Plymouth State University
Anthony Lupo, Department of Soil, Environmental, and Atmospheric Sciences, University of Missouri
Richard Mower, Department of Geology, Central Michigan University
Clint Rowe, Department of Geosciences, University of Nebraska-Lincoln
Alan Shapiro, School of Meteorology, University of Oklahoma
Jie Song, Department of Geography and Meteorology, Northern Illinois University
Bart Wolf, Department of Geography and Meteorology, Valparaiso University

Mathematics

Richard Gillman, Department of Mathematics and Computer Science, Valparaiso University
Kenneth Luther, Department of Mathematics and Computer Science, Valparaiso University
William Marion, Department of Mathematics and Computer Science, Valparaiso University
Catherine Murphy, Department of Mathematics, Computer Science, and Statistics, Purdue University, Calumet

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Social Science

CRAFTY Curriculum Foundations II Project
Virginia Commonwealth University, October 18–19, 2007

Report Editors: Jennifer A. Johnson and Patricia H. Grant
Workshop Organizer: William Haver

Summary

Social science represents a broad selection of academic disciplines that focus on a common research orientation that examines human behavior and recognizes that questions of human behavior often are ambiguous and imprecise. Faculty from the disciplines of sociology, criminology/criminal justice, political science and international relations, and psychology represented social sciences at this workshop. The overall consensus among the participants was that mathematics faculty can better prepare social science students by emphasizing conceptual understanding, proportional reasoning, standard equations (e.g., equation of a line), and basic arithmetical skills. In addition, mathematics courses could ground problems in a social science context, as well as develop and refine estimation and approximation skills. Social scientists also agree on the importance of introducing students to some basic statistical methods including measures of central tendency, variables, co-variation and standard deviation. Developing an understanding of graphical representation and interpretation, including use and manipulation of spreadsheets, also should be emphasized to enhance quantitative literacy in social science students. Technology can be a "hook" for social science students, drawing them into mathematics through creative pedagogical uses of visualization and analytical software.

Social science students often believe they are weak in mathematics or incapable of mathematical reasoning. Mathematics courses should strive to help students see mathematics as a necessary language of life that enables them to become good public citizens.

Social scientists rely on mathematics courses to assist in developing a sense of mathematical literacy among social science students by illustrating how mathematics is one of the languages that social scientists use to describe the world. Implementation ideas range from major developments such as building and requiring a basic statistics course for all students or college algebra for social science majors to minor changes such as developing word problems using actual social science data or including more statistics in the basic college algebra course.

The workshop participants suggest the following primary strategies as means through which this assistance can be provided.

- Stress the imprecision of social science data by highlighting conceptual understanding of mathematics and emphasizing estimation and approximation.
- Include more statistical skills in introductory college mathematics courses, either through statistics modules or optimally through the creation of a required introductory college statistics course.
- Situate mathematics in a "real world" social science context by using social science data to compute and conceptualize mathematics problems.

These strategies will be most effective and easier to implement through collaborations among mathematics, statistics, and social science professionals, with this report being one tool for facilitating such partnerships.

Narrative

Introduction and Background

This report will discuss the basic mathematics skills needed by social science students in their major course work. The diversity of disciplines that constitute the social sciences, their widespread locations across a given campus curriculum, and the resulting high level of diversity among social science students makes this a challenging task. For example, the level of requirements for quantitative and statistical skills in any given social science discipline will vary, depending on the location of the department within the college. In addition, colleges and universities located near research centers may see a high demand for quantitative skills from a particular major while the same major at a different institution may see a higher demand for applied skills.

However, there is a universal set of social science skills grounded in mathematics that cut across these local differences. For example, all social science students need to be able to read and critique published research, both from academic journals and public literature such as political polls and public survey data. Most students who complete a bachelor's degree in a social science will not go on to become independent researchers. Instead, many will become research analysts or public servants who will read, interpret, and disseminate published research. Social science students need a strong foundation in mathematical literacy—particularly in the area of statistics—thereby enabling them to understand the quantitative data process.

The instructional interconnections should reflect a shared knowledge between mathematics and the social sciences. Introductory college mathematics courses can help facilitate quantitative literacy through an emphasis on conceptual understanding of mathematics problems. While computational skills remain central to understanding the "hows" of quantitative reasoning, the "whys" of mathematics cut across all levels of social science requirements. All students—from all colleges and universities—need to come away with a sound ability to understand the world through both numbers and theory. If the current discussion centers on the baseline skills that all social science students need to be successful in their chosen major, the ability to conceptually understand mathematics is a unifying skill. While it is recognized that this cannot take place without computational skills, conceptual understanding should be heavily emphasized as well. Because of the imprecise nature of social science theory, a rigorous focus on exact/precise answers may limit the way in which students think and critically assess social science problems.

Understanding and Content

In response to these concerns, the participants sought to establish several levels of skill sets ranging from those that are most necessary to those that would contribute to a more advanced understanding of social science. Necessary skills are those that cut across the local differences and constitute those basic mathematics skills that all social science students should possess when they enter their major courses; desirable skills would significantly enhance most social science students' ability to be successful in their major course work; and, optimal skills are advanced mathematical, statistical, and theoretical skills that benefit advanced social science students (see Appendix A).

A central concern for the workshop participants is the development of a common understanding of variations in language, particularly as it relates to distinguishing between mathematics and statistics. For example, some mathematicians do not consider statistics to be part of the mathematics curriculum. As one mathematician stated "Statistics is the science of data; mathematics is the unique science with no data." For many social scientists, statistics *is* the mathematics of their discipline. Social scientists use statistics to describe human behavior and social observations. A central concern shared by social scientists is the need for a commonly-understood language that applies mathematics to a "real world" context. Increased collaboration between colleagues in mathematics and various social sciences is needed to develop a common understanding of the language needed to present mathematics to social science students (see Appendix B for examples and resources).

It is important that mathematics and social science language flow evenly across disciplines. There needs to be enhanced collaboration efforts between mathematics and social science departments to facilitate this flow. Ideas to continue the current dialogue include 1) a graduate student exchange in which mathematics graduate students provide statistical and mathematical support to social science students and faculty, while social science graduate students work with mathematics students and faculty; and 2) an open mathematics lab for social science students supported by mathematics graduate students.

Technology

Technology presents an avenue for “hooking” students into mathematics. Students are well-versed in using technology to communicate and learn. This presents an opportunity for unique ways of learning that may alleviate mathematics anxiety. Furthermore, social science students will use statistical programs such as SPSS, SAS or STATA to conduct data analyses. While mathematics courses should not be expected to teach students these statistical programs, they can begin by introducing students to software that facilitates mathematical calculations. For example, social science students need to be proficient with spreadsheets (e.g., Microsoft Excel) to manage data and run basic calculations.

Instructional Techniques

The central pedagogical concern for the social scientists centers on the need to enhance the mathematical confidence of social science students by dispelling the myth that mathematics is difficult or irrelevant to their lives. Unbeknownst to many who declare a social science major, numbers play a pivotal role in understanding the social world. It is one of the languages that social scientists use to describe human behavior and interpret society. Mathematics courses can help prepare social science majors by emphasizing the role of mathematics in the social sciences through the pedagogical use of social science data and problems, as well as increasing the amount of statistical work that is included in the introductory mathematics courses that are offered. While it would be optimal for students to come to the major having taken a two-course mathematics sequence which includes a college algebra and college statistics course, it is recognized that this requirement will not work for many colleges and universities. Therefore, it is suggested that a statistics module (or two) be built into every student’s introductory mathematics experience. How this is incorporated is contingent upon the structure of the particular college’s curriculum. However, it is imperative that students come away with an appreciation for statistics, and that this preparation takes priority over more typical introductory mathematics courses.

Other pedagogical suggestions include keeping mathematics class sizes small in order to enable professors to provide immediate feedback, emphasize active learning using structured group projects, and test students into a mathematics *level* as opposed to testing them out of a mathematics *course*. Additional instructional techniques that are desirable include incorporating multi-sensory learning styles, using repetition for positive reinforcement, utilizing CD-based learning, and starting with broad issues to get students invested in the learning process.

Resources

List of common data sources used by social scientists:

- Census Data (www.census.gov)
- Interuniversity Consortium for Political and Social Research (www.icpsr.org)
- National Opinion Research Center (www.norc.org)
- National Survey on Drug Use and Health (https://nsduhweb.rti.org/)
- General Social Surveys (http://gss.norc.org/)
- Department of Justice (http://www.usdoj.gov/)
- Uniform Crime Reports (http://www.fbi.gov/ucr/ucr.htm)
- Centers for Disease Control (http://www.cdc.gov/)
- World Health Organization (http://www.who.int/en/)

Social Science Methodology Reference Books and Commonly Used Software:

- Babbie, Earl. (2007). The Practice of Social Research, 11ed. Wadsworth Publishing.
- Holcomb, Z.C. (1997). Real data: A statistical workbook based on empirical data. Los Angeles, CA: Pyrczak Publishing.

- Kelly, M.R. (1992). Everyone's problem solving handbook: Step-by-step solutions for quality improvement. Portland, OR: Productivity Press.
- SPSS (www.spss.com)
- SAS (http://www.sas.com/technologies/analytics/statistics/stat/index.html)

Workshop Participants

Social Science

Richard Arum, Department of Sociology, New York University
Julio Carrion, Political Science and International Relations, University of Delaware
Angela Cole, Department of Psychology, Howard University
Zachary Goodell, Department of Sociology, Virginia Commonwealth University
Patricia Grant, Wilder School, Virginia Commonwealth University
Scott Graves, Department of Political Science, Georgia State University
Zaryab Iqbal, Department of Political Science, University of South Carolina
Jennifer Johnson, Wilder School, Virginia Commonwealth University
Joseph Marolla, Department of Sociology, Virginia Commonwealth University
Betsy McEneaney, Department of Sociology, Virginia Commonwealth University
Laura Moriarty, Criminal Justice, Virginia Commonwealth University
Nicolle Parsons-Pollard, Sociology, Social Work & Criminal Justice, Virginia State University
Lesley Reid, Department of Sociology, Georgia State University
Randolph Smith, Department of Psychology, Lamar University
Mittie Southerland, Academy of Criminal Justice Sciences, Murray State University
Hassan Tajalli, Department of Political Science, Texas State University – San Marcos
Michael Vasu, Leadership in the Public Sector Program, North Carolina State University
Jeffery Walker, Department of Criminal Justice, University of Arkansas, Little Rock

Mathematics

William Bauldry, Department of Math Sciences, Apalachian State University
Andrew Bennett, Department of Mathematics, Kansas State University
Jenna Carpenter, Department of Mathematics, LA Tech University
Aimee Ellington, Department of Mathematics, Virginia Commonwealth University
William Haver, Department of Mathematics, Virginia Commonwealth University
Deborah H. Hallett, Department of Mathematics, The University of Arizona
Henry Johnson, Department of Mathematics, Virginia Commonwealth University

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Appendix A

Necessary Skills

- Basic arithmetic skills
- Linear algebra
- Basic descriptive statistics
- Basic functions
- Proportional reasoning
- Percentages/Percent increase
- Graph analysis and representations
- Rate of change
- Variable types
- Variability
- Slope of Line
- Relative Frequencies
- Correlation/Causation

Desirable Skills

- Advance Statistical skills such as linear regression
- Data collection issues such as random sampling
- Probability Theory
- Normal Distribution
- Significance Level
- Standard Deviation
- Spreadsheet management
- Levels of Measurement

Optimal Skills

- Matrix Algebra
- Game theory
- Calculus
- Power law graphs
- Non-linear functions
- Monte Carlo methods
- Central Limit Theory

Appendix B

Social scientists focus on variables which represent social phenomenon that can be observed and measured. Variables can be classified according to their level of measurement: nominal, ordinal, interval, and ratio. Here is a brief description of these levels. A sample variable and its values are listed for each level.

- **Nominal** (the values are simply names of groups, categories, etc.):

Eye color	1 = blue
	2 = green
	3 = brown

- **Ordinal** (there is an order to the values):

Education	1 = grade school
	2 = high school
	3 = college
	4 = masters degree
	5 = Ph.D.

- **Interval** (a one-unit change in value represents the same amount throughout)

Years	1995
	1996
	1997
	1998
	(etc.)

- **Ratio** (there is a true zero – therefore, the ratio of one value to another is meaningful):

Age	10
	39
	65
	(etc.)

For most analytical purposes, both the nominal and ordinal levels are referred to as a categorical measurement level and both the interval and ratio levels are referred to as a scale measurement level.

Some examples of social science variables that are commonly used:

- Occupation (type of industry: agriculture, construction, etc.)
- Education (years of schooling completed)
- Religion (Protestant, Catholic, Jewish, etc.)
- Schooling by type of diploma (elementary, high school, college)
- Marital status (married or not married)
- Sex (Female or Male)
- Race (African American, Euro-American, Hispanic American, etc)
- Income (0-$9,999; $10,000-19,999; $20,000-39,999, etc)
- Political party preference (Democrat, Republican, etc.)
- Number and type of crimes committed

Responding to the Recommendations of the Curriculum Foundations Project

Susan L. Ganter
Clemson University

Bill Haver
Virginia Commonwealth University

The recommendations from the 17 workshops during the first phase of MAA's Curriculum Foundations Project, focusing on the needs of students majoring in "mathematics intensive" disciplines, present an opportunity and a challenge to mathematics departments. A close examination of the first series of workshops found that the stereotype that users of mathematics care primarily about computational and manipulative skills, forcing mathematicians to cram courses full of algorithms and calculations, "is largely false" (Ganter and Barker, 2004). Indeed, disciplinary representatives "were unanimous in their emphasis on the overriding need to develop in students a conceptual understanding of the basic mathematical tools." The disciplinary reports provide guidance on the fundamental skills required for each discipline and emphasize that the abilities most valued are problem solving skills, mathematical modeling, communication skills, and command of appropriate technology.

The workshops recently conducted during phase two of the project, with disciplinary representatives from social sciences and humanities, re-confirmed these universal needs—in spite of the vast differences between the disciplines represented. Specifically, the five disciplinary reports in this volume indicate a need for mathematics courses that emphasize:

- **Conceptual understanding and problem solving**—communicating solutions to diverse audiences; precise and correct use of mathematics in presentations and reports
- **Arithmetic and basic mathematical equations—**relationships between variables; percentages, proportion, and measurement; translation of words into appropriate formulas and equations; graphical representations; unit conversions
- **Problems in context**—building analytical models and testing their viability; applying theory to real problems and evaluating alternative solutions; communicating and coordinating with disciplinary faculty to develop alternative problems; using context to inspire and create a need for mathematics (i.e., mathematics as a common technical language)
- **Estimation and approximation**—use of experimentation and exploration to discover mathematical concepts
- **Statistics and quantitative data**—measures of central tendency and standard deviation; analyzing data to make inferences and draw conclusions; presenting data as pictures (such as bar graphs, line graphs, and scatter plots)
- **Appropriate use of technology**—spreadsheets; geometrical/graphical software

Although a line-by-line comparison between the stated needs of social sciences and humanities students versus those from the more traditionally mathematically intensive disciplines yields some subtle differences in the specifics, the broad categories outlined above are virtually the same as those from the initial round of Curriculum Foundations disciplinary reports. And even more striking is that this list of priorities was consistently and independently developed across multiple disciplines. Whether the workshop focused on physics, engineering, economics, or the arts, the message from these partner disciplines was repeated again and again: introductory collegiate mathematics courses should focus on giving students an appreciation and understanding of fundamental mathematical topics while grounding the discussions in context. The specific topics are not as important as 1) technical confidence; 2) the application of mathematics to a variety of contexts; and, 3) the ability to choose appropriate tools for modeling, evaluating, and communicating mathematical results. As stated in the first Curriculum Foundations reports (Ganter and Barker, 2004), such needs can be met for many disciplines through a "new and improved" version of the traditional College Algebra course. Specifically, most of the needs described by each of the partner disciplines fall easily within the mathematical purview of College Algebra, making the revision of this course a natural first step in addressing the recommendations from the Curriculum Foundations Project.

Of course, the recommendations initiated by the conversations with the partner disciplines still present a big challenge, coming from the large and diverse set of disciplines that are making ever greater use of mathematics. And it is certainly understood that no mathematics department can possibly offer a different mathematics course for majors in each of the different disciplines represented on its campus—making the need to rethink and revise the most popular introductory mathematics courses (such as College Algebra) even more critical. And since the broad categories of conceptual understanding, problem solving, mathematical modeling, and communication cut across the recommendations from all the partner disciplines, it makes sense to re-develop College Algebra in a way that incorporates these universal needs.

As such, the reports from the first round of Curriculum Foundation workshops and the CUPM *Curriculum Guide 2004* recommendations concerning general education and introductory college courses led CRAFTY to the decision to focus first on College Algebra. In addition, College Algebra was selected because: 1) 650,000 to 750,000 students annually enroll in College Algebra (Lutzer, et al 2002); 2) more than 45% of students enrolled in College Algebra nationwide either withdraw or receive a grade of D or F (Haver, et al 2007); and, 3) the content and approach of the traditional College Algebra course are diametrically opposed to the recommendations of the Curriculum Foundations workshops, CUPM's *Curriculum Guide 2004*, and the curriculum committees of national mathematics organizations (CUPM, 2004; Ganter and Barker, 2004; Wood et al, 2006).

To achieve the necessary change, CRAFTY has defined an ambitious first step: to have every college mathematics department that offers College Algebra review the course's content and approach, determine if the course best meets the needs of its students, and assess whether the success rate in the course is satisfactory. CRAFTY is convinced that if departments undertake such a review, utilizing resources made available by CRAFTY, many departments will realize that the course needs fundamental revisions and that the renewal process will result in a stronger and more effective course. In addition to the current volume, some of the resources developed by CRAFTY to assist departments in this process include:

- **College Algebra Curriculum Guide**. These guidelines represent the recommendations of CRAFTY concerning the nature of all College Algebra courses. The guidelines were endorsed by CUPM in 2006, after an 18-month development process by CRAFTY. The Guidelines are included in this volume.
- **Professional Development Workshops**. CRAFTY has co-sponsored a number of workshops, many through the MAA PREP program, that support teams of faculty members who wished to renew their College Algebra course. One such workshop series focused on Historically Black Colleges and Universities (HBCUs); the resulting report as well as reports from other institutions are included in this volume, including information on student achievement.
- **Contributed Paper Sessions at the Joint Mathematics Meetings**. CRAFTY has sponsored relevant special sessions at the Joint Mathematics Meetings since 2006, and has had large responses both in terms of numbers of contributed papers and attendees at sessions. This is one indication of the importance that individuals and departments place on College Algebra and other introductory courses.

- **National Science Foundation Pilot Project**. With support from NSF, CRAFTY invited colleges and universities to participate in a program in which the institutions offer a pilot College Algebra course that utilizes a modeling approach, and then share their experiences with the broader community. More than 210 departments expressed interest in participating, but budgetary and other resource limitations restricted the project participant sites to only eleven. A project report, including papers from six of the participating institutions, is included in this volume.

- **Individual Departmental Initiatives**. Members of CRAFTY have initiated projects to improve College Algebra at their own campuses. The results of several such efforts are included in this volume (see papers describing work at Kansas State University, Oregon State University, and Virginia Commonwealth University).

The members and "friends" of CRAFTY have spent many hours discussing the pros and cons of proceeding down various possible paths in an effort to most effectively promote the sweeping changes recommended by the Curriculum Foundations Project and others. There are many courses—including College Algebra—that deserve the attention of the mathematics community. But in the end, it became clear that making a choice—any choice—about where to start was the most important first step. Perhaps others will argue that College Algebra was not the most appropriate choice that College Algebra should not even fall within the purview of MAA's committees that focus on undergraduate education. However, the facts about enrollments and the needs of the greater academic community tell mathematicians that this is not a healthy or prudent attitude. The reality is that courses at the level of College Algebra are an important and vital part of the undergraduate mathematics curriculum. To ignore this reality places the entire college mathematics enterprise in great danger—danger of forcing other departments to teach their own mathematics courses, of losing college and community resources, and, perhaps most important, of losing the opportunity to reach and teach thousands of students about the importance of mathematics in their lives. For better or worse, College Algebra is the venue within which mathematicians must do this "reaching and teaching." College Algebra must be a course that can step up to the plate and meet the diverse needs of many disciplines, putting mathematics firmly back in its rightful place as a core part of every student's college education. The time is now.

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COLLEGE ALGEBRA

This section begins with the MAA *College Algebra Guidelines* that were developed by CRAFTY and endorsed by the CUPM. It then includes a report on the National Science Foundation supported college algebra project in which eleven institutions offered pilot College Algebra courses that utilized a modeling approach along the lines of the *College Algebra Guidelines* that were then under development and then shared their experiences with the broader community. The section also includes papers describing the results of efforts led by four different members of CRAFTY to improve College Algebra, three at their home institutions and one with a consortium of HBCUs. The section ends with a set of recommendations for departments that are considering revitalizing college algebra.

College Algebra Guidelines

These guidelines represent the recommendations of the MAA/CUPM subcommittee, Curriculum Renewal Across the First Two Years, concerning the nature of the college algebra course that can serve as a terminal course as well as a pre-requisite to courses such as pre-calculus, statistics, business calculus, finite mathematics, and mathematics for elementary education majors. They were endorsed on January 31, 2007 by CUPM, the Committee on the Undergraduate Program in Mathematics.

Fundamental Experience

College Algebra provides students a college level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, provides a foundation in quantitative literacy, supplies the algebra and other mathematics needed in partner disciplines, and helps meet quantitative needs in, and outside of, academia. Students address problems presented as real world situations by creating and interpreting mathematical models. Solutions to the problems are formulated, validated, and analyzed using mental, paper and pencil, algebraic, and technology-based techniques as appropriate.

Course Goals

- Involve students in a meaningful and positive, intellectually engaging, mathematical experience;
- Provide students with opportunities to analyze, synthesize, and work collaboratively on explorations and reports;
- Develop students' logical reasoning skills needed by informed and productive citizens;
- Strengthen students' algebraic and quantitative abilities useful in the study of other disciplines;
- Develop students' mastery of those algebraic techniques necessary for problem-solving and mathematical modeling;
- Improve students' ability to communicate mathematical ideas clearly in oral and written form;
- Develop students' competence and confidence in their problem-solving ability;
- Develop students' ability to use technology for understanding and doing mathematics;
- Enable and encourage students to take additional coursework in the mathematical sciences.

Competencies

1. Problem Solving

Goals for students include

- solving problems presented in the context of real world situations with emphasis on model creation and interpretation;
- developing a personal framework of problem solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and

parameters; use analytical, numerical and graphical solution methods as appropriate; determine plausibility of and interpret solutions);
- creating, interpreting, and revising models and solutions of problems.

2. Functions and Equations

Goals for the students include
- understanding the concepts of function and rate of change;
- effectively using multiple perspectives (symbolic, numeric, graphic, and verbal) to explore elementary functions;
- investigating linear, exponential, power, polynomial, logarithmic, and periodic functions, as appropriate;
- recognizing and using standard transformations such as translations and dilations with graphs of elementary functions;
- using systems of equations to model real world situations;
- solving systems of equations using a variety of methods;
- mastering algebraic techniques and manipulations necessary for problem-solving and modeling in this course.

3. Data Analysis

Goals for the students include
- collecting (in scientific discovery or activities, or from the Internet, textbooks, or periodicals), displaying, summarizing, and interpreting data in various forms;
- applying algebraic transformations to linearize data for analysis;
- fitting an appropriate curve to a scatter plot and use the resulting function for prediction and analysis;
- determining the appropriateness of a model via scientific reasoning.

Emphasis in Pedagogy

Goals for the instructor include
- facilitating the development of students' competence and confidence in their problem-solving abilities;
- utilizing and developing algebraic techniques as needed in the context of solving problems;
- emphasizing the development of conceptual understanding of the mathematics by the students
- facilitating the improvement of students' written and oral mathematical communication skills;
- providing a classroom atmosphere that is conducive to exploratory learning, risk-taking, and perseverance;
- providing student-centered, activity-based instruction, including small group activities and projects;
- using technology (computer, calculator, spreadsheet, computer algebra system) appropriately as a tool in problem-solving and exploration;
- conducting ongoing assessment activities designed to determine when mid-course adjustments are warranted.

Assessment

- Assessment tools will measure students' attainment of course competencies, including:
  - solving problems and interpreting results using algebraic tools;
  - building and interpreting models and predicting results;
  - communicating processes and solutions orally and in writing;
  - making quantitative and algebraic arguments;
  - reading and interpreting data presented in various forms.

- Assessment tools will include
  - individual quizzes;
  - individual examinations;
  - additional activities or assignments, such as
    - individual or group homework, projects, and activities;
    - individual or group oral presentations;
    - portfolios that demonstrate student growth;
    - group quizzes and exams.
- The course will be assessed by analyzing its effectiveness in:
  - facilitating student achievement of the course competencies;
  - positively affecting student attitudes about mathematics;
  - preparing students for subsequent courses in mathematics and mathematics-dependent disciplines;
  - preparing students for subsequent endeavors in and outside academia.

A Tale of a Change Initiative

Revitalizing College Algebra Program of the Mathematical Association of America

Barbara Edwards
Oregon State University

Each year in colleges and universities across the United States approximately 1,000,000 students enroll in college algebra; and each year approximately 50% of these students fail to pass this course with a grade of C or better (Gordon, 2008). Originally designed to meet the needs of college students with an inadequate background for studying calculus, the course syllabus for college algebra varies, but it usually contains a large number of topics including properties and operations on polynomial, rational, exponential and logarithmic functions, Today, fewer than 10% of the students who pass college algebra enroll in a rigorous calculus course (Gordon, in press). The high failure rate in college algebra and the apparently changing needs of college students indicate the need for a change in the college algebra course. CRAFTY, the Mathematical Association of America's committee for Curriculum Renewal Across the First Two Years recently released the MAA *College Algebra Guidelines* (CRAFTY, 2007). Among other recommendations, the guidelines state that college algebra should

- emphasize the use of algebra and functions in problem solving and modeling,
- provide a foundation in quantitative literacy,
- supply the algebra and other mathematics needed in partner disciplines, and
- help meet quantitative needs in, and outside of, academia.

According to CRAFTY, students in college algebra should "address problems presented as real world situations by creating and interpreting mathematical models. Solutions to the problems [should be] formulated, validated, and analyzed using mental, paper and pencil, algebraic, and technology-based techniques as appropriate" (CRAFTY, 2007).

In 2005 with support from the ILI program of the National Science Foundation CRAFTY selected faculty teams from eleven institutions in the United States to participate in a program to change their college algebra courses so that they would be more closely aligned to the College Algebra Guidelines. The eleven institutions, chosen from over 200 applications, included private and public institutions of various sizes; two-year, four-year and research institutions; and represented eleven states in the Southeast, Northeast, Midwest and Western United States. Faculty teams from these institutions attended a three-day workshop during summer 2005 designed to introduce them to the notion of a modeling-based college algebra course. Each team chose one of four modeling-based college algebra texts and agreed to participate in a project in which they would offer both modeling and control sections of college algebra during two semesters in 2006. Each team agreed to collect data in the form of student grades and copies of student work on a common portion of the college algebra final examination. This researcher visited each site at least once during implementation to observe both modeling and control sections and to do interviews and focus groups with faculty and selected student volunteers.

This paper serves as an introduction for the following six reports from participating institutions in the MAA Project. These reports represent an "inside" view of the project and reflect the process as it occurred on each campus. This paper augments those views from an outside observer's perspective. It focuses on the researcher's view of the process and challenges involved in changing the content and pedagogy in college algebra and the effects of this change process on both instructors and students. In this paper we provide the results from some of the project grade data, we discuss the factors that may have influenced the results and we suggest some implications of these results on curricular change efforts such as the MAA's.

The Data: Students' Grades in College Algebra

The following table shows the comparison between control and modeling sections of College Algebra in terms of the Passing Rate (Grade A, B, or C) at nine institutions[1] for spring semester 2006 (1) and fall semester 2006 (2). The data shows that the passing rates were better for the modeling sections at Colleges A, B, F, G, and H while the passing rates were better for the control sections at Colleges E and I. At College D the passing rates were essentially the same

Course Passing Rate Comparison

Institution	Section Type/ Semester	Number of Participants	ABC Passing Rate
College A	Control/1	132	46.2%
	Modeling/1	109	56.9%
	Control/2	103	57.3%
	Modeling/2	99	65.7%
College B	Control/1	53	73.6%
	Modeling/1	78	79.5%
	Control/2	130	79.2%
	Modeling/2	188	84%
College C	Control/1	76	74%
	Modeling/1	22	64%
	Control/2	74	62.2%
	Modeling/2	94	73%
College D	Control/1	70	57.1%
	Modeling/1	54	57.4%
College E	Control/2	296	56.3%
	Modeling/2	332	50.5%
College F	Control/2	162	38.9%
	Modeling/2	179	52.5%
College G	Control/1	95	67.4%
	Modeling/1	121	75.2%
	Control/2	155	83.2%
	Modeling/2	179	88.8%
College H	Control/1	107	47.7%
	Modeling/1	125	69.6%
College I	Control/1	375	56.3%
	Modeling/1	208	43.8%

[1] Two institutions dropped out of the study before grade data was collected. Only one semester of data was available to the researcher for five of the remaining institutions.

for both control and modeling sections, and at College C during the first semester the control sections had a higher passing rate and during the second semester the modeling sections had a higher passing rate. Over all, six of the nine institutions had at least one semester during which their modeling sections had higher or the same passing rates as the control sections. Thus the project can claim a certain level of success in increasing the pass rate in College Algebra. However, based upon data from classroom observations and discussions with the faculty involved in the project, drawing generalizable conclusions from this data is problematic.

In many ways the project was not one big project, but eleven smaller projects with some common threads: participants met together at the beginning of the project and with some exceptions at three reunion meetings; seven of the institutions used the same textbook and all were visited by one researcher/observer. We expected some variation in the approaches of each of the eleven teams, however, what a modeling section "looked like" from institution to institution (and sometimes even within an institution) seemed to vary more widely than expected.

Factors Influencing the Project Outcomes

There were both external and internal factors that seem to have influenced the project as it evolved.

External factors

Nationally, there is no general agreement on the content of college algebra. Some institutions teach college algebra as a terminal mathematics course while other institutions view college algebra as part of the pre-calculus track. In one of the participating institutions a long list of topics for inclusion in college algebra was mandated by the state. In other institutions a similar mandate came from the departments. There seemed to be a concern at many of the participating institutions that students in a modeling section of college algebra might not learn everything they needed to know to move on to the next course because modeling-based college algebra can include a reduced emphasis on skills practice. In this project, when instructors had this concern they often supplemented their text materials with skill-building assignments for the students or provided additional information during lectures. The end result of this content issue was apparent in some of the classroom observations when the pedagogy and content seemed to be very similar in both control and modeling sections.

The physical facilities in some of the participating institutions sometimes provided challenges for implementing cooperative-learning groups in class. Some classrooms had stadium-seating and/or desks that were bolted to the floor which made it difficult for more than two students to comfortably work together. Some classrooms were large enough if students were sitting in rows facing forward, but when desks were moved to form small circles for group-work the room became crowded and it was difficult for the instructor to move from group to group.

Although a condition of participation in the project was a letter of support from the administration of each faculty group's institution, interviews and informal discussions with faculty in some of the institutions revealed the existence of a financial obstacle making difficult the continuation of the modeling-based approach beyond the two semesters of the project when these sections required more resources in the form of teaching assistants and graders or reduced teaching loads for a faculty member to coordinate the course or to provide professional development for instructors. The researcher interviewed two administrators who said there was no extra money to support the college algebra effort and that, although they were supportive of the change, it would not be possible without external funding.

Some participating instructors also sensed a lack of support from their colleagues in the department either because these colleagues were opposed to offering a modeling-based college algebra course, or because they were opposed to changing how they taught the course should they be assigned to teach college algebra. In many cases, especially when they knew they would never teach college algebra, these colleagues had no opinion on the way the course would be taught. In the researcher's visits to the various sites, however, all but one of the department chairs were very aware of the low passing rates in college algebra and were interested to know if modeling-based college algebra would be a way to increase student success in the course. In all but two of the eleven institutions (one of which withdrew from the project) there seemed to be some level of support from the department chairs. In three instances the researcher met with college-level administrators who showed considerable interest in the success of modeling-based college algebra at their institutions.

Personal Factors

There is a body of literature pertinent to this study that describes the process for teachers who are involved in curricular changes and innovations. (cf. Cohen, 1990, Edwards, 2000, Shaw & Jakubowski, 1991). In particular, Shaw and Jakubowski (1991) identify six cognitive requisites for successful curricular and pedagogical change: the instructor must be 1) dissatisfied with some aspect of their courses or their teaching, 2) have a commitment to making changes, 3) construct a vision of what the change should be, 4) project themselves into that vision, 5) decide to make a change within a given context, and 6) be reflective about their efforts by comparing their new practice with their vision. All of the instructors who originally chose to participate in this project were dissatisfied with their college algebra courses, but of the two institutions who dropped out of the project, some of the instructors in one of the institutions did not seem to have a commitment to changing the course and in the second institution there was a strong commitment to change by the instructors in the group, but changing to modeling-based college algebra did not seem to match their vision.

Another personal factor is that teaching a modeling college algebra course requires more work for the instructor; there is more grading (especially if the course involves an out-of-class project whose product is a written report) and preparation for class often takes longer. Furthermore, it is sometimes not as easy to listen to and follow students' reasoning and mathematical ideas or to guide their thinking in appropriate directions. There is often the temptation to "help" students by providing solutions to template problems or giving too many hints. Many instructors are somewhat uncomfortable in this role since it is not how they were taught mathematics

Some of the students indicated that they were also somewhat uncomfortable in their new role in the modeling sections. They no longer could merely sit and take notes. Now they had to be active in class: talk with their neighbors and try to think and reason, rather than memorize rules. A number of students in the student interviews and focus groups said they thought the class was much harder than a "regular" math class. In the interviews the researcher asked the students if they had liked mathematics in the past and if they thought they were "good in math." Among the students who said they liked math, several did not like having new rules for the mathematics game. Interestingly, the number of unhappy students in the modeling classes was essentially equal to the number in control classes who said they were dissatisfied, although those in the control sections were dissatisfied for different reasons, most prominently the perceived "uselessness" of mathematics.

The impact of any sign of student discomfort in the modeling sections seemed to have an effect on the instructors who were somewhat unsure of their commitment to or efficacy of the modeling approach. One instructor said, "I'm afraid I'm not good at this kind of teaching. I'd be better off — and the students would be better off — if I went back to the traditional college algebra." Another instructor worried that her students would "fail miserably" if they tried to take a subsequent mathematics class. She said, "I am really not sure these kids are learning the things they should know."

There were, however, instructors in each of the nine institutions who seemed to be working through the requisite stages outlined by Shaw and Jakubowski. These project instructors talked about the obstacles they faced doing the project, but some seemed to decide that the good experiences they were having in the modeling sections outweighed the obstacles. Many were reflective about the project and their teaching during the second round of site visits and instructor interviews. One instructor summed up what several of them were saying when she said, "I don't know how this is going to end up [whether or not her department would continue to support a modeling approach to college algebra] but I know for myself that I will never teach the way I used to teach. I can see that some of the things I have tried to do in this project are really better for the students. Besides it's more fun for me."

There seemed to be other aspects of the project that provided a positive influence on the instructors' attitudes. For the most part these aspects seemed to be more prevalent in the institutions that reported more success in the modeling-based college algebra grades. Some of these positive influences were frequent staff meetings for the instructors involved in the project, moral support and interest from colleagues in the department and from administrators, and outside support and encouragement from the MAA staff involved in the project and from the authors of the textbooks used in the project.

Discussion and Implications

Originally the hope was to conduct a study that would involve a large number of college algebra students enrolled in both control sections and modeling-based sections which were limited to 35 students, included an assigned out-of-class modeling project, portions of the final examination that were common to both modeling and control sections, cooperative working groups during class and the availability of technology for students. However, the conditions for the experiment allowed for only a very loose research design; thus, as indicated before, the modeling courses between institutions were not necessarily comparable. Not all institutions could limit their sectional enrollment to 35 students; indeed one institution enrolled as many as 80 students per section. Not all participating institutions assigned an out-of-class project and the inclusion of technology varied greatly between institutions and even between instructors within institutions. Several of the institutions were unable to have a meaningful common portion in their final examinations, often due to resistance from the departments or control section instructors. As mentioned before, it was sometimes difficult to determine if a given section was modeling or control both in the pedagogy and the content. If these rare observations were a typical view of the classes, it seems that any difference in grade results in those instances would be based on factors other than the influence of the modeling approach.

Data gathering was a challenge due to the rules and policies of the participating institutions and the dependence upon the busy project instructors who were asked to do additional work for which they were often not compensated. For this reason there are gaps in the researcher's grade data as well as difficulties obtaining the follow-on data. The six reports from participating institutions fill in some of those gaps.

Despite all the difficulties, however, change has occurred. As of this writing, six of the institutions are still involved to some degree in teaching a modeling approach to college algebra. One of these institutions has begun teaching all their college algebra sections with a modeling approach including providing professional development support for the instructors and graduate teaching assistants who are involved. Another institution has created a new course with a modeling approach essentially replacing their previous college algebra course.[2] The faculty at a third institution decided that the modeling approach might be too challenging for their students, but they teach an honors section of college algebra with modeling.[3] A faculty member at another participating institution attended a faculty development workshop after the MAA project ended and has continued to develop ideas for teaching college algebra from the modeling perspective at his institution. Another faculty group is seeking funding to support a permanent change to modeling college algebra for all their sections. Finally, many of the faculty who were involved in the study have decided to include portions of the modeling approach in their traditional classes until a permanent shift can be made. Since change is a slow process, it is possible that this permanent change will yet occur.

It seems that studies of this kind involving multiple institutions are certainly possible, but to obtain definitive information they require more resources for research and participant support. Our experience indicates that projects like this should also involve a more concentrated initial time for faculty development. During the 2005 workshop it was apparent that many of the participants had not had previous experience with some of the pedagogical aspects of the project such as teaching with cooperative working groups, using technology to teach mathematics, or assigning out-of-class projects. For most participants the 2005 workshop was their first view of the idea of a modeling approach to teaching college algebra.

Continuing research is important both to document successful efforts to renovate the undergraduate curriculum and to tease out important factors in instructor change at the tertiary level. The experience of this project has been valuable for informing the design and implementation of future research in these areas.

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[2] See the report from Florida Southern, pp. x –x.

[3] See the report from South Dakota State University, pp. x –x.

Transforming College Algebra at Florida Southern College

Daniel Jelsovsky, Kenneth D. Henderson, Jr., and Susan Serrano
Florida Southern College

Abstract. College algebra has changed little over the years, however, the course is now evolving in a number of ways to better meet the needs of a changing society. Florida Southern College recognizes the importance of curricular transformation. As part of the Renewal of College Algebra Project, three professors at Florida Southern College conducted a research project testing the traditional approach versus a modeling approach to teaching college algebra. Students from modeling and traditional college algebra classes were tracked through their subsequent mathematics courses to see if there was a difference in their success in follow-up courses. The results were mixed. This paper provides a qualitative overview of Florida Southern College's experience, as well as a narrative from the three participating professors.

Florida Southern College

Located between Tampa and Orlando in Lakeland, Florida, Florida Southern College is a private college with a superb liberal arts and science core. Primarily an undergraduate college, Florida Southern is home to 1750 undergraduate students, as well as 135 graduate students. Florida Southern offers 50 different undergraduate majors, as well as three graduate programs. Incoming freshmen have an average SAT score of 1070 and an average high school grade point average of 3.4. Florida Southern has an average student to faculty ratio of 13:1 and an average class size of 20.[1] Carnegie classifies FSC among "Baccalaureate Colleges—Diverse Fields[2]", meaning that fewer than half of the bachelor's degrees awarded each year are in the arts and sciences.[3] In 2006 the Mathematics Department at Florida Southern had seven full-time mathematics faculty and offered twenty-eight undergraduate sections in mathematics each semester. In spring semester six of these classes were college algebra and in the fall there were eight sections of college algebra. At that time, college algebra had a firmly established place as the primary general education mathematics course. At some point in the past, college algebra had served as a springboard to precalculus and mainstream calculus. By 2006, this was no longer the case. Some students took college algebra and then took the applied calculus course, but the vast majority did not. In fact, there were no courses in FSC's college catalog that listed college algebra as a prerequisite.

The Renewal of College Algebra

In the spring and fall semesters of 2006, Florida Southern College took part in the MAA Renewal of College Algebra Project. Three professors in the Mathematics Department conducted a project to test a modeling approach versus a traditional approach to teaching college algebra. The modeling classes used a text by Crauder, *Functions and Change*,

[1] http://flsouthern.edu/about_fsc/quickfacts.htm

[2] Institutions: Florida Southern College http://www.carnegiefoundation.org/classifications/sub.asp?key=748&subkey=13836&start=782

[3] Basic Classification Technical Details, http://www.carnegiefoundation.org/classifications/index.asp?key=798

that emphasized the use of real data and technology to model real world problems. Equal numbers of control and pilot sections of college algebra were created for each of those semesters. The resulting grades for students in those classes for both semesters appear in Tables 1 and 2 below.

Table 1. Spring 2006 College Algebra Florida Southern College

	Enrollment	SAT	ACT	A	B	C	D	F	W	Average GPA
3 Control sections	85	1003	21	32	19	13	6	7	8	2.8
3 Pilot sections	31	1051	23	6	8	6	3	5	3	2.44
Total	116	1014	24	38	27	19	9	12	11	2.68

Table 2. Fall 2006 College Algebra Florida Southern College

	Enrollment	SAT	ACT	A	B	C	D	F	W	Average GPA
4 Control sections	92	1030	22	19	21	18	8	13	13	2.37
4 Pilot sections	81	1012	22	17	25	20	7	5	7	2.5
Total	173	1021	22	36	46	38	15	18	20	2.44

Note that although the numbers in the fall are evenly distributed between control and pilot sections, they were not in spring semester. This is attributable to multiple factors. First, in the fall, freshmen are placed into classes, and dropping and adding courses is difficult. By spring, students enroll themselves electronically, and dropping and adding in the first week is much easier. Second, the text for the pilot sections was a new edition. The control sections used the same book that had been at Florida Southern for years. This meant that inexpensive texts were available for the control sections in spring but not for the pilot sections. The higher cost of texts may have prompted students to want to change courses. Finally, a popular long-time professor who was to retire at the end of spring semester taught one of the control sections. Many of the students wanted to be in this professor's class, and most were accommodated. These factors may have created the imbalance between control and pilot sections.

Although the authors all embrace mathematics and statistics, we believe that sometimes numbers do not tell the whole story. Instead, we will discuss qualitative aspects of the study, and how it affected the mathematics curriculum and the Mathematics Department itself. Then we will discuss how each of the three main participants in this study, and authors of this paper, view the results of the study.

Impact of the Pilot Study on Course Offerings

In general, at the completion of the pilot study, the mathematics faculty was pleased with the new form of college algebra. Since no course at FSC required college algebra as a prerequisite, the majority of students who enrolled in college algebra did not need to focus on symbolic manipulation skills. Students needing symbolic manipulation skills were already being enrolled in precalculus. Students who took college algebra often took other mathematics courses such as a liberal-arts type math course (Contemporary Mathematics) or statistics, but neither of those courses requires symbolic manipulation skills. Few students took applied calculus where symbolic manipulations might be useful. The department felt whatever the students lost in symbolic proficiency, they would gain in problem solving skills. For these reasons, the more engaged approach to teaching algebra was adopted by our department and our traditional college algebra class was deleted.

We now have a course called Mathematical Modeling, which was designed after the pilot course. The department felt that a change of name was in order to eliminate any misunderstanding among students who expected a course similar to their high school algebra, which, in the department's opinion, was what traditional college algebra represented. The new course is taught with very little lecture and most of the learning is done through exploration using real life application problems. Problems in this new course include determining wind chill or projecting shortfalls of rice production in Asia—all using real life data. Students try to solve these problems using various methods, including using the graphing calculator to help achieve a solution to problems that they could not have mastered without technology. Most of the students who enroll in this course are not mathematics or science majors.

Impact of the Study on the Culture of the Department

The culture of the Mathematics Department was changed by this study. Learning a new teaching style fostered more camaraderie and ultimately more collaboration among many faculty members. The department had previously been collegial and willing to share teaching tips, but there had been little impetus to do so. Faculty were teaching courses that they had taught for years, whether at FSC or some other institution. The Project changed all of that. The participants were now teaching in a manner that was outside their comfort zone. This encouraged the sharing of information and discussion of teaching styles. The faculty members involved in the study have become active in the discussions of curricular reform both on campus and off. They have led discussions at regional meetings of the MAA, and some have been involved in curricular changes at other schools. For example, due to his experiences with this grant and other curricular changes, Dan Jelsovsky was asked to join in the discussion of mathematics curricular change at Foshan University in China.

Florida Southern College is now in the midst of transforming its curriculum to include an active student learning environment. This transformational initiative moves toward a more learner-centered educational approach which attends to student learning outcomes and engaged learning pedagogy. As part of this transformation, the use of technology in the classrooms is strongly encouraged, as are the ideas of group work and interactive technology such as clickers for answering questions in class. The Mathematics Department's involvement in considering a new approach to teaching college algebra has been an advantage in this reform effort – our new course is already in place.

Impact of the Study on Individual Faculty Members

This study has also impacted individual professors and the way they teach. To understand this impact, we now look at personal statements from the three main people involved in the study at Florida Southern and the authors of this paper.

Dan Jelsovsky

In follow-up courses, specifically statistics and our liberal-arts type math course, I find that students who take this 105 course are at no disadvantage as compared to students who had a more traditional algebra course. In fact, as I introduce more graphing calculator exercises in these other courses, I am finding that students who have taken modeling have a distinct advantage through their experience with the calculators.

I believe that the experiment with modeling has changed my teaching. Although I had used technology (computers specifically) in my mathematics courses in the past, this use had usually been quite limited. I had never used technology in general education math courses. Even in majors' courses, unless the course was specifically for computer applications, I often would not use technology extensively. I would assign a project or two, perhaps, but that was the limit of the graded work. I gave students limited instruction on the use of technology, but use of technology in the course was optional.

After teaching the experimental modeling course, I have more readily accepted the use of technology. I have retooled my statistics course to include the use of graphing calculators. Now, instead of stressing the application of formulas, I stress understanding of concepts. I have also been more accepting of technology besides calculators. In my Contemporary Mathematics course, I have started using clicker technology that allows students to participate in class by answering questions and getting immediate feedback. In my linear algebra class, I have expanded the use of the computer Algebra system Maple to include almost all elementary calculations. This enables me to cover more theoretical aspects of the course. My experience with the modeling course has liberated me from teaching primarily technique and enabled me to focus on concepts and problem solving.

My in-class style has also changed after this experience. I find I use group work (even group of the whole) much more often than I previously had. Instead of presenting multiple examples, I am allowing students to try multiple problems in class and get quick feedback.

Kenneth D. Henderson, Jr.

During Fall 2006, I taught one pilot section and one control section of college algebra and two sections of Applied Calculus I. I first encountered the effects of the modeling approach in two adult students enrolled in Applied Calculus I. They had both received an A in the college algebra modeling sections the previous term. Neither of them was familiar with factoring a quadratic equation. The academically stronger of the two was able to use the graphing calculator to do problems that most of the students were doing in the traditional way. The other had a much weaker algebra background and was not able to use the graphing calculator with the necessary proficiency. One received a B and the other a C in Applied Calculus I. Both would have benefited from a traditional algebra course.

The following spring, one student from my pilot section and one student from my control section enrolled in Applied Calculus I. Both received a B in their respective courses. The student from the pilot section struggled with the algebra in the Applied Calculus I course in a similar manner to that previously described. Both students received a C in Applied Calculus I. The student from the pilot section enrolled in Applied Calculus II the following term and earned an A. My conclusions are that if a student is not ready to enroll in calculus as a freshman, a traditional algebra course would serve them best. However, teaching the modeling course has changed the way that I teach Applied Calculus (and Statistics). I employ the graphing calculator to a much greater extent and use more problem solving and engaged learning in those classes

Susan Serrano

As part of the MAA Renewal of College Algebra project, two members of the Mathematics Department at Florida Southern College attended a workshop in Albuquerque to learn how to teach college algebra with a more discovery-oriented approach. This approach also incorporated a strong emphasis on technology. That was my first exposure to this different approach to teaching mathematics. I had always taught with a predominately lecture style with assigned problems being done by the students at home. Students would then ask any questions from the homework at the beginning of the next class. Even though I had been using technology, specifically, Minitab, in my statistics courses for a couple of years; I had not allowed graphing calculators in my math courses. Instead of focusing on using the calculators, I believed that the students needed to know how the formulas and computations should be applied.

After attending the workshop, I came home excited to try all the new techniques that I had learned. I spent the following semester preparing lessons and group work. After a visit from Bruce Crauder, the author of our textbook, we were ready to get started.

In spring 2006, I taught one pilot and one control class. I considered myself above average in my ability to teach mathematics to students who do not normally like math. But as the first semester went on, I found the control class becoming increasingly tedious to teach. The students in this class took a more passive role in the learning experience. They took notes as I presented many applications on how to solve the problems without using anything more than a basic calculator. The pilot section was broken into five or six groups with approximately four students per group. I typically spent five to ten minutes at the beginning of class, briefly explaining the topic and any new calculator techniques. I then passed out two to four worksheets for the students to "explore" with some guidance from me. The students were allowed to visit other groups and ask questions or to offer their suggestions on the solution. There was a lot of energy in this discovery environment. Also, many topics that would have been difficult for most students at this level to master without the use of a graphing calculator could now be introduced and acceptable solutions found. I subsequently taught two algebra courses during our summer terms using the modeling and technology-based approach. The class size is typically very small in the summer — seven to ten students — which is ideal for a modeling and active learning approach.

My experience with the students from the pilot program in subsequent math courses depended on which math course they took. The more technology-based material from the pilot program seemed to prepare the students very well for the elementary statistics course. This course also typically has a large technology component; either statistical software or the graphing calculator is used. But the students from the traditional course seemed to do better when their next math course was in one of our calculus series, since these courses are still based primarily on computational skills. I am convinced that with some adjustments, a blending of a computational and a technology based learning environment could be successful.

The course typically taken by students who are going on to higher level mathematics courses is Precalculus. I have just started incorporating the graphing calculator and group work into this course. The students prefer the group discovery portion of the class but I try to cover some of the background and computational skills to the concepts.

Conclusion

The Renewal of College Algebra project has transformed the mathematics program at Florida Southern College. It has enabled the faculty to take a learner-centered approach in the classroom. It has also helped the faculty to use more active and engaged learning. These classroom techniques have carried over into other classes taught by the faculty, which they feel will help in the facilitation of education among the millennial generation.

CRAFTY Project at Harrisburg Area Community College
Refocusing of College Algebra

Dan Fahringer and Chris Yarrish
Harrisburg Area Community College

Abstract. As in many post-secondary institutions in the country, College Algebra success rates are a major concern at Harrisburg Area Community College. Even though our pass rate (approx. 58%) is higher than the national average, we are looking for ways to increase success for our students. In 2005 and 2006 we participated in the National Science Foundation funded College Algebra Renewal study coordinated by the CRAFTY committee of the MAA. Beginning in the spring 2006 semester, we offered sections of College Algebra using both a traditional and a modeling approach. The grade data we collected seemed to show little difference between the A–C passing rates for the two approaches, but we have continued to offer and improve the modeling-based College Algebra sections. Although our work is not yet complete, we have found many interesting facts about how a modeling course works not only as a method of delivery for College Algebra, but as a means of helping our students in many areas of their lives as well.

Background

Harrisburg Area Community College is located in central Pennsylvania and has five regional campuses serving over 13,000 students. At HACC, students take College Algebra either to fulfill a three-credit core mathematics requirement for their associate's degree or as a course that will transfer to a four-year institution. For the majority of these students, College Algebra is their final mathematics course.

In fall 2000 our department began giving a common final exam in College Algebra. Data from these common final exams showed that one-third of the students who started the class were not persisting to the final, and only one-third of the original students were passing the final. Because of the low success rates we applied for inclusion in the NSF-funded College Algebra Renewal study.

College Algebra at HACC

Our College Algebra course covers fundamental algebraic operations, exponentials and radicals, systems of equations, polynomial equations, logarithms, matrices, and polynomial, exponential, and logarithmic functions. Each semester, approximately 45 sections of the course are taught by both full-time and adjunct faculty. Each College Algebra course must adhere to a department-approved list of learning outcomes to ensure transferability.

Currently, HACC offers both traditional and modeling approaches. All of the modeling courses are taught in a SmartRoom classroom with a maximum class size of 27 students. Each student is required to have a graphing calculator at the beginning of the course. At this time, there are only a few modeling sections of College Algebra offered each semester, however our refocused modeling course continues to undergo changes. As we refine the course and classroom activities, we have begun to see an increased interest in offering more sections.

Our modeling approach began as a student-centered course with the following goals:

- improving students' success,
- incorporating group work with both in-class explorations and out-of-class projects,
- using technology by requiring the use of graphing calculators and integrating computer spreadsheets,
- developing students' confidence in mathematics,
- improving students' reading, writing, listening, and presentation skills,
- improving students' appreciation of mathematics in meaningful situations using real-world data,
- improving students' attendance and classroom participation as well as decreasing the withdrawal rate, and
- giving class ownership to the students.

Initial Results

After returning from the CRAFTY workshop in the summer of 2005, we began to put together our modeling course in a way that would address our students' needs in College Algebra. During the fall 2005 semester, we surveyed 12 sections of College Algebra to determine the course of study of these students. Only 35.8% of the 282 students indicated that they planned to take either Calculus or Business Calculus. The results were as we expected. The majority of our College Algebra students are not going to take a Calculus course. This is a conception that was common among our peers and has initiated an ongoing discussion in our Math department about which learning outcomes are appropriate in our College Algebra course.

As part of participation in the CRAFTY study, we collected data during the spring and fall 2006 semesters from students in both the modeling and traditional classrooms. We offered four sections of the modeling course in the spring and five sections in the fall. Data was also collected from four sections of traditional College Algebra in the spring and four in the fall. Theses sections of modeling and traditional classes included both two-day (75 minutes) and three-day (50 minutes) a week schedules. All classes were taught by full time tenured faculty and were offered at similar times during the day. The following table, contains a breakdown of the grades for students who submitted a signed consent form to have their grades included in the study.

Spring 2006	Spring 2006		Fall 2006	Fall 2006
Pilot	**Control**		**Pilot**	**Control**
2	10	A	5	4
11	11	B	12	6
18	19	C	11	6
8	10	D	8	2
8	12	F	6	1
7	8	W	7	2
54	70	Total	49	21
57.4%	57.1%	A–C	57.1%	*76.2%

*College-wide this number was 55% for students taking College Algebra during Fall 2006 semester.

As the numbers indicate, there were no significant differences in either the retention or success rates of students between the control and pilot sections of College Algebra. Generally, in these initial two semesters of the modeling class roughly the same percentage of students succeeded (finished with a C or better) as in the course across the college.

Looking at data that we've collected since the initial two semesters, we have noticed certain trends. In the four-semester period immediately following the experimental period (spring '07–fall '08) the following observations were made. The pilot classes (190 students) had a pass rate of 57.9% and the traditional classes (3,092 students) had a pass rate of 57.3%. Another interesting factor was that the withdrawal rate for the pilot classes was 13.2% and for the traditional classes it was 17.5%. This difference was made up in the D and F grades.

Spring ‘07–Fall ‘08	Spring ‘07–Fall ‘08	
Pilot	**Traditional**	
9.5%	15.2%	A
17.9%	19.3%	B
30.5%	22.8%	C
11.1%	9.1%	D
17.9%	16.1%	F
13.2%	17.5%	W
57.9%	57.3%	A–C

Other findings included a significant increase in student attendance in the modeling sections. This was one of the major goals we hoped to attain and it was happening immediately. Also, we found there was a definite increase in some student’s interest in this course as well as an understanding in the application of mathematical concepts. For example, students felt more comfortable working with the phrases “initial value” and “rate of change” and seemed to understand how these concepts applied to both linear and exponential functions. Once the students made the connection between the application and the algebra, their attitudes toward math improved and their involvement in classroom activities increased.

Improvements and Changes Made

As we’ve continued to offer these sections of College Algebra using a modeling approach, we have identified gaps between our actual and desired results. We have continually refined classroom activities and the structure of the course to address any weaknesses in the design of the course and course materials. In order to continually assess student learning, we first used in-class “explorations”, interactive lectures and out-of-class (graded) homework assignments.

- The “explorations” are discovery-oriented activities that the students complete in groups during class. The activities immediately give students a reason for needing the material and a context in which to work with a particular mathematical topic. The activities also introduce topics in an accessible way. For example, in finding a formula where every time the input increases one unit the output triples (or is multiplied by some factor), they are consequently introduced to an exponential function.
- The interactive lectures allowed students to work both alone and in groups soon after material was presented. This not only forced students to complete some practice problems, but also allowed an opportunity for them to learn from one another and learn from explaining concepts and procedures to their classmates.
- The graded homework assignments were problem sets from the textbook. Initially students were responsible for submitting neat and orderly completed homework problems (either an entire assignment or selected sample problems.) These assignments were designed to encourage the students to work with the material from class and assess how well they have grasped the concepts. However, even with some adjustments to the structure of these assignments, these assignments never produced the desired results. We then replaced these graded homework assignments with online weekly quizzes to help the students evaluate their comprehension of the material. Again while some students used these quizzes wisely, a significant portion did not. We have since begun giving these quizzes in class rather than online. Most recently, we are testing two approaches: one instructor is giving these quizzes in class rather than online, while the other has eliminated any of these types of assignments or quizzes and instead gives more graded take-home assignments along the same lines as the in-class “explorations.” We have yet to see the results from these most recent changes.

Another significant issue was that many students showed an initial resistance to this unexpected ‘new’ approach. During the first two semesters, these sections of the course were not advertised in our course catalog any differently than the traditional sections (as required by our involvement in the study.) Some students did not like this surprise. In subsequent semesters we were able to list the courses along with a comment that “the course is taught using a modeling approach." While this advertising blurb does give students forewarning that this approach may be different from their past experiences, many do not know what is meant by "a modeling approach." To alleviate this issue, on the first day of class we explicitly state what will be expected of them in this course to our students. By letting them know that we require a lot of group work, outside projects and individual exploration, we have been able to eliminate many of their reservations about this course. One way in which we do this is that on the first day we do a classroom activity that involves group work and active student involvement. Since we have started doing this and publicizing what we do in a modeling approach with other faculty and counselors, we have found that we have had no students who wish to leave the course for the traditional approach.

Other improvements we have made include getting students to become more familiar with available online resources such as the textbook's website and the instructor's website. We have found that it is difficult for some groups to meet since our students are all commuters. By having an instructor website, groups are able to meet 'virtually' and communicate to each other via discussion boards and chat rooms about their project.

An on-going problem that we have encountered is that students have difficulty finding help outside the classroom. Tutors and former math students are often not familiar with the modeling approach and find it difficult when trying to help students. Recently we have been able to have tutors audit the course and have had small tutor training sessions to help alleviate this problem.

We realize that this course is one that will continually be changed and improved. Our hopes are that as more faculty teach with using this approach, we will get more and more ideas for making this the best course it can be.

Recent Results

One of the major questions in the CRAFTY study was whether students who took the modeling approach would succeed in courses after College Algebra just as well as students in a traditional classroom. To do this we would need to obtain students' grades in subsequent mathematics courses that have College Algebra as a prerequisite. Because we are a transfer institution, it is very difficult for us to show these results. Our students usually transfer shortly after taking College Algebra and we have not yet found sufficient data to show these results. We are hoping that over time our Institutional Research Department will be able to collect enough data so that we can investigate how our students are doing in subsequent courses. We are also relying on results from the project to give us support to share with our colleagues regarding this method.

While we have not been able to gather very much hard data from our students after their experience in our classes, they have been very willing to share their thoughts at various points throughout the semester. This data (see Appendix) shows that there are both favorable and unfavorable comments about the modeling approach, but the favorable comments seem to outweigh the unfavorable comments. One way this "better attitude" is apparent is in the students' attendance. Two of our most successful classrooms in terms of the student attendance, occurred in the fall 2006 and fall 2008. During the fall 2006 semester, one of the modeling classes attained a 94% attendance rate with only one withdrawal; and during the fall 2008 semester, in a class that started with 26 students, two students withdrew in the middle of the semester, one student stop attending near the end of the semester, and there was a 95.4% attendance rate for the other 23 students. The attendance rate in traditional College Algebra sections is not this high.

Conclusion

We feel that teaching using a modeling approach is a little more effective than the traditional approach. Although the math content is the same for students in both environments, we feel our students have a more meaningful experience working in groups with real-world data and technology. They are able to work more productively in small groups, they are more technology-savvy using the graphing calculator and Excel, and they develop a connection between mathematical concepts and their everyday life.

For the fall 2008 and spring 2009 semesters, we found that the modeling courses that were offered filled quickly. For spring 2009, we offered two modeling courses and they were the second and third sections to fill during registration. We do understand that other factors may affect this, including instructor and time of class, but we feel from talking with our students, that other students are beginning to say good things about modeling-based College Algebra.

APPENDIX

The following comments are from students at the end of the course from the Spring and Fall 2006 semesters.

Good	Bad
• "better classroom environment because I got to know people", • "it made more sense why to take math", • "most enjoyable math class since 6th grade", • "this class is easier to understand and makes sense, decreased my anxiety towards math", • "it is a great class, I never understood math before this semester", • "even though the rumor is that this class is harder, it seems to make sense (at least for me)" • "the readings and real life data helped me understand why the equations are necessary"	• "instead of giving methods to used, we weren't given help in figuring things out", • "couldn't find help", • "word problems are not my thing" • "I should not have to write a sentence to answer a question" • "this class felt like a reading class instead of a math class"

These comments are from students at the end of the third week of a modeling course in Fall 2008.

How does this approach compare to a traditional approach?

- "everything through modeling is broken down in a way that makes it easier to comprehend"
- "it's different, not bad, take getting used to"
- "it is harder than the traditional in some aspects, but it is helpful"
- "it's not just memorizing equations"
- "a lot easier to understand with groups than on your own"
- "taking steps to solve the problem is much easier than just trying to get the answer"
- "it is more focused on the reasoning behind the math rather than just the numbers"
- "I love the traditional approach, I'm a big fan of simple x's and y's"
- "had I known about having to work in groups, I wouldn't have taken this course. I don't like groups"
- "gets confusing when it goes back & forth between real life concepts and math equations"
- "the modeling approach is easier to me and seems easier to other students"
- "this class keeps my attention much more than the traditional style and it explains the biggest question I had in all the other classes I have taken that is how is what I am learning useful in everyday life"

Would you recommend this approach to a friend?

- "it is a good way to view another angle at math"
- "I think any approach to math is valuable. Especially if someone finds math difficult, this approach could be useful for him"
- "it is much more interesting to learn math this way because the old way is so damn boring"
- "it's more helpful"
- "it would be a way to explain math with things that they can relate to"

Has your interest in math improved with what you have learned up to this point?

- “I can see better that our world can be described using math”
- “I think the information being taught is easier to comprehend rather then the traditional way it is taught”
- “I have learned things yes, but my interest has wavered due to lack of interesting content. Imaginary numbers was much more interesting.”
- “I have been focusing more in this math class than what I have in previous math courses”
- “I don’t like math that much but I don’t dread coming to class!”
- “I find it easier to pay attention because we actually talk about things opposed to just straight lecture”
- “it is helping me to understand the relationships in everyday life in terms of math and that is helping me to understand better”

Modeling-Oriented College Algebra at Mesa State College

Cathy Bonan-Hamada and Tracii Friedman
Mesa State College

Abstract. In 2005, Mesa State College (MSC) was one of eleven schools chosen to participate in the Mathematical Association of America's College Algebra Renewal Project. In 2006, we developed and taught a modeling-oriented college algebra course. Students from twelve sections of College Algebra at MSC participated in the study. Six of those sections of College Algebra were taught from a modeling perspective and six were traditionally taught. More students successfully completed (C or better) the modeling course (60.9%) than the traditionally taught control sections (46.6%). Students who chose to participate in the study were followed for two semesters in subsequent mathematics courses in order to compare the performance of the students who took a modeling-based course to those who took a course that was traditionally taught. Little difference was found in the performance of students in subsequent mathematics courses.

Introduction

The Mathematical Association of America's Committee on Curriculum Renewal Across the First Two Years (CRAFTY) has determined that in many colleges and universities across the nation, retention and pass rates for students in traditionally taught, skills-based, college algebra courses are quite low. The committee also determined that students have difficulty applying the skills they have learned in college algebra in the context of other disciplines. After extensive conversations with partner disciplines, CRAFTY reported that partner disciplines recommend that intricate algebraic manipulations be de-emphasized and that college algebra courses stress problem solving, mathematical modeling and applications in appropriate technical areas. The Mathematical Association of America's College Algebra Renewal Project, funded by the National Science Foundation, is a study involving eleven institutions, including Mesa State College, to investigate the impact of modeling-oriented college algebra courses on student learning and student success.

About Mesa State College

Mesa State College is a public institution located in Grand Junction, Colorado and is the main regional education provider in western Colorado. The college provides a broad, liberal arts based curriculum and offers a wide range of baccalaureate degree programs and a limited number of graduate degree programs while also maintaining a community college role including vocational and technical programs. Mesa State College is described as a "moderately selective institution" in its mission statement that was established by the Colorado Legislature.

Goals and Objectives

A goal of the study and of the faculty at Mesa State College was to determine if a modeling approach to teaching college algebra could address the problem of low success rates in college algebra courses and at the same time, assist students in applying the mathematics they learn in future work. To address these goals, we designed our modeling-based courses to:

- have modeling as a central theme
- incorporate collaborative work on projects
- make significant use of graphing calculators
- de-emphasize (carefully chosen) algebraic and computational skills

Required Algebra Topics at Mesa State College

Mesa State College is a public institution in the state of Colorado. In order to guarantee transferability of our college algebra course to other institutions in the state, we are required to include the following topics in our course:

- algebraic properties of the integers, rationals, real and complex numbers
- techniques for manipulation of expressions
- techniques for solving linear, non-linear and absolute value equations, inequalities, and systems of equations
- Cartesian plane, relations and functions
- properties and graphs of polynomial, rational, exponential, logarithmic and inverse functions
- conic sections

Prerequisites for College Algebra at Mesa State College (during the time period of this study)

In 2006, students at Mesa State College could enroll in College Algebra if they met one of the following prerequisites:

- earn a grade of C or better in Intermediate Algebra
- earn a score of 19 or better on the mathematics portion of the ACT
- earn a score of 500 or better on the mathematics portion of the SAT
- earn a score of 42 or better on the College Level Math portion of the Accuplacer placement exam

Challenges in Creating the Modeling-Oriented Algebra Course

When developing the modeling-oriented algebra course, we chose a text that we felt was true to the spirit of a modeling-based approach to college algebra and which de-emphasizes many of the traditional skills that are typically taught in a college algebra course. Since the sections of college algebra that were taught from a modeling perspective were assigned the same course number as the college algebra course that had been approved for transferability by the state of Colorado, we were required to cover many skills topics that are either not covered or not adequately covered (according to Colorado standards) in the text. Thus, it was necessary to create supplementary materials and associated exercises for such skills topics. We found it challenging to decide how and where to incorporate these supplementary topics into the schedule to minimize the disruption to the flow of the modeling topics as presented in the text. It was also a challenge to determine which skills topics to de-emphasize so that there was sufficient time to develop modeling techniques.

It should be noted that we had many informal conversations with our colleagues in other disciplines (especially Biology and Environmental Science) to obtain their input regarding the topics they wanted their students to know and in what ways they felt the traditionally taught course was failing. This input was carefully considered as we created the course.

Overview of Mesa State College's Participation in the Project

Mesa State College agreed to run three pilot (modeling-oriented) and three control (traditionally taught) sections of College Algebra as part of the study during both spring and fall semesters of 2006. Each semester, three faculty members participated in the project (two full professors and one adjunct) and each faculty member taught one modeling section and one control section. Students were unaware whether the section they enrolled in would be taught traditionally or from a modeling-perspective until the beginning of the course. It was decided that in the control sections of the course, instructors would make no changes to the way that they traditionally teach and assess their college algebra courses.

At MSC, we do not have a graduate program in mathematics and so teaching assistants are not available for our classes. We chose to use the $5000 stipend provided by the grant to hire undergraduate teaching assistants for each pilot course. The teaching assistants' responsibilities included grading homework and assisting the instructors with facilitating group work activities. At Mesa State College, College Algebra is a four credit hour course but for scheduling purposes, classrooms are reserved five days a week. This gives each college algebra instructor the flexibility to choose which four days of the week the class will meet (or equivalently, which weekday the class will not meet). In the pilot sections, we utilized the extra (fifth) class period by offering an optional recitation hour led by the teaching assistant for the course.

A table outlining a comparison of course structure and set up for the control and pilot sections can be found on the next page of this document.

Comparison of Course Structure and Set-up

	Control sections (Traditional)	**Pilot sections (Modeling)**
Text used	• Stewart, Redlin and Watson: *College Algebra*, 4th ed.	• Crauder, Evans and Noell: *Functions and Change*, 3rd ed.
Content	• Chapters P, 1–5, Sections 6.1, 6.2	• Chapters 1–5 and supplementary skills material
Teaching style	• Primarily lecture with occasional group work consisting of computational practice problems	• Approximately 50% group work, often covering a topic in its entirety (no lecture) • Approximately 25% interactive lecture • Approximately 25% lecture
Classrooms and class size	• Lecture hall with approximately 40–60 unmovable seats. • Course capped at 40 students.	• Five of the six sections had same set-up as control for both classroom and class capacity. • One section had small classroom with 30 movable table desks & chairs and was capped at 30.
Number of class meetings per week	• Four (Monday-Thursday)	• Four meetings were required (Monday–Thursday); students were strongly encouraged to attend a 5th meeting (a recitation session) each Friday
Teaching assistants	• N/A	• Each section had a teaching assistant. • Duties included: • Attending several classes each week to assist with group work • Teaching the recitation section each week • Grading weekly homework assignments
Assessment	• Eight quizzes, three tests and comprehensive final • Participation • Homework notebooks were collected during each test for up to 5 bonus points added to the test score	• Five quizzes, three tests and comprehensive final • Participation • Homework collected each week and graded by teaching assistants

Teaching the Modeling-Algebra Course

To remain true to a modeling-based approach to teaching the course, we chose to have students work on group activities at least once a week. We faced the usual challenges associated with incorporating group work into a course including creation of the groups, determining roles for members of the group, assessment of group assignments, management of room setup and facilitation of group activities for a class size of 40. It was also a challenge to train the students to write solutions to homework assignments in a professional manner using complete sentences and correct mathematical grammar. Many students had never been required to write in a mathematics class and resisted the writing component of the course. In particular, some felt that it wasn't fair that other sections weren't required to submit lengthy homework assignments. This was particularly true of stronger students who would probably succeed doing minimal work in a traditional college algebra class. Some of them even expressed anger that they were stuck in a section that required so much writing, group work and homework, and that they were not informed of the differences before registering for the class.

There were also students in each of the pilot sections who felt that the group work and writing components of the course aided them significantly in their understanding of course material. In particular, being required to explain their answers in complete sentences enabled them to learn the concepts. Many students appreciated being exposed to real life problems and applications. There were a number of students who had failed or withdrawn from college algebra several times before, but were able to succeed in this course because they felt more connected to the material.

Comparison of Pilot and Control Sections: Retention and Final Course Grades

During the two semesters of the study, more students successfully completed (C or better) the modeling course (60.9%) than the traditionally taught control sections (46.6%). More specific data is included in the following tables:

Spring 2006	**Control**		**Pilot**		**Fall 2006**	**Control**		**Pilot**	
Grade	# students	%	# students	%	**Grade**	# students	%	# students	%
A	13	9.85	7	6.42	A	12	11.65	14	14.14
B	18	13.64	26	23.85	B	19	18.45	23	23.23
C	29	21.97	29	26.61	C	18	17.48	27	27.27
D	20	15.15	19	17.43	D	14	13.59	9	9.09
F	40	30.30	17	15.60	F	28	27.18	21	21.21
W	11	8.33	11	10.09	W	12	11.65	4	4.04
I	1	0.76	0	0.00	I	0	0.00	1	1.01
Total	132	100%	109	100%	Total	103	100%	99	100%

It is not clear to us whether the higher retention and pass rates seen in the modeling courses are due to the modeling approach or to the rather significant disparities between assessment methods as well as the use of teaching assistants in the modeling courses. In particular, graded homework and group work accounted for a significant portion (20%) of the total possible points earned in the pilot sections while homework assigned in the control sections was not graded and only minimally contributed to students' grades in the form of bonus points. In fact, because of high homework and group work scores, at least two students passed the pilot version of college algebra with final grades of C and yet had earned scores below 70% on all quizzes and exams. Also, once a week, there was an optional question/answer recitation session led by the teaching assistant available to students in each pilot section, but not to those in the control sections. It should be noted, though, that all Mesa State College mathematics students have free help available to them at our Tutorial Learning Center.

It seems to us that if students are successfully completing large quantities of well-explained homework and have more opportunity to get their questions answered, then they will be more successful regardless of whether or not the course is modeling-based. However, it also seems that the homework in the modeling course is more relevant and interesting, rather than purely computational, and that students may be more likely to complete it. It would be interesting to complete a similar study in which the same resources were available for both the traditional and modeling courses and the same assessment techniques were used in each.

In summary, we believe that the students who took the modeling oriented algebra course benefited significantly from the content as well as from the teaching methods. The modeling-based approach focused on material that is used in related disciplines in the context of those disciplines. In particular, a significant number of problems and activities in the modeling course involved analysis of data sets. We believe that students will be more likely to recognize how to use the mathematics they have learned in this course when it is needed in future courses in their disciplines. In fact, one student enrolled in a modeling section was very excited that he had been able to use techniques learned in the class on an economics test problem when he failed to remember how the economics professor taught the topic. He got that problem correct.

Data from Subsequent Courses

An important phase of this study involves tracking student performance in mathematics courses taken during the two semesters following their participation in the study. This phase is critical to assessing whether, by adjusting the content in this way, we maintain (or perhaps even improve) the level of preparedness of students who take additional mathematics courses following college algebra.

We found that 21 of 38 subsequent classes (55.3%) were passed (C or better) by students who successfully completed the modeling college algebra course while 14 of 29 subsequent classes (48.3%) were passed by students who successfully completed the traditional course. Data was collected only for students who participated in the study and only in the two semesters immediately following their participation. Thus, students who passed the modeling class performed slightly better than those who passed the traditional course. However, the number of subsequent courses taken is so small that there is little statistical significance to this outcome.

While we are pleased that students' work in subsequent mathematics courses seemed to be unaffected (or perhaps slightly improved) by the change in content/teaching style, we are disappointed that the study did not collect data from students' subsequent work in related disciplines. In fact, a major reason to change from a traditional teaching style to a modeling approach in college algebra is so that students will be more successful in these other disciplines. We hypothesize that a much clearer distinction would be made in such courses between the students in the modeling courses and the students in the traditional courses in favor of the modeling courses.

Dissemination of our Results

In addition to this report and a preliminary report made to the PI's and participants of this College Algebra Renewal Project, we have also shared our results in local, regional and national venues.

At the conclusion of the project at Mesa State College, our results were shared with our college community in an hour-long Faculty Colloquium presentation. The partner disciplines were very excited about the project and were hopeful that we would continue teaching at least some sections of the course from a modeling approach.

We gave a presentation at our regional (Rocky Mountain Section) MAA meeting and received much positive interest. One school, in particular, had planned to try out a modeling-based college algebra course and we supplied them with some of our materials. We also presented a poster at the 2007 Joint Mathematics meetings in New Orleans.

Where do we go from here?

The mathematics faculty at Mesa State College has been aware for several years that there is a problem with success rates in our college algebra course. Attempts to more clearly define and strengthen prerequisite requirements to ensure that students who enrolled in college algebra are adequately prepared for the course were rejected by college administration. In fact, recently the Office of Academic Affairs decreed that even the minimal placement requirements for all courses would be advisory rather than mandatory. Consequently, many of the students who enroll in college algebra are under-prepared for the course.

Because the modeling-oriented college algebra courses taught as part of the College Algebra Renewal Project had a significantly higher student success rate than is typical of college algebra at MSC, there have been departmental discussions about whether the modeling approach could address the college algebra success rate problem at MSC. Unfortunately, there are concerns about the viability of the modeling-based college algebra course that was designed and taught as part of the MAA study in a department with limited financial and physical support.

Discussion items have included:

- **Funding sources for and availability of teaching assistants:** Teaching assistants were a critical component of the modeling-based algebra course that was created as part of the MAA's College Algebra Renewal Project. Unfortunately, there is not money to support teaching assistants. Given our current teaching loads (12 credit hours per semester), it is not reasonable to expect faculty to hold recitations and/or to grade the large writing assignments given in the modeling courses. Also, since our teaching assistants were undergraduate mathematics majors (we graduate about 5-10 per year), it is unlikely that we would be able to staff all of our college algebra sections with a qualified teaching assistant.
- **Implications on department for reduced class sizes:** The class size for our college algebra courses is capped at 40 and classes are typically full. With group work being a critical component to teaching from a modeling perspective, we are concerned that it will be difficult for a single instructor to facilitate meaningful group activities for such a large number of students without teaching assistants. Thus, in the absence of teaching assistants, class sizes for the modeling-oriented courses would need to be reduced resulting in lower FTE for the department.
- **Curriculum issues:** There are curriculum concerns particular to MSC that need to be addressed. They include the transferability of the modeling-oriented course in the state of Colorado, satisfaction of general education requirements at MSC and whether the traditionally taught skills-based college algebra courses and modeling-based college algebra courses be different courses or simply different versions of the same course.

As of this writing, no changes have been made to our college algebra course as a result of this study. The faculty members who participated in the study were those more inclined to use group work in their courses and they continue to do so on a limited basis. There are no other faculty members at Mesa State who are employing substantial group work or a modeling-based approach in their college algebra courses at this time.

Unfortunately, a budget request for funds to continue the study was denied by our administration. For the reasons outlined above, and most importantly due to lack of funding, there are currently no plans to continue teaching the modeling-oriented version of college algebra at Mesa State College.

South Dakota State University's Participation in the MAA Renewed College Algebra Project

Donna Flint, Becky Hunter, and Daniel Kemp
South Dakota State University

Abstract. We at South Dakota State University in Brookings, SD eagerly began the modeling college algebra project in our classes in the spring of 2005. We experienced frustration early on with this new method of teaching due to several factors, some we could control and some we could not. As instructors, we found that the project challenged us to think outside the box of how to teach college algebra and stimulated us to try somewhat awkward but effective new strategies. As a whole, our students were not excited about the new ways we were attempting to teach them algebra. In the end, we decided the modeling class was not a fit for our university but instead we transformed it into an honors class that we launched in the fall of 2007.

Why we chose to enter the study

South Dakota State University (SDSU), a land-grant institution with an enrollment of 11,400, is the largest university in South Dakota and offers over 200 majors. SDSU is located on the east side of South Dakota in Brookings. Brookings has a population of approximately 19,000. The Department of Mathematics and Statistics is located in the College of Engineering and provides service courses to the University at Large (College Algebra) and to a large audience of engineering majors. To understand the preparation level of incoming freshmen, it is useful to know that approximately 28% of incoming freshmen are initially placed in remedial mathematics courses. The average ACT score of incoming freshmen is 22.8 and 45% of incoming freshmen hold an ACT Composite score of 24 or higher. Currently, College Algebra is taught by Master's level instructors and Graduate Teaching Assistants in traditional classes of approximately 40 students.

In the spring of 2005 and perhaps earlier there was discussion among the Mathematics Faculty at SDSU about how to improve our College Algebra (Math 102) class. Fortunately for us the MAA College Algebra Redesign Project (the Project) became available about this time and it was natural that we should participate. Donna Flint, Becky Hunter, and Dan Kemp agreed to be the Organizing Committee at SDSU.

The emphasis of the Project was to be Mathematical Modeling. We (the Organizing Committee) agreed that such an emphasis had the potential to lead to a 'better' course for our students. Much of our discontent with the current College Algebra course was that this was the last mathematics class for most of the enrolled students and the content was not particularly useful to them. Many had already failed at this type of mathematics in high school and it did not seem responsible on our part to make them suffer through the material again with the same traditional approach. At SDSU the traditional use of College Algebra (a terrible name, since the material has long been junior level high school mathematics!) is to prepare a student to study trigonometry and/or calculus. In addition College Algebra is a prerequisite for our beginning Statistical Inference course. We knew that only a small percentage of our students were taking College Algebra for those purposes, nevertheless we had to serve that population. Our System mandated that College Algebra was the minimum level mathematics class that students needed to pass to satisfy general education requirements. All higher level courses had to have at least College Algebra as a prerequisite. Since an alternative terminal mathematics course for our students was not available, modifying the existing course was our only choice.

We entered the Project knowing that we had the additional constraint of making the course suitable for those few students who would go on to other mathematics classes. That meant that a higher level of algebraic skill would be required than we would have preferred. We also had to include some beginning material on probability and combinatorics to satisfy College Algebra being a prerequisite for Statistics. These constraints did not turn out to be a good fit with the modeling approach of the Project.

Donna Flint and Dan Kemp were able to attend the Renewal of College Algebra workshop during the summer of 2005. They came home convinced that trying a renewed College Algebra course with a modeling emphasis would be an excellent project for SDSU. The fall 2005 semester was used for planning for the renewed course which would be offered for two semesters beginning with the spring 2006 term.

What we did

For our first semester of involvement of the project, Donna, Dan, Becky and two Graduate Teaching Assistants became the team teaching the course. We taught 5 sections of the renewed College Algebra. The team met once a week to plan the following week's curriculum, share feedback about the previous week's activities, and to share experiences. Each team member was assigned several sections for which they were to plan the class activities and we all used these activities in our classes. We quickly found that our students were not reacting favorably to the activities, usually not understanding what they were supposed to be gleaning from the activities. Many of them expressed their frustration vehemently. When we were preparing students for their first test, we found that they truly did not know what they were learning. For example, though many of our activities had involved solving quadratic equations, when we told them they should be able to solve quadratic equations on the first test, they were surprised that they had learned that! Based on this, for the second test, we created a review sheet that specifically listed those skills they had addressed during that portion of the course along with examples of basic skills they should be able to demonstrate on the test.

Part way through the semester, we were given a workshop by one of the authors of the textbook we were using. She suggested that we use worksheets to facilitate the discussion and help the students focus on what was being done in class. We immediately implemented this idea and found that students became a bit more comfortable. The worksheets helped us to maintain consistency in the course as well as helped students spend time thinking about what we were doing rather than trying to figure out what they should be writing down as "notes."

One aspect of the course with which we were particularly pleased was the projects. The team created some very interesting projects, and the students, though they grumbled about the "extra work," really found themselves enjoying projects. One project involved determining the value of their cars based on an exponential model of decay. They researched the value in the Kelly Blue book and drew conclusions about the value of their cars in future years. A Linear Regression Project helped them determine the correlation between income and education. Another project involved determining whether a flat tax or a graduated tax was better for them personally. Students began to see the relevance of the algebra they were learning through the assigned projects.

In the second semester Becky and two new GTA's took over the course and offered 6 sections. This team along with Dan and Donna continued to meet weekly. The materials were revised and improved by the new team. Though the prerequisite for the course was either a sequence of remedial algebra courses, or placement based on the ACT Compass exam, some students were not prepared to use basic algebra in class. To respond to this, for the second semester of the course, we provided a "prerequisite" sheet for each chapter which included 10–15 problems students should be able to do *before* they begin that chapter. These prerequisites included material from previous courses and previous chapters already covered in the course. This helped instructors ensure students were ready for the chapter and gave students a better idea of what they would be doing in that chapter.

How the study went and why we chose to stop teaching the new course

The faculty involved in the renewed College Algebra project was enthusiastic and worked hard to make the course work, constantly revising and improving. However, we drew the conclusion that the project was not successful at SDSU. We believe there were many factors that caused this. The main issue was student perception. Students were aware that there were two College Algebra courses offered on campus. One course was the traditional course which was strictly lecture with on-line homework and traditional tests. The other was the renewed course in which students worked through activities to learn the material, were expected to turn in well-written homework solutions, and also did several projects. Students in the renewed course felt that they had to do more work and were graded more harshly than those in the traditional course. Frequently traditional students could earn 100% on the homework, simply by manipulating the on-line homework system and without having learned anything. In the renewed course, it was likely that students lost points for incorrect answers as well as poorly written solutions. Though as instructors we believed this was more valuable because homework involved learning the mathematics rather than learning to manipulate the system, students were not convinced. In the second semester of the study, we added an on-line homework portion to alleviate this perception, but still required some (though less than before) written homework and projects, so students still were not happy.

Our hope was that those in the renewed course would be energized by the applications focus of the course and the interactive (rather than lecture) style, but instead, students felt intimidated. Their understanding of basic algebra was not strong enough to make a connection between the application introduced and the algebra they were learning. They found basic reasoning using numbers a baffling concept. Since our course, as a service course, is required to cover a variety of specific topics (which completely fill a lecture course) we didn't have time to work through the material at a slow enough pace to help students feel less intimidated.

It may be true that the course could have been successful had we been fully committed to the course. To do this, we would need to reduce the number of topics we cover in the course and eliminate the traditional college algebra course. It might have been possible to negotiate with our client departments to reduce the number of topics, and we could have eliminated the traditional college algebra course, but we had one final obstacle.

College Algebra is taught exclusively by Graduate Teaching Assistants (GTA's) and Lecturers. We believed that it would be possible to create a curriculum to be used by these instructors, but felt that the course would require first, experienced faculty and second, faculty who were committed to the approach. Our current group of lecturers had already indicated a lack of commitment to the approach and with GTA's only teaching for 3–4 semesters, they would not be able to reach the experience level we required. We felt that inexperienced or uncommitted faculty would not use the curriculum in a valuable way, making the course less effective than the traditional course.

In the first semester of the pilot, student dissatisfaction was expressed most clearly in the drop rate. For the pilot sections, there was a drop rate of 31% compared to a 21% drop rate in the control group. At the end of the semester, 77% of the students in the pilot sections earned at least a C in the course, while 75% of the students in the control group did so. In the pilot group, 33% of students registered for a subsequent mathematics or statistics course. Of these students, 50% earned a C or higher in the subsequent course. In the control group, 33% of students registered for a subsequent mathematics or statistics course. Of these students, 63% earned a C or higher in the subsequent course. These statistics solidified our decision to discontinue offering the Renewed College Algebra.

What we did with the course

Finally we decided our typical student could not handle the course and our department was really not equipped to offer the course as our standard college algebra course. However, we still felt it was a valuable course, so Becky Hunter suggested that most of the obstacles about which we concerned were exactly those things that would make the course ideal in the Honors College. An Honors instructor would be experienced and committed to the course. The honors students should be stronger and should have an understanding of basic mathematics, or at least be capable of learning some of this on their own. They should be able to make connections that other students could not make. Most importantly, Honors students expected a different type of class with more interaction, projects, presentations and higher expectations. So, Becky began offering an Honors College Algebra course at SDSU.

We launched the first ever Honors College Algebra class the next semester after finishing the NSF project. The class was based on the modeling approach and we even used the same textbook we had used in the project. We set the standards high for this class, hoping to achieve the kind of success we had anticipated during the project. We continued using the online homework, minimizing the bookwork that was assigned. However, we now required the students to type all the bookwork that was to be turned in for a grade. Instead of needing to feed the students every bit of information for them to discover one concept, we were able to be very broad in the description of a problem and give them freedom to explore possible solutions and dialogue about their thought processes. Class discussions were sometimes very fun and thought-provoking. The projects we assigned to the Honors class involved more research and presentation than we could do in the traditional algebra classes. We also focused more on problems that involved analyzing solutions and writing descriptions of those solutions. Most of the math the honors students learned was in the context of real-world problems.

Although better received as an Honors class, we did experience some frustrations throughout the semester. The placement of students into the class wasn't restricted so there were students in the class who were not at an honors level. These students found the class to be very hard and most of them dropped before the semester was over. Other students who realized the requirements of the class dropped right away, leaving only 17 students in the course initially.

Even though the majority of the students were at an honors level, they were still freshmen in college who were not necessarily equipped for the demands of a college level honors course. We used proficiency exams to assess computational skills and found that some of these students also lacked basic algebra skills, which made it difficult for them to connect the applications to the concepts they were intended to motivate. They complained about the amount of homework assigned (which was not unreasonable) and when faced with a problem they did not know how to do, most of the students just didn't try it at all. They didn't go to the instructor for help or to the free tutoring offered on campus. They lacked the discipline to follow through. This was probably one of the most disappointing characteristics of these students.

The second time teaching modeling-based algebra in the Honors course was a different and better experience. We had realistic expectations of our students this time and designed the course to incorporate basic algebra building skills while not neglecting the modeling approach. Instead of expecting our students to know how to write about mathematics, do research with mathematics, and give presentations using mathematics, we used the class to teach them how. We still ran into the problem of non-honors students being in the class and they had trouble with the style of instruction and eventually dropped the class. Many students really liked the class, though, and expressed how refreshing it was to see how the mathematics they are learning applies to their world.

Overall, the Honors class did succeed in using the modeling-based curriculum to a fuller extent. It took the students a few weeks to adjust to the style of instruction but for the most part they enjoyed the interaction and constructing their own knowledge. We hope to keep track of their success in subsequent math classes.

As educators, our ideal method of instruction is the modeling-based approach, but it doesn't currently fit with the students we teach as a whole. However, all of us have become aware of the benefits of this style of teaching and look for ways to implement its principles into the other classes we teach. Group work, small projects, writing assignments, and hands-on activities are a few of the strategies we like to use, along with finding real-world examples that speak to our students' experiences. Though not a successful project in terms of the original intent, it was highly successful for those of us whose teaching practices are now forever changed and the students we have yet to encounter.

Southeastern Louisiana University's Participation in the Renewal of College Algebra Project

Randall Wills
Southeastern Louisiana University

Abstract. Southeastern Louisiana University participated in the College Algebra Renewal Project funded by the NSF and supported by the MAA. For each of three semesters starting in 2005 and ending in 2006, Southeastern Louisiana University taught a total of 10 sections, four control and six pilot. Two individuals taught the control sections and three the pilot sections. We describe the project and then look at the performance of four sections taught in the spring of 2006, two pilot and two control. For these sections students in the control sections received higher grades in college algebra. There was little difference in the performance of control students and the pilot students in future courses. The most interesting finding from this data is that no pilot students who received an *A, B,* or *C* in college algebra failed any subsequent math courses. We then describe what two pilot section instructors learned by participation in the project and how it changed their teaching style. We end the paper by describing the college algebra course currently used by Southeastern Louisiana University and several other universities in Louisiana. Essentially, a portion of the course is taught in the classroom and the remainder of the course is taught online.

Introduction

Southeastern Louisiana University has 16,000 students who have an average ACT score of 20, and the mathematics department teaches college algebra to more than 1000 students each semester. Participation in the Renewal of College Algebra Project gave members of the Mathematics Department at Southeastern Louisiana University a chance to look at how they view the teaching of mathematics using alternatives to the standard lecture format which is so prevalent in today's classrooms. For more than 20 years the mathematics department has worked to define our college algebra course and find an effective way of delivering the material.

In 1990 placement into mathematics courses at Southeastern Louisiana University was done by ACT score. For those students who did not place directly into college algebra, the University offered three courses to prepare students for college algebra. The first two of these courses were transitional mathematics courses and were taught by faculty in the College of Basic Studies. The third course was Intermediate Algebra which was taught by the department of mathematics. By 1995, the number of these preparation courses had dropped to one of the transitional courses. The move to eliminate these two courses was driven by the fact that students were being required to take up to 9 hours of mathematics prior to college algebra. At this time, there was a great deal of discussion between the mathematics department and the rest of the university community about how to increase student success in mathematics. These discussions led to three tracks that students could follow to satisfy their college algebra requirement. Each department in the university chose which track they wanted their students to follow. Two of these tracks were the traditional college algebra and precalculus courses, and the third was a course entitled Explorations in College Algebra. This new course was developed by the department of mathematics in conjunction with members from other disciplines whom this course would serve. In 2004, the department took over the teaching of the transitional math course. Despite everything that we have tried, we still struggled with the problem of making our students successful without reducing our standards. Having struggled with this problem for so long, we were pleased to see that the MAA was addressing this problem through the CRAFTY Committee and the College Algebra Renewal Project and were glad to be chosen as a participant in this project.

Description of the Project

During the three semester college algebra renewal project, Southeastern Louisiana University taught 18 pilot sections and 12 control sections. There were 540 students in the pilot sections and 480 students in the control sections. The pilot sections were taught by two instructors, and one associate professor and the control sections were taught by two instructors. The pilot sections were taught from the text *Explorations in College Algebra* by Clark, Kime and Michaels while the control sections were taught from the text *College Algebra* by Dugopolski.

A Comparison of the Performance during the Project of the Control and Pilot Sections

Both pilot and control sections covered a common set of topics and objectives, and each semester there was a common portion of the final exam containing both skill and modeling problems. Students from the pilot sections performed better than the control students on the modeling portion of the final exam, and there was little difference in the performance of the two groups of students on the skill portion of the final exam.

For the purpose of this report, we tracked 140 students from two of the pilot and two of the control sections taught in the spring 2006 semester. Each of these four sections was taught by a different faculty member.

In the tables below we summarize the grade distribution for each of these sections.

Table 1. Grades of Students in Control Sections

Grade	Control Section 1	Control Section 2	Percentage
A	2	4	7.6
B	7	5	15.3
C	13	5	23.1
D	7	4	14.1
F	8	9	21.8
W	3	11	18.1

Table 2. **Grades of Students in Pilot Sections**

Grade	Pilot Section 1	Pilot Section 2	Percentage
A	1	3	6.4
B	2	5	11.3
C	2	3	8.1
D	2	4	9.7
F	16	14	48.4
W	7	3	16.1

It is clear from Table 1 and Table 2 that the students in the control sections received higher grades than those in the pilot sections, and the control sections had a significantly lower failure rate. One possible reason for this difference is that students are used to being taught mathematics in a traditional way, as was done in the control sections, and had never seen mathematics taught from a modeling approach. The fact that students in the pilot sections were required to not only find an answer to a problem, but to also be able to explain its meaning frustrated many of the students and caused many to give up without truly giving the new approach a chance. Also, it is not uncommon for students to simply stop coming to class rather than withdraw from the course due to external matters such as financial aid.

A Comparison of Post Project Performance of Students in Control Sections and Pilot Sections

We tracked the performance of these 140 students in mathematics courses taken after spring 2006. We have organized this data by the grades received in mathematics courses after the project as a function of the grade received in the project for both the pilot and control sections.

Table 3. Grades of Control Students in Subsequent Courses

College Algebra Grade	Number of Students with Indicated Grades						
	A	**B**	**C**	**D**	**F**	**W**	**No Math Class Taken**
A	2	1	1	0	0	2	2
B	2	6	2	1	1	1	3
C	2	0	4	2	5	11	4
D	1	3	3	1	6	4	1
F	0	0	3	0	10	2	8
W	0	0	1	4	4	5	6

Table 4. Grades of Pilot Students in Subsequent Courses

College Algebra Grade	Number of Students with Indicated Grades						
	A	**B**	**C**	**D**	**F**	**W**	**No Math Class Taken**
A	4	1	0	0	0	0	1
B	4	3	1	1	0	5	1
C	1	1	1	1	0	3	2
D	0	1	1	1	2	0	2
F	0	1	5	2	6	5	18
W	0	0	0	0	4	3	6

We see in Table 3 and Table 4 that even though the students in the control sections received higher grades than the students in the pilot sections during the project, there does not appear to be a significant difference in the performance of the two groups in subsequent courses. This suggests that the approach taken had little effect on the performance of the students in future math courses.

It is noteworthy that no pilot student who received an A, B, or C in the project course failed a future course, and there were only two grades of F among those pilot students who received a D in the project course. The only group of control students not to earn an F in any course post project was those who received an A in the project course.

What We Learned From Participating in the Project

Two of the three instructors who taught the pilot sections took away many ideas and techniques learned at the various workshops and through teaching 15 of the pilot sections. Our experiences have certainly had an impact on the way we teach and assess our students. We both use group work/projects to a greater degree in our classes, and also now put questions on our exams which are designed to see if a student can use modeling to solve a problem or if a student can solve a problem and explain its meaning in the context of the problem. For us these have been very positive outcomes that we have taken from our participation in the project. We also have the belief that students who stayed in our sections and did the work truly appreciated the modeling approach we used to teach the course.

College Algebra at Southeastern Louisiana Post Project

I do not think that the project ever had a chance to influence our college algebra course as a whole due to changes made at the state level regarding both the content and delivery of all college algebra courses taught at several schools in the State of Louisiana.

In 2007 we returned to a single track for college algebra. This was brought about in part by the department heads from all the state universities, who wrote a core curriculum to be followed by all state universities in Louisiana. To help insure the uniformity of the college algebra courses throughout the state, it was decided that schools adopt the text *College Algebra* by Sullivan which has an online component called *Course Compass*. Using the text in conjunction with the online component, homework, quizzes, and exams are performed by students on the computer. At Southeastern Louisiana University, students have two one hour lectures per week, and are required to spend at least 3 hours doing homework and quizzes in one of three computer labs devoted solely to our college algebra. We have made an effort to increase the success of our less prepared students by requiring all students with a math score of 20 or less on the ACT to take college algebra as a 5 hour course. This 5 hour course is not currently taught using the computer, but does require group work and projects.

I find it ironic that a few years after our participation in the College Algebra Renewal Project, the State of Louisiana did a sort of College Algebra Renewal of its own. While the two approaches to the teaching of college algebra are very different, they have common goals. The first goal is to increase student success in college algebra, and the second is to see this success continue in later math and science courses. It is my belief that there is not one single approach to the teaching of college algebra which will always be successful. Are either of these approaches the right approach? I doubt it, but only time will tell.

A Modeling Approach to Teaching College Algebra: What we Learned at UND

Michele Iiams and Richard P. Millspaugh
University of North Dakota

Abstract. Through our participation in the Renewal of College Algebra project we sought to determine whether teaching College Algebra through mathematical modeling affects students' basic algebraic skills, ability to solve algebraic application problems and to interpret information presented algebraically. In addition, we were also interested in the impact it might have on College Algebra success rates and the success of students in their subsequent mathematics course. We also gathered instructor and student impressions about teaching College Algebra in this manner. Based on the data collected and analyzed we decided not to adopt the mathematical modeling approach for our College Algebra courses.

Introduction

The University of North Dakota (UND) is the second largest university in the Dakotas. UND enrolls approximately 13,000 students in its 9 colleges: Aerospace Sciences, Arts & Sciences, Business & Public Administration, Education & Human Development, Engineering & Mines, the Graduate School, Law, Medicine, and Nursing. For the five years preceding this project an average of just under 5,000 students per year enrolled in math courses at UND. Nearly 1,100 of them enroll in College Algebra, which is taught in sections of approximately 35 students.

The audience for our College Algebra course changed several years ago when we stopped using it as part of our preparation for Calculus. We no longer have mathematics, science, and engineering students in this course. Instead, we have students from business, education, nursing, the social sciences, and aviation, etc. Many are preparing for business calculus or for a statistics course in their major, but many others are taking this as a terminal mathematics requirement. When we were invited to participate in the Renewal of College Algebra project, it seemed like the right time to rethink what we were doing in College Algebra. In addition, a couple of us have experience teaching a general education course which is largely a problem solving course. This approach to College Algebra sounded like it might be compatible with that course.

If our department was going to consider changing how we teach College Algebra, there were several questions we needed to consider.

- Does teaching College Algebra in a non-traditional way have any effect on students' basic algebraic skills?
- Will students in the project section be better able to solve algebraic application problems and interpret information presented algebraically than students in the control sections?
- How are the differences in teaching and learning mathematics with mathematical modeling perceived by our graduate teaching assistants, faculty, and students?

As would be expected, we were also interested in the effect of teaching College Algebra with mathematical modeling on our success rate in the course. Unfortunately, upon inspection of this data it was determined that extraneous factors influenced the results, which inhibited our ability to conclude the effects of this new approach on this success rate.

The control and project sections of College Algebra at UND were taught in dramatically different fashions. The project sections used *College Algebra: Modeling Our World,* (COMAP, 2002), incorporated graphing calculators, and relied heavily on group work. The control sections used the more traditional *Fundamentals of College Algebra* (Swokowski & Cole, 2005), did not incorporate technology, and instructors used more traditional teaching methods. For each pilot section taught by a graduate student, lecturer or faculty member we had a control section also taught by a graduate student, lecturer or faculty member.

Data

We measured the impact of this new approach on basic algebraic skills in two ways: 1) a common skills problem was included on the final exams for all project and control sections and 2) we tracked the students from our project and control sections as they took Applied Calculus, which is commonly taught in a large lecture format. To answer our question about application and interpretation of algebraic situations we also included two common problems on the final exams, for all project and control sections, which required students to demonstrate their abilities in these areas. We measured College Algebra success rates two ways. One common way of defining student success in a course is that they earn a grade of C or better. Since most students can satisfy major and/or general education requirements by receiving a D or better in College Algebra, and since we suspect that that is the goal for many of these students, we have also computed the percentages of students in both groups who have satisfied requirements. Finally, at the end of the semester we asked the instructors to write down their thoughts about teaching College Algebra with a modeling approach; and we gathered student opinions about learning in this way by posing the following questions on the course evaluation form:

- What advice would you give to students who enroll in the modeling-based College Algebra next semester?
- Do you think learning College Algebra through mathematical modeling has given you a better understanding than if you had taken a standard algebra course? Why or why not?

Basic algebraic skills

The problem involving symbolic manipulation was more difficult than the average problem students might encounter in this course. They were asked to solve a quadratic equation that first required them to square a binomial and collect like terms to write the equation in an appropriate form. Students from both control and project sections found this to be very difficult, with only 26% of the students from control sections and 21% of the students from project sections completing the problem correctly. Many of the difficulties involved squaring the binomial. Students from project sections were somewhat more likely, 33% versus 20%, to try to square the binomial by adding (or subtracting) the squares of the individual terms.

Applied Calculus is designed for students majoring in business, the social sciences, and a few others. The course can also be used to satisfy part of a student's general education requirements. Over two semesters, a total of 219 students agreed to participate in this study; 125 from the project sections and 94 from the control sections. Of these students, 109 students from the project sections and 81 students from the control sections took the final exam. In the end, 103 students from the project sections and 73 students from the control sections had earned a grade of D or better and were eligible to enroll in Applied Calculus. We are aware of 72 of them who have taken Applied Calculus at UND, 48 from project the sections and 24 from the control sections. We have not been able to determine how many of these students might have taken an equivalent course at another institution. There are 15 students in these groups who have attempted Applied Calculus at least twice since completing College Algebra, not including one student who currently has an incomplete in their second attempt. We are not able to determine whether students withdrawing from Applied Calculus were passing or failing at the time they withdrew. Since grades for the current semester have not been determined yet, this data does not reflect any Fall 2008 enrollments in Applied Calculus.

We report two sets of grades for these students, one based on their first enrollment in Applied Calculus and the other for their most recent enrollment. We normally define successful completion of a course as receiving a C or better, and that is how we have determined the success rates reported below. Since most students can satisfy their major requirements by receiving a D or better in Applied Calculus, and since we suspect that that is the goal for most of these students, we have also computed the percentages of students in both groups who have satisfied requirements.

Table 1 presents Applied Calculus grades for these students based on their first enrollment in Applied Calculus. Of the 48 students from project sections, a total of 16 received at least a C in Applied Calculus and 19 received at least a D. In their first attempt at Applied Calculus 33% of the project students were successful and 40% of them satisfied requirements. For the 24 students from control sections, 10 received at least a C and 15 at least a D in their first attempt in Applied Calculus, so 42% were successful and 63% satisfied requirements.

Table 1. Course Grades Earned in First Attempt of Applied Calculus.

	Course Grades							
Section	A	B	C	D	F	W	A–C	A–D
Project	2	6	8	3	8	21	33%	40%
Control	1	4	5	5	6	3	42%	63%

We next consider the most recent grades students have received in Applied Calculus, which are summarized in the Table 2. Before looking at these grades we note that, consistent with our hypothesis that students are largely interested in satisfying major requirements, only 1 of the 8 students who received a D in their first attempt in Applied Calculus has taken the course again. Of the 29 students from project sections who received an F or W in Applied Calculus, 9 (31%) have enrolled in the course at least one more time. Of the 9 students from control sections who received grades of F or W, 4 (44%) have enrolled in the course again.

In their most recent enrollment in Applied Calculus, 17 of the 48 students from project sections received at least a C and 23 received at least a D, so 35% have been successful in the course and 48% have satisfied requirements. Of the 24 students from control sections, 12 have now received at least a C and 17 at least a D, so 50% have been successful in the course and 71% have satisfied the math requirements for their majors.

Table 2. Course Grades Earned in Most Recent Attempt of Applied Calculus.

	Course Grades							
Section	A	B	C	D	F	W	A–C	A–D
Project	2	7	8	6	7	18	35%	48%
Control	2	4	6	5	5	2	50%	71%

To the extent that success in Applied Calculus indicates mastery of basic skills in College Algebra, these numbers give us cause for concern. We are especially concerned that so few of the students from project sections have been able to satisfy the math requirements for their major programs two or three semesters after taking College Algebra.

Algebraic applications and interpretation

The common word problem was a mixture problem involving health club memberships. Both groups of students did fairly well on this problem, with 81% of the students from control sections and 79% of the students from project sections giving correct responses. We did notice a significant difference in the way students approached this problem. Among students from control sections, 75% set up and solved a system of linear equations, 16% found the solution by “guessing and checking,” while 9% used some other method. Students from project sections were far less likely to use a system of equations. Among these students 24% set up and solved a linear system, 65% used “guessing and checking,’ and 12% found another method. We are not sure whether this difference indicates that students from the project sections were unable to set up and solve a system of equations, or whether it is due to some other factor. For example, some of the students from control sections may not have had access to a calculator during the exam, making checking possible answers more difficult.

There were three parts to the questions involving interpretation of graphs. Students were first asked to describe a sequence of events that might lead to the graph they were asked to interpret, the graph was either speed as a function of time during a trip or weight as a function of time over the course of a lifetime. Students from the project sections were slightly better at completing this task. Among those students, 61% came up with a description that accounted for all of the features of the graph and 86% were able to account for at least some of the features of the graph. Among the students from control sections, 55% accounted for all of the features and 83% accounted for at least some of them.

The second part of the questions asked students when they thought the dependent variable was increasing most rapidly and to explain their reasoning. Students from the control sections did slightly better on this question; 71% were able to identify a reasonable time and explain their choice, and another 15% were able to identify a reasonable time without an adequate explanation. Among students from the project sections, 66% were correct with an adequate explanation and another 11% chose a reasonable time but with an inadequate explanation.

Finally, students were asked to use the graph to estimate the value of the dependent variable at a particular time. Most of the students, 84% from the control sections and 88% from the project sections, were able to do this correctly.

Student and instructor impressions

Student comments about the modeling based College Algebra sections ran the gamut from "I found it interesting and fun" to "This style is LAME." Several students noted that they had to think more than in other algebra classes, which students do not always find to be a positive. As might be expected, some of our students were uncomfortable with a class that made them do anything other than memorize formulas, while others seemed to be happy that they were seeing some reason for knowing and applying the concepts. At least a few students didn't seem to think that this approach made them think that algebra was any more important or useful than more traditional courses. As one student put it, "It gave me practical applications but in certain places it didn't enhance my understanding of why we had to learn certain things."

When asked whether or not they thought this approach gave them a better understanding than a standard algebra course, student reactions were again mixed. Overall 54% said yes, but this number varied greatly from one semester to the next. Only 32% responded positively the first semester (Spring 2006), while 67% responded positively the next semester. We are not sure if this was because we had a different mix of students or because the instructors were better prepared and more comfortable with the class the second time they taught it.

We also kept track of the impressions and concerns of instructors who taught the modeling sections. These instructors valued the opportunity for students to learn mathematics through real-life contexts in a student-centered classroom and hoped that students would view mathematics as relevant outside the classroom. To some extent they believe that they achieved this goal. They were concerned that the text seemed more appropriate for good high school students than for college students and that they had to create so much of their own material to supplement the textbook. This group of instructors agreed that they would not choose to use this textbook again in the future. Instructors were concerned about the amount of preparation time the course required of them, especially the first time they taught it. This is a particular concern for the department because College Algebra is usually taught by graduate teaching assistants. Instructors found that students needed more technology support than they expected. They also weren't sure whether their students were motivated or persistent enough to take responsibility for so much of their own learning.

Our instructors felt the following questions remained unanswered for them.

- Are the modeling based approach and the use of student-centered learning activities necessarily related? Can more lecturing be used successfully in a modeling based course? Conversely, can more student-centered learning activities be used successfully in a more traditional course?
- Are we replacing symbolic manipulation without understanding with calculator manipulation without understanding?
- What do we need to do differently to help our students make connections from one context and mathematical concept to the next?

Conclusions

We didn't see any evidence on final exams to indicate that students in the modeling based sections of College Algebra had significantly worse algebraic skills than students in other sections. We also didn't see any evidence that they were significantly better, or worse, when it came to solving applied problems or interpreting graphical information. We have serious concerns about the poor performance in Applied Calculus of students from the modeling based sections.

The department also has serious concerns about the amount of effort exerted by instructors of the modeling based sections, especially given that College Algebra is typically taught by graduate teaching assistants.

From our point of view, adopting a modeling based approach for all of our College Algebra sections would involve more work for our graduate teaching assistants and more work for our faculty in support of this course. Since there doesn't seem to be any significant payoff for most of our students, and since some of them might actually take longer to complete the math required in their programs, we have decided not to change the way we are teaching College Algebra at this time.

Modeling Based College Algebra at Virginia Commonwealth University

Aimee J. Ellington and Bill Haver
Virginia Commonwealth University

Abstract. The Virginia Commonwealth University mathematics department offered pilot sections of a Modeling College Algebra course beginning in fall 2004, carefully assessed the experiences of students in this and subsequent courses, and continually modified the course to meet the needs of our students and instructional staff. Based on this experience, the department decided to offer, beginning in spring 2008, a Modeling Based College Algebra course to all of the approximately 2000 students who enroll in College Algebra in 65 sections each calendar year. Overall the instructors have been very pleased with the results. The DFW rate was reduced from 38% in traditional classes in 2004–2006 to 22.1% in the modeling sections in fall 2008 and, even though more students completed the modeling course, they were more successful in subsequent courses than students who completed the traditional course.

Introduction

Virginia Commonwealth University is a state supported university with an enrollment of 32,000 students. Students are required to take a placement test, and those that do not demonstrate the required achievement level are required to complete a course at the level of college algebra. Those who perform least well are placed in a "stretch" section that has mandatory attendance requirements and meets for an extra class day with the additional time devoted to reviewing basic concepts. Students typically enroll in sections of 30–35 students. Almost all sections of these courses are taught by graduate teaching assistants, part-time adjunct instructors, or full-time instructors on appointments of limited duration.

Prior to 1993 all students at this level enrolled in a College Algebra course that had the primary goal of increasing students' abilities to perform algebraic computations. The course emphasized adding algebraic fractions, solving equations for unknown variables, exploring exponential and logarithmic functions, and solving systems of equations. The end of each chapter of the textbook included some applications of skills the students had practiced, but these were not emphasized and were rarely tested. Computers and calculators typically were not utilized by the instructor, but students were permitted to use graphing calculators for homework and during testing.

Beginning in 1993 approximately 50% of the students who are enrolled in a course at this level – including most of those who are not required to complete a calculus course – have been taking Contemporary Mathematics instead of College Algebra. The Contemporary Mathematics course (Haver and Turbeville, 1995; Haver and Hoof, 1996) aims to enable students to study and comprehend quantitative situations that they have not previously encountered. The textbook is *Excursions in Modern Mathematics* (Tannenbaum, 2009).

Therefore, only students who intend to take any calculus course (business calculus, engineering calculus, etc.) enroll in a College Algebra course. Until 2008 the College Algebra course taken by most students focused on algebraic computations as described earlier. The three credit course met four hours per week, with those in the stretch sections meeting five hours. The non-stretch sections met four hours a week in classes of 35 students. Three of these hours are conducted in a traditional classroom and the fourth in a mathematics/computer laboratory.

Modeling-Based College Algebra

Modeling-based College Algebra sections were first offered on a pilot basis in fall 2004, with the same class size and meeting schedule. The modeling course uses *Contemporary College Algebra: Data, Functions and Modeling* (Small, 2008) as a text and is designed to meet the recommendations of the MAA College Algebra Guidelines (CRAFTY, 2007). The goals of the approach are to develop problem solving abilities, provide a foundation in quantitative literacy, focus on mathematics needed in other disciplines, and meet the quantitative needs of the workplace. Students are expected to address problems presented as real world situations and then create and interpret mathematical models. In the process, students make use of algebraic techniques and employ linear, exponential, polynomial, radical, and logarithmic functions. Students make extensive use of graphing calculators and Excel spreadsheets. A great deal of class time is devoted to small group exploratory activities and projects. Students are assigned two long-term projects. Approximately one half of a student's grade is based upon small group projects, homework, in-class activities, quizzes, and the long-term projects. The other half of the grade is determined by performance on hour exams and the final exam.

Offering Modeling-based College Algebra on a Pilot Basis

Because the Modeling-based course is so different from the traditional course, we offered the course on a pilot basis for a total of seven semesters. On the average, approximately six pilot sections and 31 traditional sections were offered each semester. During the first semester we held a four-day workshop for all instructors who would be teaching the course in future semesters. During the workshop we discussed our goals for the course, the content of the Precalculus and Business Mathematics courses students would be taking in subsequent semesters, and worked through many of the assignments students would be undertaking. The author of our text, Don Small, attended a portion of the workshop and discussed why various topics and assignments were included. We also held weekly meetings of all of the modeling instructors to discuss progress, refine the approaches we were taking, and design tests and other assignments. These weekly meetings continued throughout the seven semesters of the pilot program. Because the number of sections offered each semester was small and because different instructors taught pilot sections different semesters, many instructors developed ownership of the course that evolved at VCU. The co-authors of this paper each taught pilot sections as did the department chair, five other tenure/tenure-track faculty, many adjunct instructors and graduate teaching assistants. The pilot course was revised each semester, and as it evolved we developed new ways to address weak student computational skills through the use of practice and review assignments, without changing the major thrust of the course. We also struggled to refine the two longer term modeling projects that we assign each semester. Typically the students were asked to model world population or use of text messaging or deer population or housing prices over time and to use their models to predict future behavior. Our struggle was, and is, how to encourage student initiative on the projects by not being too prescriptive while at the same time making expectations for high quality work.

Assessment of Pilot Offerings

From fall 2004–fall 2007, the time frame in which different versions of college algebra were being offered at VCU, we continually evaluated our program using a variety of assessment measures. We gathered and analyzed grades in College Algebra each semester. During this time, we consistently found that more students in modeling sections received an A, B, or C in the course compared with their counterparts in traditional sections. College Algebra is a prerequisite for two courses at VCU: (1) a Precalculus course with a focus on skill-oriented content and (2) a Business Mathematics course with a focus on modeling-based content and applications. We conducted an analysis to determine whether or not modeling College Algebra was effectively preparing students to successfully complete these courses. During this pilot phase we found that students were more successful in precalculus after they completed a traditional section of College Algebra. On the other hand, the method of instruction (modeling or traditional) did not have an impact on the success rate of students taking Business Mathematics. Interestingly, students in modeling-based sections had a higher success rate in completing two mathematics courses when compared with students in traditional sections. In particular, 30% of modeling students and 26% of students in traditional sections completed College Algebra and passed a subsequent mathematics course. Interested readers can find more information about these assessment measures as well as others we conducted during the pilot phase including student performance on final exam questions and student attitudes toward mathematics as a result of taking College Algebra in several other papers (Ellington, 2005a; Ellington, 2005b; Ellington & Haver, 2007; Haver et al., 2007).

The faculty involved with lower level courses continued to value the exploratory learning, team work, and applications orientation of the modeling-based course. We were disappointed that students in the modeling sections appeared to perform less well in the traditional Precalculus course. We were pleased that the modeling students were much more successful in the College Algebra course. In the final analysis, the fact that we valued the goals of the modeling course and the fact that a higher percentage of students entering a modeling course ended up being successful in a College Algebra course and a subsequent course were critical for the decision to offer all sections with the modeling approach.

Offering Modeling-based College Algebra to 1144 Students in Fall 2008

Therefore, with the encouragement of the VCU administration, the mathematics department began offering all sections of College Algebra in spring 2008 using the modeling approach. But because enrollment in spring and summer sessions is small, our first major challenge was in fall 2008. After the add/drop period ended, we had 1144 students enrolled in 31 regular and 10 stretch sections of modeling College Algebra. The 41 sections were taught by two tenure/tenure-track faculty members, five full-time instructors, seven graduate teaching assistants and 20 part-time adjunct instructors. Of the 24 instructors, 12 had taught at least one section of the modeling class in previous semesters.

During the pilot period, the program had developed a substantial set of materials and approaches including skill review assignments, classroom activities, quizzes, and exams. Before the semester started a mandatory two-day workshop was conducted for all instructors introducing them to these materials and approaches with an emphasis on how to fully engage students in class activities and on grading exams and other student work. Five weeks into the semester another mandatory two-day workshop was conducted for all instructors, featuring an opportunity to review the goals of the course and to focus on how to provide support to students on their group projects. In addition, weekly meetings were held during which instructors critiqued the activities of the previous week and planned for future classes.

Results of the Current Assessment — Spring 2008–Spring 2009

With the switch to one instructional approach, we conducted another round of assessment to determine the impact of this change. This time we analyzed several questions from the student evaluation forms conducted at the end of each course at VCU, course grades, and grades in subsequent courses. For statistical analysis, we needed a group of students who took College Algebra using the traditional approach to compare to the current group of students learning through the modeling-based approach. In order to control for differences between semesters we compared data gathered from students in traditional sections for our final fall semester of the pilot phase (fall 2007) with data gathered from students from the first fall semester that all sections were taught using the modeling approach (fall 2008).

Questions from Student Evaluations

The VCU evaluation form contains 34 questions—most in the form of statements with which the students use a Likert scale with five options (*strongly agree, agree, neutral, disagree,* and *strongly disagree*) to provide their level of agreement. Many of the statements are specifically about the course instructor and classroom conditions. These would not be relevant for evaluating the College Algebra experience as a whole. However, we found several statements that we felt could shed light on the experience students have in the College Algebra course. The statements were about interest in the subject as a result of completing the course, competence in the area covered by the course, the value of classroom discussions, the ability to learn from other students, and the overall amount of learning achieved in the course.

For statistical analysis purposes, a numerical value (strongly agree = 5, agree = 4, etc.) was assigned to each response. We used the numerical values to calculate the mean and standard deviation for the student responses to each statement. We did this for all of the evaluations completed in fall 2008 and all of the evaluations completed by students in the traditional sections in fall 2007. T-tests were used to compare the student responses to each statement. The response rates for students enrolled in all of the sections ranged from 56.7% to 66.5%.

The mean values for these statements ranged from 3.38 to 4.21. Therefore, regardless of the method of instruction (modeling or traditional) the students were engaged in, they tended to respond with a moderate level of agreement to the statements. The results of the statistical analysis were mixed. For most of the statements, the students in traditional non-stretch sections responded more positively than students in non-stretch modeling sections. In contrast, students in stretch modeling sections responded to the statements more positively than students in stretch traditional sections of College Algebra.

The reasons behind the differences in stretch and non-stretch responses to these statements are unclear. With a high level of agreement for all students regardless of method of instruction, in general students were pleased with what took place in the College Algebra classroom. We plan to continue to explore reasons for the differences and analyze evaluation questions in the future to see if the differences continue to exist.

Course Grades

Table 1 contains the ABC rates for students enrolled in traditional sections of College Algebra in fall 2007 and in all sections in fall 2008. There was a statistically significant difference in the ABC rates for students in both stretch and non-stretch sections of the course. In particular, the modeling approach resulted in significantly higher rates. Also, the withdrawal rates for these two instructional formats were different. In fall 2007, 9% of students enrolled withdrew from a traditional section of College Algebra while in fall 2008 the withdrawal rate for the course was 7%.

Table 1. ABC Rate in College Algebra

	Traditional (Fall 2007)	**Modeling (Fall 2008)**
Non-stretch	69.7	80.8
Stretch	55.6	70.5

We have gathered data for spring 2009 where all sections offered again used the modeling approach. The ABC rates for students in non-stretch sections are consistent with those presented in Table 1 above. Specifically, 78.7% of students in non-stretch sections received an A, B, or C in the course. The ABC rate for stretch sections in spring 2009 was 66.4% which is slightly lower than the rate for fall 2008. The withdrawal rate of 8% in spring 2009 is also similar to the rate for the prior semester.

Grades in Subsequent Courses

With the switch to modeling-based instruction as the sole method of instruction in College Algebra, we needed to determine if this change had an impact on the success of students in courses for which College Algebra is a prerequisite. We compared the grades of students in traditional sections of College Algebra in fall 2007 who took a course for which College Algebra is a prerequisite (Precalculus or Business Mathematics) in spring 2008 with the grades of students who took College Algebra in fall 2008 and then took a subsequent course in spring 2009.

Table 2 contains the percent of students in non-stretch sections who were in one of these categories and received an A, B, or C in Precalculus or Business Mathematics. Based on a two sample test of proportions, there is a significant difference in the two percents for Precalculus. The students from modeling sections had a higher ABC rate ($p < .01$) in Precalculus than the students from traditional sections. With respect to Business Mathematics, while the ABC rate for students from modeling sections is slightly higher than the ABC rate for students from traditional sections, the two sample test of proportions revealed no significant difference ($p = .13$) between the two percents. Therefore, the ABC rates in Business Mathematics are similar for students from the two different College Algebra formats. For students in stretch sections of College Algebra, the ABC rate in Precalculus is 40% regardless of the instructional method of their College Algebra course. For Business Mathematics, 61% of students from traditional stretch sections received an A, B, or C in the course while the ABC rate for students from modeling stretch sections is 71%.

With respect to overall success (see Table 3) in two mathematics courses, 35.66% of students who enrolled in non-stretch modeling-based College Algebra in fall 2008 received an A, B, or C in that course and one of the subsequent courses in spring 2009. In the previous school year, 27.59% of students enrolled in non-stretch traditional sections of College Algebra in fall 2007 passed that course with an A, B, or C and one of the subsequent courses in spring 2008.

Table 2. ABC Rate in Precalculus and Business Mathematics for Non-stretch College Algebra Students, Spring 2008 vs. Spring 2009

Course	Traditional (Fall 2007)	Modeling (Fall 2008)
Precalculus	48.63	66.90
Business Mathematics	73.33	75.36

Based on a two sample test of proportions, the percent based on students in modeling sections is significantly larger than the percent based on traditional sections ($p < .01$). There was no significant difference in the percents calculated from the grades of students in stretch sections. Twenty-one percent of students from stretch sections (modeling and traditional) completed College Algebra and a subsequent course.

Table 3. ABC Rates for a Subsequent Course, Spring 2008 vs. Spring 2009

	Number enrolled in College Algebra	Percent who completed Precalculus or Business Mathematics
Modeling F08	830	35.66%
Traditional F07	522	27.59%

Conclusions

During the pilot phase, our assessment revealed that the modeling-based approach to College Algebra did not have negative effects on our students taking the course. In fact, in many respects, students in modeling-based sections performed as well or better than their counterparts in traditional sections of the course. In particular, more students from modeling sections were successful at passing College Algebra and a subsequent course when compared with their counterparts in traditional sections.

The change in method of instruction to a modeling-based College Algebra course has had a significant, positive impact on our students. There has been a meaningful increase in the number of students passing College Algebra since we started only offering modeling sections of the course. An even more profound comparison can be made if we consider the students enrolled in large lecture sections of College Algebra prior to fall 2004. At that time, the ABC rate for students in large lecture sections (i.e., 175 students per section) was 36.1%. When we reduced the class size but used a traditional approach to teach the course, the ABC rate was 62%. With an ABC rate of 77.9% in fall 2008, we have more than doubled the percent of students who are successful in College Algebra (see Table 4).

Table 4. ABC Rate in College Algebra

	Large Lecture (Fall 2003)	Traditional (2004–2006)	Modeling (Fall 2008)
ABC	36.1	62.0	77.9

The percent of students passing a subsequent course has increased as well with the impact being most profound on Precalculus. During the 2008-2009 school year, 35.66% of students who enrolled in modeling College Algebra successfully completed that course and the subsequent course for which College Algebra is a prerequisite. We are encouraged by the fact that more students successfully completed both courses in one school year as compared to any other point in our assessment of College Algebra.

We plan to continue to evaluate our College Algebra program. We plan to investigate the differences in responses on the student evaluation forms. We also plan to continue to analyze the grades of students in our course and in subsequent courses. We will provide further details of the assessment outlined here and of our future efforts in other publications.

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Addressing the College Algebra Problem at Oregon State University

Barbara Edwards and Ching-chia Ko
Oregon State University

Gulden Karakok
Umea University, Sweden

Abstract. Oregon State University is a land, sea, sun and space grant institution with an enrollment of approximately 20,000 graduate and undergraduate students. For many years we have struggled with a College Algebra course that enrolls about 1600 students per year but has had a D, F, W rate of approximately 50%. In 2006 we instituted a new course, Algebraic Reasoning, based upon the CRAFTY Guidelines for College Algebra. The three authors of this paper were part of the group that created this course and in this paper we will briefly describe that course and show some results from grade data gathered between 2007 and 2009. Although this course is only part of an overall plan, it seems to have been successful and has resulted in a higher pass rate for the College Algebra course itself.

Introduction

The College Algebra Problem is, by now, well-known: the 50% failure rate, students' views of pointless content, the crushing schedule of trying to "cover" so much material in so little time… all factors that guarantee that many students will leave their undergraduate experience with few fond memories of mathematics (Gordon, 2008). At Oregon State University, although College Algebra is the largest course, in terms of enrollment— approximately 1600 students enroll annually—the reputation of this course among students is not good. The course is taught in large sections of 200–300 students meeting three times a week with addition once-a-week 35-student recitation sections led by graduate teaching assistants. Both the text and pedagogy used in College Algebra are traditional and teacher-centered (Boaler, 2002). Until recently the failure rate for this course at Oregon State matched the national average of approximately 40–50%.

College Algebra as a course is not well defined nationally. At Oregon State our academic year consists of three ten-week terms and College Algebra is the first course in a two-course sequence that is designed for students who will take Calculus. Typically the two courses have been taught from the same Pre-Calculus textbook and the second term is the trigonometry content. The first term of the sequence is deemed sufficient as preparation for students who will take "non-engineering" calculus, a one-term course primarily for business and life-science majors. Both courses in the sequence are necessary for students who will take engineering calculus.

Addressing the College Algebra Problem

During the 2005–6 academic year the Mathematics Department and the College of Science began to study issues related to freshman-level mathematics courses on campus and the high failure rate in College Algebra emerged as a problem that needed to be addressed. Many students who enter Oregon State have taken the SAT; those who know that they will take a mathematics course are encouraged to take a Mathematics Department placement test before choosing which course to take. Scores on the math placement text and the Math SAT are “advisory” for students; they do not lead to mandatory placement. Based upon these two scores approximately 35 percent of entering freshman each year would be advised AGAINST taking College Algebra (or anything higher) as their first mathematics course. Before 2006 these students were advised to take Intermediate Algebra from Extended Studies (outside the Mathematics Department) or at the local community college; however, since students do not receive college credit for Intermediate Algebra and there is an extra tuition fee not covered by financial aid, many chose to take College Algebra anyway. The anecdotal strategy was that they would take College Algebra multiple times, if necessary, to pass it.

Historically college algebra was created as a quick review of important ideas from high school algebra for students who needed this review before taking calculus in college. However, the current situation is that college algebra is a requirement for students who may not plan to take calculus. In fact data from the fall 2008 college algebra class in Table 1 show an interesting picture. In 2008–9 more than 70% of students who passed College Algebra with an A–C did not take either “Business” Calculus or “Engineering” Calculus during the two terms that followed their College Algebra course. It is possible that the content of College Algebra is no longer appropriate for most of the students in the class.

Table 1. Students’ Course Following College Algebra

Math Course Following College Algebra 2008–9	**Number of Students (percent)**
Calculus[1]	90 (16%)
Calculus for Business and Life Science	67 (12%)
Another Math Course[2]	193 (33%)
No Subsequent Math Course	225 (39%)
Total A–C in College Algebra Fall 2008	575 (100%)

There may have been many reasons for the high failure rate and the student dissatisfaction with College Algebra, but we identified three: many students were not adequately prepared for a brisk review of high school algebra, there were far too many students in each section, and the goals of the course might not match the needs of the students. Our plan was to create two courses to replace College Algebra in such a way that some students would take both courses while the other more well- prepared students would take only the second course.

In spring and summer of 2006 we began by creating the first course for students who had weaker mathematics backgrounds (determined by SAT scores and math placement test scores). We hoped this course would be attractive to students—it would carry college credit, meet financial aid requirements, and be limited to 35 students per section so that students could know and be known by their instructors. It also seemed appropriate that the course should be based upon sound educational research and meet the then developing MAA-CRAFTY Guidelines for College Algebra (MAA, 2008). We determined that the course would not be a skills-only course, but that it would be one that emphasized modeling using the ideas of rates of change, functions and function representations. Algebraic skills would be addressed as needed but not in isolation. In creating our activities we concentrated on the student difficulties in these areas reported in the research literature (Janvier, C, 1996, Monk, S., 1992, Trigueros M. & Jacobs S., 2008) We also decided that the course would be student-centered rather than lecture-based (Boaler, 2002) and that an important aspect of the course would be weekly meetings for instructors to discuss the progress of student learning and to address related issues.

The resulting course, Algebraic Reasoning, is a student-centered, modeling-based course. It is student-centered because faculty rarely lecture, and students are encouraged to think deeply about mathematical ideas, to grapple with their own developing understandings, and to communicate mathematics with each other in small or larger groups. It is modeling-based because students work with data and model real situations mathematically rather than memorize facts

[1] These students took the trigonometry component of Pre-Calculus before enrolling in Calculus.

[2] Not Pre-Calculus. These were courses such as Contemporary Mathematics, Discrete Mathematics, Math for Elementary Teachers.

and formulas with little understanding in order to pass the next exam and survive the course. We assign one out-of-class project each term for which students work in small groups to gather data, analyze the data, write a professional-looking report and give a short oral presentation. Our goals are:

- to help students develop the self confidence, independence and strategies for learning mathematics that will help them do well in later mathematics courses and in life,
- to help students realize that mathematics is useful in the world and that real situations could be modeled using mathematical functions,
- to help students develop a deep understanding of important key concepts related functions and rates of change, and
- to help students understand the logic of symbolic manipulations.

For economic reasons we have been unable to proceed with the second course in our plan. Currently students who pass Algebraic Reasoning go on to enroll in the traditional large-lecture College Algebra. However we seem to have fixed part of our problem since the average SAT or placement test score has increased in the College Algebra course and the failure rate has decreased. Our assessment of the success of the course has involved analyzing grade data, analyzing student responses on short-answer questionnaires to assess student attitudes and beliefs about mathematics and our course, and conducting and analyzing interviews to determine student understanding about the main ideas of the course. In this paper we will discuss our analysis of the grade data and our attitude surveys. First we discuss the grade data.

Grade Data Analysis

We offered eight sections of Algebraic Reasoning in fall 2006 and five sections in winter 2007. During the summer all freshmen whose scores indicated they were underprepared for the regular College Algebra course were strongly encouraged to take Algebraic Reasoning. During the first year of the Algebraic Reasoning course (2006–7) we noted that our withdrawal rate (after the first week of classes) was consistently below 5% and that 80–90% of our students received an A–C. In fact in fall 2006 we began with 280 students and 275 of those remained in the course until the end of the term. Our first comprehensive look at grade data was from the 2007–8 academic year. We wanted our students to be successful in Algebraic Reasoning but we also wanted them to do well when they took College Algebra even if it was the traditional version. We studied the grade data from the winter 2008 College Algebra class. Table 2 shows those results. The overall A–C passing rate for the course that term was 63% but the A–C rate for students who had taken Algebraic Reasoning is much higher than that of students who had not taken Algebraic Reasoning.

Table 2. Students Receiving A–C in College Algebra, Winter 2008

Students (Number)	Number of A–C (%)
All Students (454)	286 (63%)
With Algebraic Reasoning (110)	86 (78%)
Without Algebraic Reasoning (344)	201 (58%)

The data for the 2008–9 academic year is more robust. We looked at all students who, in the fall, enrolled in either College Algebra or Algebraic Reasoning. We noted their SAT math scores and the grades they received in the fall. Then we followed them through the next two terms noting the classes in which they enrolled and their final grades.

Table 3 gives a summary of Fall 2008 enrollment in Algebraic Reasoning and College Algebra, students' average Math SAT scores, those who received an A, B, or C in their respective course and the average Math SAT score for the A–C students.[1] Note that the A–C pass rate for College Algebra is 69.6%, an improvement over previous years. We have assumed that this improvement is due to fewer underprepared students enrolling in the class.

[3] In this report we have used SAT scores for a comparison of the three groups of students. In the past we have used Math Placement Test scores but in 2008 the placement test was given on-line for the first time and there are some issues to be addressed before we can determine the validity of those scores. It should be noted however, that we did not have SAT scores for all students.

Table 3. Grade/SAT score Data for Fall 2008

Course	Number of Students after Add/Drop Date[4]	Average Math SAT Score	Number of Students Receiving A–C	Average Math SAT Score for A–C Students
Algebraic Reasoning	261	486.7	214 (81.9%)	493.3
College Algebra	575	527.2	400 (69.6%)	543.8

Table 4 shows the students who were “eligible” to take College Algebra after Fall 2008 (those who passed Algebraic Reasoning and those who received a D, F, or W in College Algebra fall term), their average Math SAT scores, the number who enrolled in College Algebra (either winter or spring) and the number who passed College Algebra with an A, B, or C the first time they took it.

Table 4. Algebra Grades Winter and Spring 2009

	Students “Eligible” from Fall	SAT Average	Enrolled in Algebra Winter or Spring 2009 (%)	Received A–C in Algebra (%)	A–C Students To Column 1 (%)
Students with A–C from Algebraic Reasoning	214	493.3	181 (84.6%)	104/181 (57.5%)	104/214 (48.6%)
Students with D,F,W from Fall College Algebra	175	490.1	59 (33.7%)	31/59 (52.5%)	31/175 (16%)

Table 4 shows that that in 2009 slightly more students from Algebraic Reasoning earned A–C’s in College Algebra after one attempt than did students who were taking College Algebra for a second time. However, the last column of the table shows that students with essentially the same math SAT score were much more successful if they took Algebraic Reasoning before College Algebra than if they took College Algebra and failed. Although 52.5 % of those who took College Algebra a second time passed it compared to 57.5% for students from Algebraic Reasoning, only about one-third of those who got a D, F, W in College Algebra in the fall attempted to take the course again during the year. One should question the feasibility of the “student strategy” that calls for taking College Algebra multiple times until one receives a passing grade.

A second aspect of the assessment of our new Algebraic Reasoning course involves the attitudes of our students toward mathematics in general and the course in particular. In the next section we provide some examples of student comments from surveys that have been completed by students at the end of the Algebraic Reasoning course.

Student Attitudes

The A–C pass rate for Algebraic Reasoning could indicate that most of our students are happy about our course and willing to consider mathematics in a new way. This is sometimes not the case. Many students struggle with working in groups, having to think hard about non-standard problems without being given right answers or lots of hints, or realizing that there might be several correct ways to do a single problem.

[4] The number enrolled on the first day of the term in College Algebra was 840 students. Thirty-two percent of these students did not appear on the final grade list. We compared this data to an original 290 students who were enrolled in Algebraic Reasoning at the first of the term and for that course 29 students or 10% of the students were not on the final grade list. In the future we will determine if the high early withdraw rate is a pattern in College Algebra, and if so, find why it occurs.

The attitude surveys reveal that while some students like certain aspects of the class there are others who equally strongly dislike the same aspects. On the question of the student-centered nature of the course, students weighed in on both sides.

- I like being able to work in groups to bounce ideas off each other & check our work.
- [I didn't like] group teaching. It was different. I think I like lecture better.
- I like interacting with my group and working things out with them. I liked being able to present to the class.
- I didn't like the teaching style at all. I prefer to be taught how to do something and not prompted to figure it out myself.
- [I didn't like] always doing everything in groups.
- I disliked having to "discuss" the math with peers instead of being taught the section, and then doing the work.
- I think the group project really helped me apply what we learned in class and stretch my knowledge.
- [I liked] working on the chalkboard to show the class ideas.

Student comments also varied on the notion of what doing mathematics means. This is perhaps one of the most difficult ideas we promote: that doing mathematics means making assumptions, trying things that may not work very well, finding several ways to reach an answer, and having to justify ones answers.

- [I] didn't like the structure of this class. Math is either right or wrong, so giving long explanations for each problem…. I can learn the material without those structured course "rules."
- Always having to justify/explain my work helped me understand it better.
- Being able to explain why I was doing the things I was doing helped a lot.
- This class helped me think about math in a different way, I am now able to think about many different ways to solve equations that just one.
- I am a believer in math that there is 1 right answer.

We continue to address student attitudes and although we feel that if we had students for twenty weeks we might make more progress we are slowly finding ways to ease students into our pedagogical style.

The Future of College Algebra at Oregon State

There is a mismatch between the traditional nature of our College Algebra course and the non-traditional nature of Algebraic Reasoning. Our philosophy for creating the Algebraic Reasoning course as we did was that if students' attitudes toward and beliefs about mathematics could change and especially if they could experience mathematics as a personally useful and interesting area of study, they would persevere even if their later courses were traditional. Ten weeks is not enough time to accomplish this goal for many of our students and we continue to believe that a two-term student-centered, modeling-based College Algebra course would be the preferable answer to our College Algebra situation. The current financial situation has delayed our hopes to investigate that possibility but we have been encouraged by the administration to continue to plan and revise.

Conclusion

Preliminary data indicates that the newly developed Algebraic Reasoning Course has been successful in many ways. As reported in Table 2, students who enrolled in College Algebra after completing Algebraic Reasoning were significantly more successful than those who enrolled in College Algebra without completing Algebraic Reasoning. And as indicated in Table 4, students with low SAT scores have a much better chance of ultimately being successful in College Algebra if they first enroll in Algebraic Reasoning before enrolling in College Algebra.

There are many more aspects to consider in evaluating our courses and the College Algebra situation at Oregon State in general. There is an articulation problem between courses in the present situation. Algebraic Reasoning adheres to the CRAFTY Guidelines for teaching College Algebra, but our College Algebra course does not. Our engineering Calculus Course is also somewhat nontraditional so students could spend one term in our non-traditional course, two terms in traditional pre-calculus and then back to a non-traditional approach for Calculus.

The challenge of keeping up with the necessary professional development for our instructors and graduate teaching assistants who help us teach these courses is another aspect of this project that deserves assessment and discussion. The people who teach Algebraic Reasoning meet weekly during the term to discuss ideas and problems related to the course. Since instructors and GTA's who are unfamiliar with our approach are reluctant to be involved with the course without some introduction to the way in which it is taught, we have scheduled a course and seminar for fall 2010 to provide a comprehensive introduction to teaching college algebra according to the CRAFTY Guidelines.

Finally more research is needed to determine exactly what is working in the Algebraic Reasoning course and why it works, and how we can fix the things that are not working. Since we taught eight sections of the course for the first time in fall 2006, the course has continually developed and changed. In spring 2008 we taught five sections of an experimental second term (students received credit for College Algebra) of the course, but as previously mentioned financial constraints have put the two-term model on hold. We are currently collecting data addressing this experimental course and what happened to the students from these sections after they completed it.

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Studio College Algebra at Kansas State University

Andrew Bennett, Rachel Manspeaker,
Rekha Natarajan, and Jennifer Paulhus
Kansas State University

Background and Introduction

College Algebra is a required course for almost all students at Kansas State University. While many students place into higher level courses, students from every major enroll in College Algebra. Students in this course may be heading to engineering calculus, business calculus, or statistics courses; or College Algebra may be a terminal course. Placements have historically been made based on high school records and ACT scores. This process is not flawless; often a few college algebra students will have already taken calculus in high school while other students would possibly be better served by taking Intermediate Algebra first. The course serves approximately 2000 students per year, offered as two large lectures a week (about 300 students per lecture) and one recitation hour with a Graduate Teaching Assistant (GTA) with 20–40 students in a recitation. The C or better rate has historically ranged between 55% and 66% depending on the semester. This is slightly above the national average for College Algebra, but is low compared to most other freshmen courses at KSU. Student evaluations in the past have been lower than in other freshmen courses as well, and attendance in the large lectures would often drop below 20%. In addition, the New Student Services office noted that the poor reputation of the course among freshman had a negative impact on recruitment.

The mathematics department and central administration undertook a revision of the course with the goal of making College Algebra a more successful and satisfying course while maintaining its academic rigor. With the number of interested parties, there were a variety of expectations and constraints. Given the wide range of students, it was decided to offer two different types of College Algebra. Traditional College Algebra would continue in the same format as before to serve the needs of those who were anticipating enrolling in engineering calculus. This addressed the concerns of those who felt their students were well served by the current course (and there were some of those individuals) and also calmed the fears generated by making major changes to the largest course on campus. Studio College Algebra would be geared toward preparing students for business calculus and statistics courses, and would cover slightly different material. The expectation was that the new course would utilize modern technology. The focus of the class was to be on how to use mathematical ideas, emphasizing applications to various other disciplines.

Resource limitations placed several restrictions on the new design. The course would eventually need to accommodate 700 or more students a year. It would not be possible to offer the course solely in small sections (both because of limited budget and limits on the number of classrooms available). The math department would need to be able to assign new GTAs to teach the non-lecture portions of the course in their first semester of graduate study and so the instructional techniques would need to be successful even in inexperienced hands. Resources were available to hire a full-time College Algebra coordinator to concentrate on the course, and this has proven quite beneficial.

Inspirations

The CRAFTY Guidelines for College Algebra and conversations with members of CRAFTY and participants at several of the Curriculum Foundations II workshops provided a base for ideas about how to meet our goals within the limitations that existed. The quote that provided our main inspiration for the course however was from Galileo: "Mathematics is the language in which God has written the universe.” Students are required to take College Algebra so that they can understand when ideas are presented in mathematical language in their other courses. So we approached teaching Studio College Algebra as a language course. Learning a language has several components. Students need to learn vocabulary, grammar, and spelling. These are generally learned through direct instruction and repetition. In our context, we considered the various standard algebraic manipulations and skills, such as factoring, finding the vertex of a parabola, etc., to be the basics that would be mastered this way. However, most people can not learn a language just by studying word lists and grammar rules. Language courses also include discussion sections where students use the language. This provides a context for understanding and also additional repetition of the basic elements. To address this need we had to provide situations where students would work through more involved and often “real-world” problems using their mathematical tools, and working in small groups (usually two or three students) where they would need to converse mathematically. These situations would be chosen to help students develop a conceptual understanding of the mathematics they were learning.

Course Structure and Development

The new Studio College Algebra format includes one large lecture (300 students), one recitation (20–40 students), and one *studio* (40–60 students) each week. All sessions are 50 minutes long. During large lecture, concepts, procedures, and examples of required skills are addressed, usually covering about 2 sections of the text in one session. During recitation, students meet with a GTA to work on homework problems and ask questions. Students are welcome to ask questions about all parts of the course. They also have a chance to work together on assigned and sample problems. During studio, students apply the concepts learned in lecture by working with real world data and modeling techniques. Primarily using spreadsheets to manage large amounts of data, students work in groups of two or three, while two instructors and one undergraduate assistant walk around the studio room addressing questions. This format allows new GTAs to be paired with more experienced mentors as they adapt to teaching in a studio environment.

During the first year of development, we taught one or two experimental sections of 20-30 students per semester, using borrowed facilities from the Department of Statistics for the studios. We moved to the large lecture format in fall 2007 with a limited number of sections, since we were restricted in how many students could be accommodated in our borrowed facilities. We were originally scheduled to have our own facilities in fall 2008, but construction was delayed, not least by a tornado that struck the math building on June 12 of that year. To accommodate the larger enrollment due to expectations of the new facilities, we were able to borrow additional space from the Department of Elementary Education. However two studios were forced to be run in regular classrooms that had no computer facilities. We asked students to bring their own laptops, provided several laptops for students who did not have their own, and asked students to share their laptops and to work together. We rewrote all the instructions for the various spreadsheet programs and operating systems that students brought with their own laptops. Working with the students’ personal laptops led to some interesting experiences. One studio included students with the Spanish, Arabic, and Japanese versions of Excel.

In the middle of spring 2009, new studio classrooms were inaugurated in the math department, with 20 computer stations and a two unit computer projection system with attached visual presenter (an Elmo). Primarily intended for Studio College Algebra but now used for a variety of math and physics classes, the new studio rooms have facilitated the goals of Studio College Algebra very well.

Student Activities and Assignments

We provide an active learning environment during lecture by the use of “I-clicker” technology. Students are required to bring a clicker (which are used in other classes besides College Algebra) to lecture. They answer three multiple-choice questions each lecture and receive a point for each correct answer, as well as an attendance point for submitting their answers, right or wrong. Students may work together on clicker questions, though the time to answer such questions is usually about 1 minute. The lecturer gets immediate feedback on how many students clicked each answer and can address common misunderstandings immediately. This system has helped with both attendance and student engagement.

To provide as much practice with basic skills as each student needs, they are assigned online homework from a locally developed system. There are 19 such assignments in the 15 weeks of the course. For each assignment, the system algorithmically generates 2–7 problems covering the topic (e.g., graphing parabolas, linear inequalities, matrix operations). Each student gets a different set of problems. Depending on the topic, answers may be submitted as numbers, formulas, or graphs (students can draw graphs in a Java applet). All problems are automatically graded by the computer. The first time students submit their answers, they are told which answers are correct and are given a chance to correct any errors. The second time, they are told the correct answers and given a link to a page that shows step-by-step instructions on how to solve their problem. This page is also algorithmically generated and gives the solution to their specific problem, not a generic example. In some cases, there is also a link to a video of the section of lecture where the relevant topic was discussed. After this, the students can have the system generate another problem set and try again. Their best score over all attempts completed before the due date is recorded. After the due date, students can still generate and work problems but they no longer count toward their grade. Some students find this helpful in preparing for exams.

In addition to the online homework, there is weekly written homework from the textbook. These assignments consist of 5–10 problems, with almost all problems being "word problems". These are chosen to give students practice with applying the skills in a clear and more limited situation than the studio material. While students can ask about any material in recitation, most of the time in a typical recitation is devoted to the written homework. Different recitation teachers handle their sections differently, with some having students work together or present material at the board, while others use a more traditional approach where the instructor works similar problems at the board.

Since conceptual understanding is one of the main goals of Studio College Algebra, several of the studios require students to look for patterns and answer questions that address why certain procedures work. Many of our studios are based on an inquiry approach, and students work together in groups of two or three. Students mainly use spreadsheets instead of graphing calculator technology, which is one of the main differences between this course and Traditional College Algebra. A brief description of some representative studios is given below.

Linear Models

This is one of the first studios. Students are introduced to linear functions in the context of baseball. They analyze Major League Baseball's regular season statistics and calculate run differentials (runs scored – runs allowed). They plot actual number of wins versus run differential to determine how many wins one run is worth. Depending on which season's data is used, a typical best-fit line is *Wins* = 81 + 0.1**Run differential*. Students are asked why it is reasonable to have an 81 in the formula (given the hint that the season is 162 games long) and also asked to determine how many additional runs are needed to produce 10 additional wins. This requires them to recognize the role of the y-intercept and that slope is a rate-of-change. Students also plot residuals to determine whether teams are lucky or unlucky. With this studio, students have a chance to move beyond just finding a line to interpreting what a linear model says about a situation. At the end of the studio, we try to discuss how these general notions of finding and interpreting lines apply to other settings in business and social science. The choice of baseball as the context for the studio owes both to the interests of the first author and to the fact that the data is easily available and, in most recent years, gives a nice slope that rounds to 0.1, where most real-world situations lead to "uglier" numbers. There is plenty of time for students to discover that actual data leads to messy formulas in later studios; for the early studios we try to keep things simple.

Transformations of Graphs

When we start discussing transformations of graphs, students explore shifts and stretches with the use of sliders that let students interactively adjust values in the spreadsheet by moving a marker with their mouse. They are given the graph of a fixed function $f(x)$ along with $a\,f(x + \mathrm{c}) + s$, where the values of a, c, and s can be adjusted by manipulating sliders on the spreadsheet. The experience they have in this studio is more tactile; as they move the slider to the right to increase c, for instance, students can watch the graph of $f(x + c)$ move to the left. Having completed this studio, students are then required to answer some online homework questions on translations where they must use what was learned in studio to complete the assignment.

Polynomial Models

The goals of the polynomial models studio are to give students an opportunity to explore polynomial models, to determine the appropriateness of a model via scientific reasoning, and to fit an appropriate curve to a scatter plot, then use the resulting function for prediction and analysis. Students look at a pre-programmed (3rd degree) trendline with randomly generated errors and fit 3rd and 6th degree polynomials to the observed data (trend+error=observed). They are guided to recognize that while the higher degree polynomial fits the observations more closely, it is "modeling the noise," while the lower degree polynomial is usually (but not always) a better model of the actual trendline. They also explore the effect of adding an erroneous value to the data set, demonstrating that the lower order polynomial provides a more robust model. We discuss how polynomial models are useful when there is a need to model data with turning points, but that with the spreadsheet the degree of the polynomial must be chosen and this is a non-trivial task. After this, the students try to find the "best" model for the FBI's Uniform Crime Statistics reports of "murder and non-negligent manslaughter" in the United States for the years 1985–2004. The studio is generally successful in getting students to recognize the role of turning points and polynomial models, but is unfortunately much less successful in teaching the principle of parsimony in choosing the degree.

Complex Graphs Studio

After being introduced to rational functions and the notions of zeros and poles, students work on a studio about complex graphs. To visualize these graphs, students use the Complex Function Grapher (Bennett 2001). With input and output each involving two dimensions, the grapher uses color to represent one of the output dimensions. Students can enter various functions, click on a top view and a side view to find poles and zeros, and answer various questions about multiplicities of zeros and patterns in the rainbows that are seen. In the end, students are asked to create their own "pretty picture," identifying the poles and zeros of their own function. In addition to strengthening conceptual understanding, we also hope that students see an instance of where mathematics is indeed beautiful. We tell the students that just as in a French class they might look at French art and eat French food to appreciate French culture, we want them to get a sense of mathematical culture and why we consider mathematics the most beautiful of subjects.

Student Reactions

Some students from the studio sections of College Algebra were interviewed concerning their reactions to the course. As part of a separate study, all students in the studio version of the course were separated into five clusters based on data collected from the first month of the semester. Ten students were selected as representatives of each of the clusters to be invited for interviews, ensuring a diverse range of opinions and skill levels. Nineteen students accepted the invitations, and were interviewed about topics ranging from general opinions about mathematics and Studio College Algebra, to study habits and conceptual strength.

Several common trends in student reactions emerged during the interview process. First, students tended to react strongly to the Studio part of the course. Many students very much enjoyed working in the labs and found the assignments insightful and challenging. One student liked studio because he could "*work a computer and like, a program like Excel. Put in information, have it extrapolate out of that information, change it — do all kinds of cool stuff*." Others thought studio assignments were confusing and disjoint from the rest of the course or what they thought of as mathematics. For example, when one student was describing the studio assignment about complex graphing, she said "*I kind of get what goes with the lesson, but mostly not — it was like, imaginary numbers with real numbers and colors and that was just like, numbers don't equal colors. I was so upset, I couldn't stand it*." Another was equally frustrated with Studio, saying, "*It just seems like busy work. I don't — nothing I learn in there seems to really help me learn any other concept that we learned in lecture that week. It doesn't help me do my homework*."

Almost all of the students appreciated using Excel and computer technology in general, even if they were not particularly comfortable or familiar with computers. One student said, "*I enjoy my studio. It's nice to do something on the computer and I didn't know how to make graphs and stuff on Excel as well as I do now*." Another asserted, "*Lab is also beneficial for, like I said, I mean — nowadays, I sound like such an old person, though. Nowadays, you know, a lot of stuff is using computer programs, so — you know, in the future that will be the only way to do it*."

The online homework assignments received mixed reviews. Many students appreciated the opportunity to practice and achieve a score of 100%, while others were annoyed that the algorithmically produced problems changed with each attempt. Examples of these reactions include, "*the online homework's a really good setup too, 'cause it really… it simulates test questions, but if you don't get it right off it doesn't penalize you. And it lets you really — it — it forces you to understand it.*" and "*I don't like the way if you do something they keep making you do it until you get it right — then you have to start all the way back over and do the problem over*." Survey data shows that the students who liked online homework outnumbered the ones who did not. On the most recent course survey, 73% agreed or strongly agreed with the statement, "I find the usual online homework helpful in learning algebra." By comparison, only 66% agreed or strongly agreed with the matching statement about the written homework.

One of the most surprising results that came out of the interviews was the issue of placement. More than half of the students interviewed said they were placed in Studio College Algebra because their "*advisor said it would be easier*" than the regular course, or they heard from friends that "*you could get, like, extra help if you, like, struggled with math*." The rest of the students did not know there were two varieties of College Algebra, and so they did not know they were making a choice in signing up for the studio version. Because the Studio and Traditional versions of College Algebra are designed to be equally challenging but with different goals, neither of these methods is appropriate for placing students. Advisors need to be provided with more detailed descriptions of the course differences and expected outcomes.

Results

The studio version of College Algebra has proven successful both in terms of student achievement and student satisfaction. In view of the different environments we have been forced to use, as described above, it is interesting that the outcomes have been fairly consistent between semesters.

Semester	Type of Class	Enrollment	C or better	Percentage
Fall 07	Studio	105	83	79.0%
	Traditional	1149	763	66.4%
Spring 08	Studio	78	50	64.1%
	Traditional	438	264	60.3%
Fall 08	Studio	385	277	71.9%
	Traditional	700	481	68.7%
Spring 09	Studio	204	163	79.9%
	Traditional	257	194	75.5%
Total	Studio	772	573	74.2%
	Traditional	2544	1702	66.9%

In each of the last four semesters, the C or better rate has been higher for Studio College Algebra. Over all of the last four semesters, the C or better rate for Studio College Algebra is 74.2% while the average for Traditional College Algebra is 66.9%. This difference is statistically significant with a p-value of 1.24×10^{-4}. We feel it likely the differences here are the results of instruction rather than differences in the underlying populations. An analysis of ACT scores shows no difference in preparedness among the students in the two versions of the course.

Of students who take another math class immediately after College Algebra, the most common choice is Business Calculus. In comparing students who took Studio College Algebra to those who took Traditional College Algebra over all of the last three semesters, the students in the studio version slightly outperformed those from the traditional version when they took Business Calculus the succeeding semester. However, the differences are not consistent between semesters and the overall difference is not statistically significant. We are certainly not claiming studio students are better prepared, but it does appear that the greater success in the studio version is not being achieved by lowering academic standards and passing students who are unprepared for the next class. Since our goal in developing Studio College Algebra was not necessarily to teach successful students more but rather to teach more students successfully, we are satisfied with these results.

Semester	Prior Algebra Course	Enrollment in Math 205	C or Better in Math 205	Percentage
Spring 08	Studio	21	19	90.5%
	Traditional	98	77	78.6%
Fall 08	Studio	11	9	81.8%
	Traditional	39	32	82.1%
Spring 09	Studio	37	29	78.4%
	Traditional	60	50	83.3%
Total	Studio	69	57	82.6%
	Traditional	197	159	80.7%

Pre/post test scores in the studio version show modest, but statistically significant gains. The pre/post exam has not been offered in the traditional version as it covers slightly different material from the studio version and so the test would be inappropriate. The post-test was offered to the students in most semesters as a diagnostic preparation for the final and they did little studying. During one semester in the development of the new version when we encouraged them to study for the post-test, the students' scores jumped from 27% on the pre-test to 77% on the post-test.

Semester	Count	Pretest	Std. Dev.	Post-test	Std. Dev.
Fall 07	91	39%	14%	56%	16%
Spring 08	45	35%	14%	56%	16%
Fall 08	260	33%	13%	52%	16%
Spring 09	128	35%	15%	52%	18%

Student satisfaction has also increased as measured by student evaluations. While evaluations on a scale from 1 (low) to 5 (high) traditionally ranged from 2.5 to 4.0 depending on the instructor, evaluations in the studio version now consistently come in above 4.0. Anecdotally, the number of complaints being fielded about the course has dropped substantially as noted by both the department and central administration (including New Student Services). It should be noted that evaluations have improved and complaints have decreased in the traditional version as well. Two different explanations have been proposed, both of which have merit. The new College Algebra coordinator has taken on significant amounts of the lecturing in both versions and the students are happy with her work in both versions. In previous semesters, student comments indicated they felt the department did not take College Algebra seriously but treated it as a burden. It appears the students in both versions perceive the changes as indicating the department is indeed serious about the quality of College Algebra and are thus more willing to give the department the benefit of the doubt when they encounter difficulties. This may provide more than just the benefit of not having to fend off complaints. It is possible that some of the improvements in student success come from students who keep working when they encounter difficulties instead of giving up.

Future Plans

While this article has focused on Studio College Algebra, we have transferred what we have developed to the traditional version when appropriate. Our online homework system has proven very popular and is now used in both versions of the course. The major difference between the versions is the studios, and because of the different scheduling these cannot be transferred to the traditional version. That probably would not be appropriate even if possible, as interviews with students show the studios attract strong feelings, both positive and negative.

In talking to students and advisors, currently students seem to be placed in the different versions largely at random depending on which version fits their schedule better, with some advisors indicating they lean toward placing weaker students in the studio version. We are working to develop profiles of which students do best in each version to improve advising. In addition, we have recently introduced an online placement exam for incoming students. When this exam is properly normed, we hope it will also aid in placing the students into the course they are best suited for.

In addition to trying to better understand which students belong in which version, we are also applying data-mining techniques to try to classify students according to their work and behavior in the class (who attempts homework how many times, who gets which questions right in lecture, who made which mistakes on exam 1, etc.). Our goal is to be able to identify students' conceptual development in real-time during the course, both to provide effective feedback to the instructors and to identify situations where interventions may benefit identifiable groups of students. This last goal is not one we expect to achieve in the near-term.

Acknowledgments

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Refocus College Algebra HBCU Retreat and Follow-On Program

Don Small and Kathleen Snook
United States Military Academy

Abstract. Beginning in 2006, the United State Military Academy hosted a series of three retreats aimed at assisting schools to refocus their college algebra offerings. The Historically Black Colleges and Universities (HBCU) Retreat and Follow-On Program were funded by the National Science Foundation (NSF) and the Army Research Office (ARO). The goal of this program was to: Provide a structured opportunity for participating HBCUs to reform their college algebra or calculus curriculum and develop and pilot test reform programs. Seventeen colleges participated in the retreats. Thirteen are actively involved in refocusing their college algebra course. All of these colleges report use of in-class activities and out-of-class projects, and all but one report increase in ABC pass rates.

> "A group of eight people meet and each person shakes hands with each other person exactly once. What is the total number of handshakes?"

The handshake problem is a classic, but perhaps not one you might consider using on the first day of a College Algebra course. In a refocused College Algebra course it is most appropriate.

Retreat Phase

A series of three retreats were conducted. The objective of each retreat was for teams to develop a reform program for their institution along with implementation strategies. Seventeen schools have participated in the retreats and follow-on programs. Most of the schools sent two or three faculty members to participate in one of the three summer retreats. During the four days of the retreat each school worked with a mentor. The mentors had all worked at institutions that stress academic reform and had all been involved in faculty development programs.

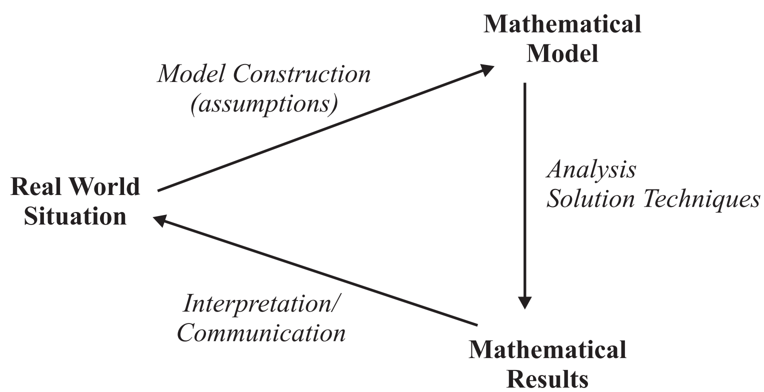
The retreats began with an afternoon program on the day of arrival. Participants quickly learned that the retreat would model the pedagogical practices desired in a refocused College Algebra classroom. After a very brief welcome, participants engaged in problem solving by addressing the handshake problem. They moved between acting as students in solving the problem, and acting as teachers in analyzing the problem and discussing strategies for using the problem in a classroom. This one problem led to a lively and energetic discussion about how to refocus a college algebra class to make it the most exciting and interesting course in the mathematics department.

The discussion included the following:

- Most of everyday mathematics is college algebra (e.g., displaying data, figuring, commissions, sales percentages, drug dosages, credit card payments, buying on time, averages, medians, etc.)
- The course is "open-ended," since the large majority of students view it as their terminal math course. Thus there is no pressure to cover "Chapter 5."
- The material can be real-life; relevant and practical.
- Inexpensive technology exists for plotting and computing.

- As in the real world, graphs and plots can be used to convey information (look in a newspaper, walk down an aisle in a store).
- Technology makes exploring easy.
- We live in a world of relations, not fractions or factored polynomials.
- Problem-solving skills, ability to communicate and ability to work as a team member are the important aspects that employers look for when hiring, not mastery of computational techniques.
- College algebra is a college gateway course.

The goal of a refocused classroom is to develop students to become exploratory learners. This requires a change from the traditional pedagogy of teaching skill using a prescribed scope and sequence to one that centers on identifying interesting problems in the real world, gathering information, questioning facts and assumptions, modeling the problem mathematically, learning the concepts and skills required to solve the model, working in small groups, and communicating results. Instead of evaluating a pre-packaged function as described in a particular section of a particular chapter of a text, students may analyze data to develop a function and then answer questions about the particular situation modeled. Finally, they evaluate their model and their answers for reasonableness. The following diagram illustrates the problem solving / modeling process suggested in a refocused course.



Toward the end of the first day of the retreat, teams have the opportunity to meet with their mentor. Mentors discuss the environment at the participants' school, any ongoing efforts, the goals of the participants, any concerns they have, and their vision for their institutions. In the evening, participants attend a talk where a current professor talks about his or her personal experience with refocused college algebra.

The second day of the retreat is a workshop bursting with hands-on modeling and problem solving activities. In the morning, participants engage in a series of classroom activities again playing the role of both student and instructor. Modeling and problem solving are integrated with lively discussions about pedagogy and curriculum. Active engagement continues in the afternoon with the focus on the use of small group projects within the curriculum.

Example Class Exercise 1. Make up a data set of 10 entries for which the mean is 7. Now, while keeping the mean 7, change your entries so the median is 8. Finally, while keeping the mean 7 and the median 8, change the set so the mode is 3.

Example Class Exercise 2. Doris is driving east on Interstate 20 with her cruise control set on 65 mph. If Doris' car is 16 feet long and it takes her 7 seconds to pass a 70 foot tractor-trailer rig, how fast is the truck traveling? (1 mile = 5280 feet). Explain how you interpret the meaning of the statement "Doris' car passes the truck."

Sample Small-Group Projects.

"Pepsi, Coke, or …?" An investigation of what varieties of soda to offer on campus and what portion of the capacity of a machine to devote to each variety.

"Packaging." An exploration of the relationships between area and circumference, and between volume and surface area of various containers.

"The Cost of Driving." An analysis of the real-world costs of owning and operating a vehicle.

The afternoon wraps up with a mini-workshop on assessment, class management, and course goals and objectives. Teams work with their mentors during this interactive session to develop course goals and objectives, as well as an assessment plan, for a refocused course at their institution. Teams work to determine what they desire a student to "look like" at the end of their course. More specifically, they identify what students should "know," "understand." and "be able to do." This vision assists teams in designing their course in order for students to develop toward these end goals. In the closing session of the day another current professor discusses his or her personal experience with a refocused course.

During the third day, teams continue to develop materials for their own institution. Drawing from the previous day's assessment session, participants examine and discuss sample tests and sample syllabi. Teams work with their mentors to develop a syllabus and to outline a final exam for their course. Each team drafts plans for a refocused course for their institution to include course goals, a syllabus and an implementation plan. They prepare a briefing to share with the entire retreat group.

Discussions on the final day of the retreat turn to opportunities for networking, support, and interdisciplinary cooperation. Additionally, each school reported on their plan of action. When participants leave the retreat, they are prepared to return to their institution and effect a change in their college algebra program.

Follow-On Phase

The Follow-On portion of the program involves continued mentoring, a $5,000 mini-grant, and activities at the Joint Mathematics Meetings. Mentors maintain contact with their teams through the telephone, e-mails, and onsite visits. During the on-site visits, mentors help build support for the school's refocusing project by meeting with the Dean or Provost, the Chairman, as well as with team members and other faculty. Visiting or conducting a class, meeting with students, helping develop a funding proposal to support the project, and consulting on curriculum development are some of the additional activities that mentors engage in during their visits.

The purpose of the mini-grants is to support the refocusing effort. Purchasing classroom sets of graphing calculators, traveling to professional meetings to disseminate their programs, paying the food costs for in-house workshops, hiring student assistants, and establishing a tutoring program are among the activities supported by the mini-grants. Because projects differ in their needs, the guidelines for how the mini-grants could be spent are very general. This helps teams to individualize their projects to their own school.

Partial financial support is offered to team leaders to attend the Joint Mathematics Meetings during the two or three year Follow-On period. Several of the leaders have represented their team by presenting posters at the National Science Foundation Poster Session held during the Joint Mathematics Meetings. In addition, a two-hour Reunion Session is held during the Joint Meetings for faculty who have participated in a college algebra workshop. The majority of the participants attending these reunion sessions are from the HBCU Retreat and Follow-On Program. The purpose of the Reunion is to allow individuals to share experiences, resources, and student results. The Session also serves to rejuvenate and inspire faculty to expand their refocusing efforts.

> "Some success stories heard at the Reunion Meetings motivated the refocused team to continue to try the refocused approach." (Faculty member at Paine College)

Dissemination of the Program has been through Contributed Paper talks at national meetings by both participants and mentors, panel sessions at conferences of the National Association of Mathematicians (NAM), the *Vision-Potential* Newsletter, and reports to the National Science Foundation and the CRAFTY committee of the Mathematical Association of America.

Preliminary Results After Three Years

> "I have never seen students so involved in their own learning, so confident, so fluent with communication of their knowledge." (Faculty member at Lincoln University)

The Program's PI maintains communication with the teams and mentors via e-mails, phone calls, and meetings at professional conferences. He monitors the program primarily through semester reports submitted by the schools and an occasional survey. The results from these reports and surveys are encouraging and document the success of the Program in refocusing college algebra. Although results vary from school to school, the following statements represent the state of the Program:

- Thirteen of the seventeen schools participating in the Retreats are actively involved in refocusing their college algebra course.
- A refocused course requires more time and effort on the part of both instructor and student compared to the traditional program.
- All thirteen schools report improved student attitudes toward mathematics.
- The ABC pass rate increased from that in the traditional program. (7 schools reported an increase of greater than 15 percentage points, 5 schools reported an increase between 0 and 15 percentage points, and 1 school reported a drop in the ABC pass rate.)
- There is an increase in students taking responsibility (10 schools reported perceived increase, 3 reported no change).
- Schools report an increase in students' communication skills compared to the traditional program.
- The withdrawal rate decreased from that in the traditional program.
- Approximately the same number of students transfers into the program as out of the program.
- All (13) schools use out-of-class, small-group projects (I or 2 per semester).
- All (13) schools use in-class, small-group activities.
- All (13) schools use graphing calculators in their programs.
- All (13) schools report decreasing lecture time in order to allow more small-group activities.
- The colleges report that students in the refocused class do as well with skill work as students in traditional classes.

> "If your class is productive, the teacher doesn't have to teach all the time. The students can teach each other and as students we sometimes learn better from our peers." (Student at Bethune-Cookman Univ.)

Challenges and Responses

> "Change in pedagogy from lecture to exploratory learning is beneficial to students and is time well spent. The department is not ready to change course content to the extent that it deviates from the description of the course in the catalogue. The department also wants to ensure that students are prepared for external exams such as GACE and institutional assessments that are done annually." (Faculty member at Paine College)

Change is never easy and critics abound. A sampling of major challenges encountered in our efforts to refocus college algebra and their responses are:

Challenge: Departmental (or school) requirement that all sections of college algebra be taught from the same text with student assessment based on common tests.

Response: Propose a *pilot* program for a limited time, say three years, with the option of selecting a different textbook and devising an assessment plan to include small group activities/projects, class presentations, technology use, etc. Propose a shared portion (e.g., skill work) of the final exam with the traditional sections. Mentors can be very helpful in addressing this challenge. In one school, the faculty member leading the refocusing effort had resigned himself to the Dean's policy on requiring that all sections of a course use the same text and common tests. When the mentor visited with the Dean and raised the issue, the Dean's immediate response was: "That makes no sense. If you are going to run a pilot project, you need to have control over the choice of text and your testing."

Challenge: Sometimes critics will assert that the refocused course does not prepare students to go into pre-calculus or calculus as well as the traditional course does. This is usually said in reference to skill work, even though the evidence refutes this claim.

Response: Designate the refocused course for a special audience. One school, after negotiating for two years, designated their course for potential business students and was then given permission to develop and implement their own pilot sections. Other schools have successfully responded to this challenge by offering two courses, one for potential STEM students and one for everyone else.

Challenge: The advocated approach requires giving up the control and ease of a teacher-centered, lecture-based pedagogy for an inquiry-based student-centered pedagogy.

Response: Participate in faculty development workshops that showcase a student-centered pedagogy, team teach, establish a schedule of instructor meetings to discuss experiences and future class preparations, confer with mentors, and network with other instructors.

Challenge: Lukewarm or no support by the Chair and/or Dean.

Response: Understand and respond to the viewpoints of the Chair and Dean. The primary concern of the Chair is probably the retention rate and the number (and success) of college algebra students in follow-on courses. For the Dean, the principle concern is often the ABC pass rate.

Challenge: The reality, or at least faculty perception, that investing more time and effort into a college algebra course is not valued by the school's administration.

Response: Mentors can be particularly helpful in addressing this challenge by engaging with Chairs and Deans on the topic of how to value a person's efforts in a non-financial manner.

Willingness to accept a traditional college algebra course as a filter course, change in Deans and/or Department Chairs, ballooning class enrollments, and heavy teaching loads are further examples of challenges different colleges experienced.

Lessons Learned

The major lessons learned from the HBCU Retreat and Follow-On Program apply to curriculum reform in general. A program of this type benefits greatly from: (1) A leader with a vision and the ability to communicate the vision to others with a sense of urgency; and (2) A manager at each institution who can interpret the vision in light of the idiosyncrasies of his or her school and who can effectively communicate the vision to both the administration and the faculty. The leader, although not necessarily a member of any particular school in the program, must establish a channel of communication with each school. The Retreat portion of the HBCU Retreat and Follow-On Program provides a platform to develop these channels of communication and to engage school teams in the vision. Each school needs to have its own manager to guide the local development of the program. The primary role of the mentor in the Follow-On phase is to support the efforts of the manager.

Patience and persistence are needed in refocusing both content and pedagogy. The teams of two of the thirteen schools struggled for over two years before receiving permission to pilot their programs. The manager in a third school has been very successful in her own sections, but has not been freed from her department's policy of common textbook and common exams. At another school, the team has been required to cover all the traditional topics and only now after two years are they getting permission to modify the traditional curriculum. Three or four (or more) years are usually required for a pilot program to become established.

Schools differ, and just as with shoes, "one size does not fit all." Programs such as the HBCU Retreat and Follow-On Program need to provide flexibility to allow each school to individualize the project to their situation. For example, one school invites a faculty member in their Business School to teach Linear Programming in their college algebra course, while another school omits any reference to Linear Programming.

On-going faculty development is the spice that brings out the richness and potential of the Program. The most vibrant projects are those that supplement hallway conversations with weekly or bi-weekly team meetings. Reflecting on the past, planning for the future, building morale, and providing support are the primary functions of these meetings. A one or two-day in-house workshop at the start of the semester serves to energize team members and integrate new faculty into the project. Typical topics include dealing with small-group activities/projects, assessing class work, holding students accountable, assessing group work, lecturing less, giving more responsibility to the students, constructing exams, and how to integrate calculators and projects into classroom instruction. Regional or national workshops conducted by the Program or a professional organization such as the Mathematical Association of America's PREP workshops (Professional Enhancement Program) link local programs to national movements. They provide for expanded sharing and networking opportunities.

A team approach greatly increases the possibility for success, particularly if its members represent a spectrum of the department faculty. The team provides a faculty core from which to build departmental acceptance. In addition, team members provide each other with encouragement and support to experiment, to integrate successful activities, and to learn from mistakes.

The final Lesson Learned to be noted is the importance of a Follow-On program. This unique feature of the HBCU Program has been instrumental to the success of the Program. Instituting meaningful change is a multi-semester process, it is an organic process. On-going support and encouragement is necessary to ensure that the successes and failures along the way lead to improvements in the project.

> "The Retreat enabled us to understand the basic philosophy of the project and enabled a critical mass of faculty to begin plans for implementing the project upon return to our campus. Having this faculty core group (inclusive of senior faculty) was important to our later gaining full acceptance of the project on campus." (Faculty member at Fort Valley State University)

Recommendations for Departments that are Considering Revitalizing College Algebra

Barbara Edwards
Oregon State University

Bill Haver
Virginia Commonwealth University

Don Small
United States Military Academy

Abstract. The three authors of this paper have each been deeply involved in college algebra courses and have become convinced that the course described in the *MAA College Algebra Guidelines* (CRAFTY, 2006) provides a much more valuable experience for students than the computational skill-centered college algebra course that was offered by our institutions and many others in recent decades. It has been our experience that the focus on modeling leads students to understand why various topics are included in the curriculum and that the projects, activities and writing assignments actively engage students. Generally, attendance and the classroom environment improve dramatically. Most importantly, in our view, students develop habits of mind necessary for life-long learning while learning the mathematics that will be most important to them in future endeavors. We are firmly convinced that it is well worthwhile to offer courses that satisfy the *Guidelines*. In this paper we share our recommendations for departments that are considering revitalizing college algebra.

Developing Habits of Mind Rather Than Habits of Memorization

The success of a *Guidelines* based course usually requires paradigm shifts for both the instructor and the students. A major goal is to encourage students to question why something is true rather than to determine the rule to memorize. In order to achieve this goal the instructor needs to shift to a student-centered pedagogy. This often involves limiting lecturing to fifteen minutes or less in order to provide for in-class, small-group activities. For the student, the shift is to become an exploratory learner. One who is actively engaged in the class, who questions and what-ifs results, who creates models and interprets results, who is willing to take risks, and who takes on the responsibilities for his/her own learning.

These shifts require more time and effort for both instructor and the students. This said, significant additional resources are often needed as institutions put in place a course meeting the *Guidelines*. Depending on the ongoing level of support of the traditional college algebra course, additional resources are often required: smaller class size, a conducive classroom configuration, and ongoing professional development and support for course instructors. In addition, course instructors need to devote more time to preparation and grading.

Natural Allies

In most college and universities there are a number of natural allies available to individuals who wish to offer an applications-oriented, modeling-based college algebra course that actively engages students: faculty from partner disciplines who are interested in programs for majors in their fields; college and university administrators who are committed to student engagement/high quality teaching/increased student learning; college and university administrators who are interested in student retention; colleagues in mathematics departments who teach and/or do research in applied mathematics who will find the content of such a course very appealing; instructors and part-time faculty who find the courses much more interesting to teach, particularly those with research and development or business experience outside of academia; and teacher educators and mathematics education researchers. These allies can be instrumental in obtaining the resources needed to offering a high quality College Algebra program.

Faculty from Partner Disciplines. Faculty from partner disciplines who are interested in the course of study required for majors in their discipline generally tend to be very supportive of a course that meets the *Guidelines*. This is particularly true for the business faculty. The Curriculum Foundation reports (CRAFTY, 2004) have proven to be a very useful tool for initiating a discussion with faculty in a partner discipline. An excellent starting point is to invite interested faculty members from a particular discipline to read the report from their discipline's workshop. Then faculty from mathematics and the partner discipline can meet to discuss the degree to which there is agreement with the Curriculum Foundation disciplinary report. This discussion can then lead naturally to a comparison of the curriculum of the institution's current college algebra course with that of a possible alternative. Our experience is that the faculty from the partner disciplines will strongly favor the latter.

Administrators who are committed to student engagement/high quality teaching/increased student learning. The location of such individuals varies from campus to campus. These individuals are guaranteed to be strong allies and it is very worthwhile to garner their active support. For a variety of reasons many individuals in such positions have given up on the idea that mathematics faculty value student creative thinking, active student involvement, team projects, and student writing. In many cases once they learn of this interest by mathematicians on their campus, they become very enthusiastic supporters. Some are in administrative positions from which they can provide on-going financial support, e.g., additional instructors to permit smaller classes or undergraduate assistants. Although many administrators with these interests can't provide this type of help, many of them can help in providing appropriate classroom space, perhaps tables or better technological support. They also often can provide one-time support for professional development, e.g., stipends for part-time faculty to attend local workshops, or registration and travel support for faculty members to participate in a PREP workshop.

Administrators who are interested in retention. There are often institutional pressures that value increased student retention, low DFW rates, and high graduation rates. Often individuals who are in administrative positions that experience this type of pressure control resources that can affect class size. There is ample data that show that a course that meets the *Guidelines* can result in significantly lower DFW rates (see for example Ellington, Haver 2009). Administrators with these institutional resources are often in a position to support a pilot study, and then to follow up with long term support if the pilot-study indeed demonstrates that the DFW rate can be impacted in a major way on their own campus. Of course some individuals are interested in both engaging classroom experiences and retention and these individual scan be especially valuable allies.

Mathematics faculty who do research in applied mathematics or teach applied mathematics courses. Many faculty members whose major research interests are in applied areas find a course based on the *Guidelines* very appealing, and indeed obviously appropriate. For example, it is natural for individuals with this experience to reach the conclusion that studying topics such as inverses or compositions of functions out of context appears useless, but studying these same ideas in context yields much deeper student understanding.

Teacher Educators and Mathematics Education Researchers. These faculty allies can help out in a number of ways. Their students, especially future high school teachers, make willing and well-prepared undergraduate tutors and/or aides in the college algebra classes; and teacher educators often think that involving these students in college algebra faculty planning meetings is an excellent experience for their students. Mathematics education researchers and graduate students might be eager to conduct research related to the outcomes of your renewed course.

Faculty who have taught sections of a course based on the Guidelines. It has been our experience that in many cases the biggest allies are full-time and part-time faculty members who have taught sections of both computational and modeling-based courses. Many instructors find it exciting that their students are more engaged, attend class much more regularly and are able to bring their own experiences into the classroom. A typical positive reaction of instructors is that students often will consider a proposed answer to a question and exclaim "this doesn't make sense." During a period of offering sections of a *Guidelines* based course on a pilot basis, an achievable goal is to build up a cadre of instructors who volunteer to teach the course once and end up as strong proponents who are no longer interested in teaching computational college algebra. It is particularly important to have at least one tenured faculty member involved in your project. This individual often has valuable connections to other areas of your institution, including administration, and can add a sense that the project is endorsed by the department and add credibility to your efforts.

Advice For Getting Started

Based on experiences at our own institutions and our observations of others we offer a number of suggestions to departments as you begin to revitalize your college algebra course:

Begin with a pilot project involving instructors who want to try this approach. For institutions with multiple sections of college algebra we would suggest first offering a small number of sections on a pilot basis. It is important to only involve faculty who want to participate in the pilot. If most of an institution's sections are taught by part-time instructors or graduate teaching assistants, individuals from these groups should be involved in the pilot sections. Some institutions have taught pilot sections for five or six semesters before adopting a *Guideline* based course for all sections.

Designate a Course Coordinator to manage the pilot program. This person needs to have a vision for a course based on the Guidelines and be able to communicate that vision and its urgency to other faculty and the administration. The Coordinator should be given released time to develop a syllabus and a Teacher's Guide, to be in charge of the instructor meetings, to oversee the developments of tests, projects, and activities, and to be a spokesperson for the program.

Be flexible and don't require all participating instructors to incorporate all features of your new course. The *Guidelines* call for many instructional approaches that are unfamiliar to most faculty: use of technology such as computers, calculators, spreadsheets or computer algebra systems; assignments such as oral presentations, portfolios, group projects or portfolios; activity based instruction. Of course, no institution will decide to include all of these features in its course. By the same token once an institution has decided what to include in its course, it is not reasonable to expect that each instructor will be able to make all adjustments immediately. Accommodations can be made. For example, if an instructor does not feel comfortable assigning activities that make use of Excel, an undergraduate assistant could be found to handle or assist with this aspect of the course. Do everything you can to avoid making the revitalized college algebra initiative seem like an exclusive club. Keep non-involved instructors and new GTAs up to date on what is going on with college algebra. If there is a poster-making assignment in the course, put the good ones up in the halls of the math department.

Collect data. It is important to gather data concerning grades in your college algebra course, grades of students in subsequent courses, student evaluation ratings, response of instructors to questions concerning their courses, etc. This data can often play an important role in convincing administrators, and your colleagues, of the impact of the *Guidelines* based college algebra course. Of course, you need to check with your Institutional Review Board (IRB), particularly if you plan to publish results concerning your work.

Invite Colleagues and Administrators to attend classes. This is an excellent way to recruit new allies. A classroom with engaged students actively thinking about mathematics leaves a great impression. Even if your invitation is not accepted, no harm is done and perhaps some good.

Most Important Advice When Implementing on Large Scale

At most institutions college algebra is taught by a wide variety of different individuals: tenure/tenure-track faculty, short-term instructors, adjunct instructors hired on a semester to semester basis, and/or graduate teaching assistants.

For most of these faculty members, teaching a *Guidelines* based course will be a new experience. Appropriate support is needed, and with this support teaching this course can be a very rewarding experience.

Workshops are very valuable. One approach that has proven effective is to have a two-day workshop before the semester begins and a Friday afternoon and evening and Saturday workshop about a third of the way through the semester. Having an outside "expert" lead a portion of the program can help provide some perspective. Important topics include grading the type of non standard assignments to be given in your course, getting comfortable with group work, fully engaging students, and overall goals of the course. We have found that math instructors are most receptive if the suggestions and advice are directly tied to the course and to the mathematics to be studied.

Weekly meetings are worthwhile. Faculty can share experiences in their classes and discuss what is going well and what isn't. A one hour meeting could be divided into three 20 minute structured segments: a focused presentation/ activity on some particular topic or classroom activity coming up next week; information and questions on schedules, tests, group projects that will occur in future weeks; general discussion of what is going well and what isn't in individual sections.

Centrally provided activities, quizzes and exams cut down on faculty time. There is no question that a *Guidelines* based college algebra course requires significantly more faculty time than a lecture class in which the students are graded by their performance on multiple choice or short answer, computational questions. The department can make up for some of this time by providing work sheets for in-class activities, assignments for long-term individual or group projects, quizzes and exams. These materials should be presented for what they are: tools to save faculty time.

Don't expect to get the course right the first time. The authors of this paper have each tried methods that did not work well in our efforts to create a college algebra course based on the *Guidelines*. In fact, we believe these experiences helped to create a course that was more successful in the end.

Final word of advice. Above all else, try not to do everything at once. As noted previously, for many faculty and students a Guidelines based course involves a big paradigm shift. Successful changes require time, patience, and the willingness to keep trying when things don't turn out exactly right the first time.

It has been our experience that with this type of support faculty members very much enjoy teaching a *Guidelines* based courses and that student achievement definitely justifies the additional commitment.

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