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Charles A. Dana Center  
The University of Texas at Austin  
1616 Guadalupe Street, Suite 3.206  
Austin, TX 78701-1222

Fax: 512-232-1855  
danaweb@austin.utexas.edu  
[www.dcmathpathways.org](http://www.dcmathpathways.org)

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## About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We work with our nation’s education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at [www.utdanacenter.org](http://www.utdanacenter.org).

## About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—*Statistical Reasoning*, *Quantitative Reasoning*, and *Reasoning with Functions I* and *Reasoning with Functions II* (a two-course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point—a developmental math course that helps students develop foundational skills and conceptual understanding in the context of college-level course material.

In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies—and tenacity—necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.

In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

### Acknowledgments

The development of this course began with the formation of the DCMP **Curricular Design Team**, who set the design standards for the curricular materials of individual DCMP courses would be designed. The team members are:

Richelle (Rikki) Blair, Lakeland Community College (Ohio)  
 Rob Farinelli, College of Southern Maryland (Maryland)  
 Amy Getz, Charles A. Dana Center (Texas)  
 Roxy Peck, California Polytechnic State University (California)

Sharon Sledge, San Jacinto College (Texas)  
 Paula Wilhite, North Texas Community College (Texas)  
 Linda Zientek, Sam Houston State University (Texas)

The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The **Usability Advisory Group** members are:

Ioana Agut, Brazosport College (Texas)  
 Eddie Bishop, Northwest Vista College (Texas)  
 Alma Brannan, Midland College (Texas)  
 Ivette Chuca, El Paso Community College (Texas)  
 Tom Connolly, Charles A. Dana Center (Texas)  
 Alison Garza, Temple College (Texas)  
 Colleen Hosking, Austin Community College (Texas)

Juan Ibarra, South Texas College (Texas)  
 Keturah Johnson, Lone Star College (Texas)  
 Julie Lewis, Kilgore College (Texas)  
 Joey Offer, Austin Community College (Texas)  
 Connie Richardson, Charles A. Dana Center (Texas)  
 Paula Talley, Temple College (Texas)  
 Paige Wood, Kilgore College (Texas)

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Unless otherwise noted, all staff listed are with the Charles A. Dana Center at The University of Texas at Austin.

#### *Project Lead, Authors, and Reviewers*

Richelle (Rikki) Blair, lead author, professor emerita, Lakeland Community College (Ohio)  
Scott Guth, professor, Mt. San Antonio College (California)  
Rob Kimball, professor emeritus, Wake Tech Community College (North Carolina)  
Andrea Levy, mathematics instructor, Seattle Central Community College (Washington)  
Maura Mast, associate vice provost for undergraduate studies, University of Massachusetts Boston (Boston)  
Aaron Montgomery, Professor, Central Washington University (Washington)  
Jeff Morford, Henry Ford Community College (Michigan)  
Victor Piercey, Assistant Professor, Ferris State University (Michigan)  
Connie J. Richardson, project lead and lead author, *Quantitative Reasoning*  
Jack Rotman, Lansing Community College (Michigan)  
Jan Roy, mathematics department chair, Montcalm Community College (Michigan)  
Erin Sagaskie, mathematics instructor, Carbondale Community High School (Illinois)

#### *Project Staff*

Adam Castillo, graduate research assistant  
Heather Cook, program coordinator  
Ophella C. Dano, lead editor  
Amy Getz, strategic implementation lead, higher education  
Monette McIver, manager, higher education services  
Brandi M. Mendez, administrative assistant  
Erica Moreno, program coordinator  
Dana Petersen, independent consultant  
Phil Swann, senior designer  
Sarah Wenzel, administrative associate

#### Pearson Education, Inc. Staff

Vice President, Editorial Jason Jordan  
Strategic Account Manager Tanja Eise  
Editor in Chief Anne Kelly  
Senior Acquisitions Editor Marnie Greenhut  
Digital Instructional Designer Tacha Gennarino  
Manager, Instructional Design Sara Finnigan  
Senior Project Manager Dana Toney  
Director of Course Production, MyMathLab Ruth Berry  
MathXL Content Developer Bob Carroll

Project Manager Kathleen A. Manley  
Project Management Team Lead Christina Lepre  
Product Marketing Manager Alicia Frankel  
Senior Author Support/Technology Specialist Joe Vetere  
Rights and Permissions Project Manager Martha Shethar  
Procurement Specialist Carol Melville  
Associate Director of Design Andrea Nix  
Program Design Lead Beth Paquin  
Composition Dana Bettez

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-	-	Suggestions for Prep Week	xxxi	-	-	-
<b>Lessons 1-7: Complex Numerical Summaries; Graphical Displays</b>						
1.A		Data for Life <i>Collect data that will be referred to throughout the semester; supplemental spreadsheet provided</i>	1	1	1	-
1.B		Our Learning Community <i>Student success focus Establish a sense of shared responsibility; provide key information about course content and policies</i>	4	5	4	-
1.C	1.C	Instant Runoff Voting schemes	6	7	13	1.C
1.D	1.D	Borda Count Voting schemes	11	11	18	1.D
2.A	2.A	Graphical Displays <i>Analysis and communication; dotplots, histograms, boxplots; mean; median</i>	14	13	22	2.A
2.B	2.B	Forming Effective Study Groups <i>Student success focus Taking responsibility for own learning and supporting learning of others; setting norms</i>	17	15	26	-
2.C	-	Mini-Project: Graphical Displays <i>Write formal, contextual analysis on compared data; research-related data; sample rubric provided</i>	20	17	32	-
3.A	3.A	Who Is in the Population? <i>Populations; sampling</i>	23	19	38	3.A
3.B	3.B	How Much Water Do I Drink? <i>Analyzing class data; Central Limit Theorem</i>	27	21	43	3.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
3.C	3.C	How Much Water Does Our Class Drink? (Optional) <i>Sample standard deviation</i>	30	23	47	3.C
4.A	4.A	What Are the Risks? <i>Theoretical probability of two or more independent events</i>	35	27	50	4.A
4.B	4.B	Calculating Risk <i>Conditional probability of two or more dependent events</i>	39	31	54	4.B
5.A	5.A	Cost of Living Comparisons <i>Conversion to create equivalent units; supplemental spreadsheet</i>	44	35	58	5.A
5.B	5.B	Index Numbers <i>Using indices such as Consumer Price Index; supplemental spreadsheet</i>	48	39	63	5.B
5.C	5.C	Polls, Polls, Polls! <i>Weighted averages</i>	52	43	67	5.C
5.D	5.D	Average Income <i>Weighted averages and expected value; supplemental spreadsheet</i>	56	47	72	5.D
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6.B	-	Mini-Project: Income Disparities (Optional) <i>Written analysis of graphical display of weighted moving average</i>	63	51	81	-
7.A	7.A	U.S. Budget Priorities <i>Part-part vs. part-whole ratios</i>	68	53	86	7.A
7.B	7.B	Understanding U.S. Budget Priorities <i>Decimals, percentages, and part-whole ratios</i>	72	57	90	7.B
7.C	7.C	Changes to U.S. Budget Priorities <i>Absolute and relative change</i>	78	61	95	7.C
7.D	7.D	Percent of Total U.S. Budget <i>Dotplots used to introduce symmetry and skewness</i>	82	63	99	7.D

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
7.E	7.E	What's My Credit Score? <i>Application of ratios; Practice assignment can be mini-project. Collect data for Lesson 8, Part D; schedule lab for 8.D and 10.A.</i>	87	65	102	7.E
7.F	7.F	U.S. Incarceration Rates <i>Applications of ratios; comparison</i>	91	69	107	7.F

### Lessons 8-12: Mathematical Modeling

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8.B	8.B	What's My Car Worth? <i>Distinguishing proportionality and linearity</i>	99	75	118	8.B
8.C	8.C	How Money Makes Money <i>Non-linear models</i>	103	79	123	8.C
8.D	8.D	Have My Choices Affected My Learning? <i>Regression using student data. Computer lab day, if possible.</i>	109	83	129	8.D
8.E	8.E	Mini-Project: Progressive and Flat Income Tax Systems (Optional) <i>Informal piecewise linear function</i>	113	87	135	-
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Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
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<p>You may wish to consider various configurations with the upcoming modeling lessons. For example, you may wish to consider having different groups complete and present the various logistic lessons or having some groups do logistic models while other groups do the periodic models. You may also choose to omit either logistic or periodic models.</p>						
11.A	11.A	Population Growth (Optional) <i>Logistic models</i>	163	125	177	11.A
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<b>Lessons 13-15: Statistical Studies</b>						
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Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
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You may wish to consider various configurations with the upcoming lessons on analyzing and writing about graphical displays. For example, you may wish to consider having different groups complete Lesson 16, Parts B, D, E, and F, and present to the class.

### Lessons 16-18: Complex Quantitative Information and Graphical Displays

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16.B	16.B	Looking for Links <i>Analyzing comparative stacked columns graphs</i>	271	217	295	16.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
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