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Chapter 2

Re-envisioning the Pathway to Calculus:
Supporting All Students

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Abstract
STEM occupations are expected to grow at a rate 1.4 times faster than non-STEM occupations, and the United States will need approximately one million more STEM professionals between 2014 and 2024. However, the declining number of students prepared to succeed in college-level calculus in their freshman year significantly reduces the pool of students likely to graduate with a STEM degree in four years. To broaden participation in STEM fields to meet the future demand for STEM professionals, institutions of higher education should reconsider how they prepare students for calculus in order to meet the needs of all students. By leveraging the opportunities presented by the mathematics pathways movement, institutions of higher education can make significant gains in student success and retention by attending to non-cognitive factors, identifying areas in calculus where students struggle, making mathematics meaningful for students through contextualization, focusing on developing a process view of function, and developing students' covariational reasoning skills.

Introduction: The Need for More STEM Professionals

Professionals in Science, Technology, Engineering, and Mathematics (STEM) fields help drive our nation's innovation and competitiveness. According to estimates from the Department of Commerce, STEM occupations are expected to grow at a rate 1.4 times faster than non-STEM occupations, and the United States will need approximately one million more STEM professionals between 2014 and 2024 (Noonan, 2017). At the same time, decreasing college readiness in mathematics coupled with poor success rates in developmental mathematics courses are negatively impacting the number of STEM degrees awarded. The declining number of students prepared to succeed in a college-level calculus course in their freshman year significantly reduces the pool of students likely to graduate with a STEM degree in four years (Kreysa, 2006). In response to poor success rates in developmental mathematics sequences, some state legislatures across the country are calling for reform by mandating that postsecondary institutions reduce the time that underprepared students spend in developmental mathematics courses. As a result, postsecondary institutions are left to grapple with addressing poor success rates for developmental mathematics students, while simultaneously needing to increase enrollments in calculus and the number of STEM graduates. These factors, combined with recent advances in mathematics education and the learning sciences, indicate the time has come to re-envision the pathway to calculus, which is essential to STEM majors. This chapter presents research on non-cognitive factors that are barriers for students, identifies mathematics education research about content and pedagogy to guide decisions about curriculum, and provides a framework for re-envisioning the pathway to calculus to broaden student participation in STEM fields.

Mathematics Pathways as a Catalyst to Re-envision the Pathway to Calculus

The mathematics pathways movement (see the Preface in this monograph) represents a significant shift in the approach that institutions of higher education are taking towards the content and sequencing of undergraduate mathematics education. In a growing number of states, mathematics departments are reallocating existing resources to increase student success in mathematics by decreasing enrollments in developmental mathematics, reducing the number of courses in traditional mathematics sequences, and providing mathematical content that is aligned to students' intended programs of study. A national study of mathematics departments found that 58 percent of two-year colleges implemented a mathematics pathway with redesigned courses in foundations, quantitative reasoning, and statistics in Fall 2015 (Blair et al., 2018, Table TYE.11, p. 176). Enrollment in introductory statistics at two-year and four-year institutions increased 45 percent between 2010 and 2015, signaling a significant change towards alignment (Blair et al., 2018, Table S.1). These shifts represent an opportunity to improve student outcomes. By providing meaningful mathematics pathways for students whose programs do not require calculus, institutions can focus their attention on designing appropriate mathematics pathways that will prepare STEM students for success in traditional calculus.

Supporting All Students: Attending to Non-Cognitive Factors

The task of supporting students on the pathway to calculus requires reflection about non-content issues that create barriers for underrepresented STEM students like women and underrepresented minority students. To broaden participation in STEM fields and fully realize the potential of mathematics pathways, mathematics faculty should work to minimize the negative impacts of three critical non-cognitive factors: lack of sense of belonging, lack of self-efficacy, and stereotype threat. Although these non-cognitive factors are relevant to student success across disciplines, strategies to reduce their negative impacts can be applied effectively in mathematics courses.

A sense of belonging reflects the feeling that one fits in, belongs to, or is a member of the mathematics community. A healthy sense of belonging is a significant predictor of one's intent to pursue mathematics in the future (Good et al., 2012). Strategies that enhance students' sense of belonging can be as simple as an instructor noticing that a student is absent and then contacting the student. Slightly more involved strategies include holding class discussions about effective work groups and developing classroom norms for working in collaborative groups. Self-efficacy, or one's belief in their ability to succeed, also plays a role in broadening participation in STEM programs, especially in the retention of women and underrepresented minorities. Women are 1.5 times more likely to leave STEM after completing calculus due to a lack of self-efficacy (Ellis et al., 2016).

To further enhance students' feelings of belonging and self-efficacy, institutions should leverage an important feature of the mathematics pathways movement: alignment of college algebra and precalculus courses to STEM programs that require calculus. Successfully aligning mathematics to programs of study leverages the use of contextualized mathematics that is meaningful to students. Contextualized mathematics provides opportunities for students to explore different approaches to problem solving at different levels of formality and makes mathematics more accessible and more likely to engage students in learning (Van den Heuvel-Panhuizen, 1999; Widjaja, 2013). From a cognitive perspective, contextualization promotes transfer of learning and retention of information (Boroch et al., 2007), which increases the probability of success in calculus and, consequently, student self-efficacy.

Stereotype threat contributes to the underperformance of women, African Americans, Latinos, and other minorities in mathematics (Aronson & Steele, 2005). At its core, stereotype threat is characterized by activated stereotypes that, when left unchecked, trigger a number of disruptive psychological processes that can undermine student performance (Croizet et al., 2004). The experience of being in a numeric minority in academic environments where stereotypes are part of the dominant culture reduces individuals' self-efficacy, especially in the face of difficulty, even if their actual performance is objectively the same as majority-group members (Dasgupta, 2011). A learning environment that utilizes group work, makes student learning visible, and showcases different student approaches to solving challenging mathematical problems can have a significant positive impact on student self-efficacy by making it evident that everyone must work hard to succeed. This in turn may diminish stereotype threat (Asera, 2001).

A recent study of the calculus redesign at Boise State University indicates that the core elements of frequent group work, making learning visible through active and collaborative learning, and contextualization produced sizable, sustainable, and statistically significant gains in Calculus I pass rates and grades (Bullock et al., 2016). In addition, the university's redesigned Calculus I course significantly improved STEM retention for women and underrepresented minorities. After implementation of the redesign, STEM retention rates of women and underrepresented minorities increased by more than nine percentage points (Bullock et al., 2017). This large effect significantly broadened participation in STEM programs at Boise State.

Pathway to Calculus: Content Considerations

While attending to non-cognitive factors can produce sizable gains in retention of women and underrepresented minorities in STEM programs, it is important to recognize that additional significant gains in student success can be made by vertically aligning the mathematical content used to prepare students for calculus. A process to facilitate discussion and action around curriculum redesign involves identifying areas in calculus where students struggle most, examining current research in mathematics education to anchor content and pedagogy decision in evidence-based research, and using the resulting information to create a relevant and engaging curriculum for students. As possible outcomes of this process are reviewed, specific content and other pedagogical choices will vary based on individual institutional and department needs. To begin the process, the authors of this chapter asked mathematics education researchers and calculus instructors, during a variety of professional development workshops, to give examples of areas of mathematics content where their students struggle in calculus courses (Boersma et al., 2015). The surveys produced the following list of potential problem content areas for calculus students:

- Algebra skills
- Functions and function notation
- Concepts of inverse functions and function composition
- Communicating about change and rates of change
- Limits and approximations
- The definite integral as an accumulator
- Dynamic geometric reasoning
- Overall problem-solving skills
- Working with open-ended problem structures

"Algebra skills" was identified as one of their students' major weaknesses even though many students enter college-level calculus classes having had years of algebra coursework in high school or college. Research in mathematics education indicates that some of students' algebraic difficulties are rooted in their superficial understanding of functions (Carlson et al., 2010; Oehrtman et al., 2008). This research has shown that when students develop a process view of a function, some problematic areas, such as inverting and composing functions as well as using function notation and symbols correctly, can be effectively addressed. Thus, what instructors identify as poor algebra skills may in fact be rooted in the lack of a deep understanding of functions.

To create a more meaningful approach to algebra, students should see, or discover for themselves, why each algebraic skill is useful. This can be accomplished by striving to contextualize many of the problems students encounter. Contextualization could entail engaging students with a self-contained meaning through modeling or showing explicitly how the skill will be used in later STEM courses. Context is key. It provides immediate meaning for students, a trigger to help them recall useful knowledge in later courses, and additional access points for students. Context also allows students to engage more quickly with the material (Perin, 2011; Schoenfeld, 2014). Additionally, instructors should strive to anchor algebraic skills with conceptual understanding of basic mathematics. For example, when addressing factoring, instructors could encourage students to realize the importance of factoring by seeing how a factored expression quickly yields different information than an unfactored expression, by exploring the connections between factors and roots, and so on.

To help build students' confidence and ability to persevere, instructors should help students discover their own strategies for performing certain algebraic manipulations and encourage students to share successful strategies with their peers (Abell et al., 2017; Schoenfeld, 2014). Most important, instructors can facilitate their classes in such a way that students realize that there is no single right way to "solve" a problem. The power of having students explore rich problems that allow for alternate approaches can also empower a broader range of students. Often if students simply use the skills they know, and use them correctly, they can eventually make progress and be successful in performing algebraic manipulations. Contextualizing content can provide immediate meaning for students and simultaneously increase their desire and/or willingness to struggle and persevere with a task.

Given students' repeated algebra experiences, faculty should keep three strategies in mind during students' pathway to calculus: (1) be aware of the mathematics education research about teaching and learning algebra; (2) take a meaningful approach to algebra in the classroom; and (3) provide more opportunities for students to build up confidence and perseverance in their algebraic abilities (Abell et al., 2017; Perin, 2011; Schoenfeld, 2014).

Additionally, research in mathematics education suggests that success in calculus is tied to students' ability to both gain a deep understanding of the function process and become proficient in covariational reasoning (Carlson et al., 2010; Oehrtman et al., 2008). Students usually exhibit either an "action view" or a "process view" of function. When students have an action view of a function, students tend to see functions as static entities and are typically focusing on computations involving evaluating functions at a single point (Carlson et al., 2010). Students who have developed an understanding of the process view of a function perceive functions as processes that can be composed and inverted; they understand that functions are processes that take a continuum of input values and produce a continuum of output values and that functions are used to model dynamic situations. Alternatively, students who lack the process view of a function:

- Have difficulty composing and inverting functions.
- Exhibit an inability to use functions effectively in word problems.
- View graphs of functions as fixed curves and not as a representation of a relationship between input and output values.
- View points on graphs and slopes of tangent or secant lines as fixed geometric properties of graphs and not as properties of the underlying function.
- Conflate the shape of a graph with the physical situation being modeled.
- Exhibit an inability to use symbols meaningfully.

In addition to developing a deep understanding of functions as processes, research shows that students should also develop the ability to apply covariational reasoning to a variety of situations (Carlson et al., 2010; Oehrtman et al., 2008). Covariational reasoning is the ability to consider how one quantity changes while imagining or visualizing changes in another quantity. Clearly this is an essential concept for understanding the major concepts in calculus. However, students who complete rigorous precalculus preparation may still have difficulty keeping two covarying quantities in their mind at the same time. These reasoning skills need to be intentionally introduced, developed, and mastered in any course or series of courses preparing students for calculus.

Rich Curricular Ideas, Pedagogical Strategies, and Student Success

Calculus instructors are challenged to decide how all of the mathematics education research presented above can best be realized in their classrooms. Based on research by Carlson et al. (2010), Oehrtman (2008), Perin (2011), and Schoenfeld (2014), the authors suggest the following eight instructional strategies:

- *Explore concepts with multiple representations:* Allow students to explore quantitative relationships represented with tables, formulas, words, and graphs as processes that can be represented in many different ways.
- *Emphasize that functions are processes and not algebraic formulas:* Whenever possible, focus students' attention on the underlying process that maps input values to output values. Allow students to try reversing the process (identify output values with input values) or composing it with other processes. Regardless of the representation being used, intentionally emphasize the difference between the process (function) and the representation.
- *Allow students to practice describing the behavior of functions on entire intervals:* Whenever students are evaluating functions at specific values, supplement this work with prompts that require students to consider an interval of input values or output values.
- *Develop the language and inclination to describe how one quantity changes with respect to another:* Contextualization provides an inclination to describe how quantities change. Use a curriculum that gives students daily practice in describing change such as average rate of change and the notions of increasing and decreasing.
- *Practice dynamic reasoning:* Provide students with many opportunities to describe dynamic situations such as heights of objects falling, distances and velocities of moving objects, geometric objects with changing dimensions, and so on.
- *Explore algebraic concepts within authentic STEM contexts and models:* Allow students to see the positive effects of algebraic manipulations within contexts that are related to their future fields of study. These modeling experiences reinforce the usefulness of algebra and encourage the exploration of multiple problem-solving strategies.
- *Build in frequent opportunities for students to practice communicating both orally and in writing:* Provide designated time in class for students to share results among themselves as well as with the entire class. Support and require written explanations, and give students formative feedback on how to improve.
- *Make the algebra meaningful!* Provide many contextualized problems as well as connect algebraic procedures with situations in which they are immediately meaningful to the students.

The above suggestions inextricably combine curricular ideas and pedagogical strategies. The success of a re-envisioned pathway to calculus is undoubtedly affected by both. Examining a student's preparation for calculus should not simply entail checking off a list of algebraic skills or function types they have been exposed to. Rather, faculty need to take into account specific methodology of how functions have been introduced and internalized, the amount of time students have had communicating about changing quantities and describing the relationships between changing quantities, and the opportunities available to students to explore algebraic concepts within an engaging context. These notions coupled with the non-cognitive factors identified earlier should be central to a student's pathway to calculus.

Conclusion

A well-designed pathway to calculus that focuses on the following can better serve a broad audience of students seeking STEM degrees:

- Minimize the negative impacts of critical non-cognitive factors.
- Make mathematics meaningful through contextualized mathematics.
- Understand the central role of algebra, functions, and change in the learning of calculus.
- Use research-based pedagogical strategies.

By attending to non-cognitive factors and providing appropriate content and support, the pathway to calculus can simultaneously remove barriers for vulnerable populations such as women and underrepresented minorities, provide the deep conceptual understandings of functions and covariational reasoning skills required for success in calculus, and allow all students to actively engage and persevere in contextualized and meaningful problem solving. The pathway to calculus should become a well-travelled highway that allows all interested students to enjoy the growing number of opportunities that come with the successful completion of a rigorous calculus course.

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Frank Savina leads the design, development, and implementation of the pathway to Calculus for the Dana Center Mathematics Pathways (DCMP). He presents findings at national and regional conferences of professional mathematics organizations to inform the mathematics community about best practices in preparing students for calculus. Additionally, Frank collaborates with faculty at two-year and four-year institutions to inform revision of the Charles A. Dana Center's higher education courses to better serve gateway mathematics students. Before joining the Dana Center, Frank spent 17 years teaching mathematics at the high school, community college, and university levels.