Foundations of Mathematical Reasoning Course Outcomes

Dana Center Mathematics PATHWAYS

About *Foundations of Mathematical Reasoning*

Foundations of Mathematical Reasoning is a semester-long, quantitative literacy-based course designed to provide students with the skills and conceptual understanding to succeed in a college-level statistics, quantitative literacy, or STEM path algebraic reasoning course.

Foundations of Mathematical Reasoning is organized around big mathematical and statistical ideas. The course will help students develop conceptual understanding and acquire multiple strategies for solving problems (as described in the DCMP Curriculum Design Standards). The course will prepare students for success in future courses and will help them develop skills for the workplace and as productive citizens.

Foundations of Mathematical Reasoning is not for college-level credit. The Dana Center recommends that students enrolled in this course also take a co-requisite student success course*, Frameworks for Mathematics and Collegiate Learning*.

Course structure and contact hours

Foundations is designed to be taught in a one-semester course with 4 student-contact hours per week or in a quarter system with an equivalent number of contact hours. Colleges may choose to offer this as a 4-credit course or as a combination of course credits and lab credits. Regarding lab credits, it is important to note that the curriculum is not designed for instruction in separate and distinct lab meetings. Rather, the intent is for the instructor to use all 4 contact hours for classroom instruction.

Active learning design standards are evident in each lesson. Students will be expected to actively do mathematics—such as analyzing data, constructing hypotheses, solving problems, reflecting on their work, and making connections among and between mathematical concepts.

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Structure of the curriculum

The curriculum is designed in 25-minute learning episodes, which can be pieced together to conform to any class length. These short bursts of active learning, combined with whole class discussion and summary, produce increased memory retention.[1]

[1] Sources: Buzan, T. (1989). *Master your memory* (Birmingham: Typersetters); Buzan, T. (1989). *Use your head* (London: BBC Books); Sousa, D. (2011). *How the brain learns, 4th ed.* (Thousand Oaks, CA: Corwin); Gazzaniga, M., Ivry, R. B., & Mangun, G. R. (2002). *Cognitive neuroscience: The biology of the mind, 2nd ed.* (New York: W.W. Norton); Stephane, M., Ince, N., Kuskowski, M., Leuthold, A., Tewfik, A., Nelson, K., McClannahan, K., Fletcher, C., & Tadipatri, V. (2010). Neural oscillations associated with the primary and recency effects of verbal working memory. *Neuroscience Letters*, 473, 172–177.); Thomas, E. (1972). The variation of memory with time for information appearing during a lecture. *Studies in Adult Education,* 57–62.

Prerequisite assumptions

Students enrolling in *Foundations of Mathematical Reasoning* should be able to do the following:

- Demonstrate procedural fluency with real number arithmetic operations (e.g., basic operations, comparing, contrasting), use arithmetic operations to represent real-world scenarios, and use those operations to solve stated problems.

- Use graphical representations on a real number line to demonstrate fluency when ordering numbers, to represent operations (e.g., addition, subtraction, doubling, halving, averaging), and to represent fractions and decimals.

- Demonstrate a basic understanding and familiarity with fractions, decimals, and percentages. Procedural competency for representing each number form and moving from one to the other is desired upon enrollment in this course, but the student may also need to work with materials outside of class to review basic concepts and build basic skills.

Learning goals

The following five learning goals apply to all DCMP mathematics courses, with the complexity of problem-solving skills and use of strategies increasing as students advance through the pathways.

For each course, we define the ways that the learning goals are applied and the expectations for mastery. The bullets below each of the five learning goals specify the ways in which each learning goal is applied in the *Foundations of Mathematical Reasoning* course.

Each DCMP course is designed so that students meet the goals across the courses in a given pathway. Within a course, the learning goals are addressed across the course's content-based learning outcomes.

Communication Goal: Students will be able to interpret and communicate quantitative information and mathematical and statistical concepts using language appropriate to the context and intended audience.

In the *Foundations* course, students will…

- Use appropriate mathematical language.

- Read and interpret short, authentic texts such as advertisements, consumer information, government forms, and newspaper articles containing quantitative information, including graphical displays of quantitative information.

- Write 1 to 2 paragraphs using quantitative information to make or critique an argument or to summarize information from multiple sources.

Problem Solving Goal: Students will be able to make sense of problems, develop strategies to find solutions, and persevere in solving them.

In the *Foundations* course, students will...

- Solve multi-step problems by applying strategies in new contexts or by extending strategies to related problems within a context.

Reasoning Goal: Students will be able to reason, model, and make decisions with mathematical, statistical, and quantitative information.

In the *Foundations* course, students will...

- Make decisions in quantitatively based situations that offer a small number of defined options. The situations will not be limited to contexts in which there is a single correct answer based on the mathematics (e.g., which buying plan costs less over time), but will include situations in which the quantitative information must be considered along with other factors.
- Present short written or verbal justifications of decisions that include appropriate discussion of the mathematics involved.

Evaluation Goal: Students will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information.

In the *Foundations* course, students will...

- Identify mathematical or statistical errors, inconsistencies, or missing information in arguments.

Technology Goal: Students will be able to use appropriate technology in a given context.

In the *Foundations* course, students will...

- Use a spreadsheet to organize quantitative information and make repeated calculations using simple formulas.
- Use the internet to find quantitative information on a given subject. The topics should be limited to those that can be researched with a relatively simple search.
- Use internet-based tools appropriate for a given context (e.g., an online tool to calculate credit card interest).

Content learning outcomes

The content learning outcomes include both mathematical and contextual topics in keeping with the role of this course as a quantitative literacy course. The topics for the *Foundations of Mathematical Reasoning* course are:

- Numeracy
- Proportional Reasoning
- Algebraic Competence, Reasoning, and Modeling
- Probabilistic Reasoning to Assess Risk
- Quantitative Reasoning in Personal Finance
- Quantitative Reasoning in Civic Life

Numeracy

Outcome: Students will develop number sense and the ability to apply concepts of numeracy to investigate and describe quantitative relationships and solve real-world problems in a variety of contexts.

Students will be able to:

N.1 Demonstrate operation sense and communicate verbally and symbolically with real numbers.

For example: Know when and how to perform arithmetic operations with the use of technology. Use order of operations to identify an error in a spreadsheet formula. Predict the effects of multiplying a number by a number between 0 and 1.

N.2 Demonstrate an understanding of fractions, decimals, and percentages by representing quantities in equivalent forms, comparing the size of numbers in different forms and interpreting the meaning of numbers in different forms.

For example: Write a fraction in equivalent decimal form and vice versa. Compare growth expressed as a fraction versus as a percentage. Interpret the meaning of a fraction. Interpret the meaning of percentages greater than 100% and state if such a percentage is possible in a given context.

N.3 Solve problems involving calculations with percentages and interpret the results.

For example: Calculate a percentage rate. Explain the difference between a discount of 30% and two consecutive discounts of 15%. Calculate relative change and explain how it differs from absolute change.

N.4 Demonstrate an understanding of large and small numbers by interpreting and communicating with different forms (including words, fractions, decimals, standard notation, and scientific notation) and compare magnitudes.

For example: Compare large numbers in context, such as the population of the US compared to the population of the world. Calculate ratios with large numbers such as water use per capita for a large population. Interpret a growth rate less than 1%.

N.5 Use estimation skills, and know why, how, and when to estimate results.

For example: Identify and use numeric benchmarks for estimating calculations (e.g., using 25% as an estimate for 23%). Identify and use contextual benchmarks for comparison to other numbers (e.g., using US population as a benchmark to evaluate reasonableness of statistical claims or giving context to numbers). Check for reasonableness using both types of benchmarks.

N.6 Solve problems involving measurement including the correct use of units.

For example: Identify the appropriate units for perimeter, area, and volume. Calculate the amount of paint needed to paint a non-rectangular surface. Interpret measurements expressed in graphical form.

N.7 Use dimensional analysis to convert between units of measurements and to solve problems involving multiple units of measurement.

For example: Convert between currencies. Calculate the cost of gasoline to drive a given car a given distance. Calculate dosages of medicine.

N.8 Read, interpret, and make decisions about data summarized numerically (e.g., measures of central tendency and spread), in tables, and in graphical displays (e.g., line graphs, bar graphs, scatterplots, and histograms).

For example: Critique a graphical display by recognizing that the choice of scale can distort information. Explain the difference between bar graphs and histograms. Explain why the mean may not represent a typical salary.

Proportional Reasoning

Outcome: Students will use proportional reasoning to solve problems that require ratios, rates, proportions, and scaling.

Students will be able to:

PR.1 Represent, and use ratios in a variety of forms (including percentages) and contexts.

For example: Interpret a rate of change within a context using appropriate units. Interpret a percentage as a number out of 1,000. Compare risks expressed in ratios with unequal denominators (e.g., 1 in 8 people will have side effects versus 2 in 14).

PR.2 Determine whether a proportional relationship exists based on how one value influences another.

For example: Simple versus compound interest. Analyze whether an estimated percentage is reasonable based on proportions. Analyze the effects of scaling and shrinking that are proportional and non-proportional (e.g., the impact of changing various dimensions on perimeter, area, and volume).

PR.3 Analyze, represent, and solve real-world problems involving proportional relationships, with attention to appropriate use of units.

For example: Use individual water-use rates to predict the water used by a population. Use the Consumer Price Index to compare prices over time. Use a scale to calculate measurements in a graphic.

Algebraic Competence, Reasoning, and Modeling

Outcome: Students will transition from specific and numeric reasoning to general and abstract reasoning using the language and structure of algebra to investigate, represent, and solve problems.

NOTE: The goal is to capture the broad concept of a function but not in a formal mathematical manner. The focus is on solving problems and modeling, not on defining or exploring the concept of function in detail (e.g., no domain or range concepts). Students will represent relationships between quantities using multiple strategies to solve problems.

Students will be able to:

A.1 Demonstrate understanding of the meaning and uses of variables as unknowns, in equations, in simplifying expressions, and as quantities that vary, and use that understanding to represent quantitative situations symbolically.

For example: Understand the different uses of variables and the difference between a variable and a constant. Be able to use variables in context and use variables as placeholders, as in formulas. Write an algebraic expression to represent a quantity in a problem. Combine simple expressions. Use notation with variables (e.g., exponents, subscripts) in simple and moderately complex expressions.

A.2 Describe, identify, compare, and contrast the effect of multiplicative or additive change.

For example: Compare and contrast the rate of change and/or behavior of a linear and an exponential relationship in context. Recognize that a multiplicative change is different from an additive change. Explain how the rate of change of a linear relationship differs from an exponential rate of change, as well as the

ramifications of exponential change (growth can be very slow for a time but then increase rapidly).

A.3 Analyze real-world problem situations, and use variables to construct and solve equations involving one or more unknown or variable quantities.

For example: Demonstrate understanding of the meaning of a *solution*. Write a spreadsheet formula to calculate prices based on percentage mark-up. Solve a formula for a given value. Identify when there is insufficient information given to solve a problem.

A.4 Express and interpret relationships using inequality symbols.

For example: Use inequalities to express the relationship between the probabilities of two events or the size of two groups. Interpret a histogram based on intervals expressed with inequality symbols.

A.5 Construct and use mathematical models to solve problems from a variety of contexts and to make predictions/decisions.

Representations will include linear and exponential contexts.

For example: Given a statement of how the balance in a savings account grows because of monthly interest, construct a table of months and balances and then write a mathematical model that provides the balance for a given month.

A.6 Represent mathematical models in verbal, algebraic, graphical, and tabular form.

For example: Be able to move from any one representation to another. Given an initial value and information about change, create a table, graph, and/or algebraic model. Given an algebraic model, create a table of values.

A.7 Recognize when a linear model is appropriate and, if appropriate, use a linear model to represent the relationship between two quantitative variables.

For example: Given a set of data, make an informal, intuitive evaluation of the applicability of a linear or exponential model with a focus on recognizing the limitations of the model and identifying an appropriate domain (do not need to use this term with students) for which the model might be used to make accurate predictions. Describe the rate of change using appropriate units: slope for linear relationships, or average rate of change over an interval for nonlinear relationships.

Probabilistic Reasoning to Assess Risk

Outcome: Students will understand and critically evaluate statements that appear in the popular media (especially in presenting medical information) involving risk and arguments based on probability.

Students will be able to:

R.1 Interpret statements about chance, risk, and probability that appear in everyday media (including terms like unlikely, rare, impossible).

For example: Interpret statements such as "for a certain population the risk of a particular disease is 0.005". Compare incidences of side effects in unequal group sizes.

R.2 Identify common pitfalls in reasoning about risk and probability.

For example: Identify inappropriate risk statements, such as when the size of reference groups is unknown (e.g., California, 2009, 88% of motorcycle accident fatalities were helmeted, 12% unhelmeted).

R.3 Interpret in context marginal, joint, and conditional relative frequencies in context for data summarized in a two-way table and identify which relative frequency is appropriate to answer a contextual question.

For example: Distinguish between reported relative frequencies that are marginal, joint, or conditional. Choose the relative frequency that is the most informative for a given purpose. Choose the appropriate direction of conditioning for a given context (the chance of cancer given a positive test result is not the same as the chance of a positive test result given cancer).

R.4 Demonstrate understanding of absolute risk and relative risk (percentage change in risk) by describing how each provides different information about risk.

For example: Interpret the different information conveyed when comparing the magnitude of the absolute risks and percentage change in risk (e.g., an 80% increase in risk associated with taking a particular medication could mean a change in risk from 0.001 to 0.0018 or from 0.1 to 0.18).

Quantitative Reasoning in Personal Finance

Outcome: Students will understand, interpret, and make decisions based on financial information commonly presented to consumers.

Students will be able to:

PF.1 Demonstrate understanding of common types of consumer debt and explain how different factors affect the amount that the consumer pays.

For example: Calculate the interest paid on credit card debt based on a credit score; explain how the length of the pay-off period affects the total interest paid; demonstrate the relationship between a percentage rate and the amount of interest paid; define basic terminology such as principal, interest rate, balance, minimum payment, etc.

PF.2 Demonstrate understanding of compound interest and how it relates to saving money.

For example: Demonstrate the different impacts of the saving period and the amount saved on the accumulated balance; use a given formula to calculate a balance; demonstrate an understanding of the meaning of a compounding period and use the appropriate terminology for different periods (e.g., quarterly, annually, etc.).

PF.3 Identify erroneous or misleading information in advertising or consumer information.

For example: Explain why statements about "average" benefits of a product such as a weight loss plan are misleading; identify misleading graphs that create an appearance of greater impact than is warranted.

Quantitative Reasoning in Civic Life

Outcome: Students will understand that quantitative information presented in the media and by other entities can sometimes be useful and sometimes be misleading.

Students will be able to:

CL.1 Use quantitative information to explore the impact of policies or behaviors on a population. This might include issues with social, economic, or environmental impacts.

For example: Calculate the effects of a small decrease in individual water use on the amount of water needed by a large population over time; determine if the minimum wage has kept pace with inflation over time.

CL.2 Identify erroneous, misleading, or conflicting information presented by individuals or groups regarding social, economic, or environmental issues.

For example: Explain how two statements can be both contradictory and true (e.g., the "average" amount of a tax cut expressed in terms of the mean and the median); identify how two pie charts representing different populations can be misleading.