Reasoning with Functions I
Course Outcomes

Dana Center
Mathematics
PATHWAYS

About *Reasoning with Functions I*

Reasoning with Functions I prepares students to enter Calculus and succeed in future STEM coursework. The course is for students who have completed *Foundations of Mathematical Reasoning* or placed at the Intermediate Algebra level. Additionally, these students plan on taking Calculus or pursuing other technical or STEM coursework that requires a thorough knowledge of functions and algebraic reasoning.

Course structure and contact hours

Reasoning with Functions I aligns with Math 1314/1414 *College Algebra* in the Texas Academic Course Guide Manual. It gives students a strong foundation in functions and their behavior by using multiple representations and explicit covariational reasoning to investigate and explore quantities, their relationships, and how these relationships change. Additionally, this course provides students with the algebraic tools necessary to analyze a variety of function types, including linear, quadratic, polynomial, power, exponential, and logarithmic functions.

Active and collaborative learning form the basis of the in-class lessons, while independent learning and strong study habits are fostered through out-of-class assignments. The curriculum adheres to the DCMP Curriculum Design Standards and presents students with meaningful problems that arise from a variety of science, technology, engineering, and mathematical contexts. After completing this course, students are prepared for the second course in this pathway, *Reasoning with Functions II,* which will prepare them for Calculus and other STEM courses.

Reasoning with Functions I requires 5 student-contact hours per week or in a quarter system an equivalent number of contact hours. Colleges may offer this as a five-credit course or as a combination of three course credits and two developmental credits within the same semester. Regarding developmental credits, it is important that the two developmental credits and three course credits be taught sequentially.

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Structure of the curriculum

The curriculum is designed in 25-minute lessons, which can be pieced together to conform to any class length. These short bursts of active learning, combined with class discussion and summary, produce increased memory retention.[1] The instructor lesson plans contain

[1] Sources: Buzan, T. (1989). *Master your memory* (Birmingham: Typersetters); Buzan, T. (1989). *Use your head* (London: BBC Books); Sousa, D. (2011). *How the brain learns, 4th ed.* (Thousand Oaks, CA: Corwin); Gazzaniga, M., Ivry, R. B., & Mangun, G. R. (2002). *Cognitive neuroscience: The biology of the mind, 2nd ed.* (New York: W.W. Norton); Stephane, M., Ince, N., Kuskowski, M., Leuthold, A., Tewfik, A., Nelson, K., McClannahan, K., Fletcher, C., & Tadipatri, V. (2010). Neural oscillations associated with the primary and recency effects of verbal working memory. *Neuroscience Letters*, 473, 172–177.); Thomas, E. (1972). The variation of memory with time for information appearing during a lecture. *Studies in Adult Education,* 57–62.

facilitating questions to guide class discussions or help struggling students, suggestions to classroom pedagogy (individual work, small group experiences, think-pair-share, class discussion, or direct instruction), language and literacy support, possible student misconceptions, and explicit connections from the day's learning objectives to future course work in a STEM discipline. Some lessons will suggest alternative pathways through the content and instructors should feel welcome to modify their own approach to the lesson based on their personal experience and their understanding of their students.

When students are working independently outside of class, they will be offered a variety of problems that range from easy to more challenging. By having access to hints, answers, and explanations, students will receive immediate feedback on their understanding and skill mastery. This portion of the student's learning will be facilitated through the use of an online learning platform.

In addition to assignments facilitated by the technology platform, students will be expected to work regularly in an environment without instant feedback or the assistance of tutors and classmates. During this time, students will work on problems based on recent in-class activities as well as actively prepare for the next class meeting. Students will prepare for upcoming lessons by performing tasks such as learning new terminology, learning and practicing a new skill, or starting to immerse themselves in the scenarios or problem situations that will be central to the next lesson. Before every lesson, students will read and perform a self-assessment on a set of prerequisite knowledge.

Readiness competencies

Students enrolling in *Reasoning with Functions I* should be able to do the following:

- Demonstrate a basic understanding and familiarity with fractions, decimals, and percentages.
- Demonstrate number sense, including dimensional analysis and conversions between fractions, decimals, and percentages. Determine when approximations are appropriate and when exact calculations are necessary.
- Interpret and evaluate expressions involving variables.
- Create and interpret linear models within a variety of contexts.
- Solve linear equations and apply formulas.

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Learning goals

The following five learning goals apply to all DCMP mathematics courses, with the complexity of problem-solving skills and use of strategies increasing as students advance through the pathways.

For each course, we define the ways that the learning goals are applied and the expectations for mastery. The bullets below each of the five learning goals specify the ways in which each learning goal is applied in the *Reasoning with Functions I* course.

Each DCMP course is designed so that students meet the learning goals across the courses in a given pathway. Within a course, learning goals are addressed across the course's content-based learning outcomes.

Communication Goal: Students will be able to interpret and communicate quantitative information and mathematical and statistical concepts using language appropriate to the context and intended audience.

In *Reasoning with Functions I*, students will…

- Interpret statements containing function notation, communicate about function processes, and use function notation.
- Interpret and communicate the behavior of functions on entire intervals in addition to single points.
- Read graphs, tables, and verbal descriptions of dynamic scenarios.
- Communicate their conclusions in both spoken and written form and support their conclusions by providing appropriate mathematical justifications.

Problem Solving Goal: Students will be able to make sense of problems, develop strategies to find solutions, and persevere in solving them.

In *Reasoning with Functions I*, students will...

- Develop a predisposition to consider a variety of approaches to a mathematical problem, identify an appropriate strategy, persist in applying that strategy, and reflect on the outcome of that strategy.
- Practice solving multistep problems in a variety of contexts related to science, technology, engineering, and mathematics.

Reasoning Goal: Students will be able to reason, model, and make decisions with mathematical, statistical, and quantitative information.

In *Reasoning with Functions I*, students will...

- Examine and explore functions using multiple representations and dynamic reasoning.
- Acquire covariational reasoning strategies by exploring patterns of change between two related quantities in various contexts and representations.
- Create mathematical models in a variety of meaningful mathematical applications and use these models to make decisions.

Evaluation Goal: Students will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information.

In *Reasoning with Functions I*, students will...

- Identify constraints and limitations for mathematical models in a variety of contexts and representations.

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- Critically reflect on the reasonableness of their solutions.

Technology Goal: Students will be able to use appropriate technology in a given context.

In *Reasoning with Functions I*, students will...

- Develop proficiency with appropriate technology and understand when technology use is most appropriate.
- Use technology to generate graphs of functions, find roots and intercepts, and locate points of intersection.

Content learning outcomes

The content learning outcomes ensure students develop a firm foundation in functions and algebraic reasoning. The learning outcomes for *Reasoning with Functions I* are organized around three topics:

- Foundations of Functions
- Analysis of Functions
- Algebraic Reasoning

Foundations of Functions

Outcome: Students will use multiple representations of different function types to investigate quantities, describe relationships between quantities, and attend to how two quantities change together.

Students will be able to:

FF.1 Conceptualize quantities and define variables that are present in a given situation.

To include: Measurement and association of units with numerical values and "delta" notation to denote the changes in quantities.

FF.2 Use multiple representations of functions to interpret and describe how two quantities change together.

To include: Justifying the presence of a relationship, identifying constraints on quantities and domains, distinguishing between dependent and independent variables, attention to domains and ranges, and drawing diagrams of dynamic situations.

FF.3 Measure, compute, describe, and interpret rates of change of quantities embedded in multiple representations.

To include: Constant rates of change, average rates of change, and intuitive treatments of instantaneous rates of change.

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FF.4 Effectively communicate with function notation.

To include: The justification/motivation for function notation and the multiple ways to represent functions.

FF.5 Use appropriate tools and representations to investigate the patterns and relationships present in multiple function types.

To include: Linear, quadratic, exponential, logarithmic, rational, periodic, piecewise, and absolute value functions.

Analysis of Functions

Outcome: Students will describe characteristics of different function types and convert between different representations and algebraic forms to analyze and solve meaningful problems.

Students will be able to:

AF.1 Create, use, and interpret linear equations and convert between forms as appropriate.

To include: Ability to read important values (e.g. slope & intercepts) from multiple representations, calculating equations of lines given 1) point & slope, 2) two points, or 3) statements about proportional relationships and/or first differences being constant.

AF.2 Create, use, and interpret exponential equations and convert between forms as appropriate.

To include: Modeling constant percent change (over multiple and fractional units of change in input), half-life, doubling time, similarities and differences with linear functions (first differences), rate of change is exponential (with the same base), rate of change is proportional to amount, the role of "e" as a natural base, describing long-term behavior, inverting the exponentiation process (logarithms).

AF.3 Use and interpret polynomial, power, and rational functions.

To include: How power functions are different from exponential functions (first differences of quadratics), sum of rational functions, introduction of basic limit ideas as they pertain to horizontal and vertical asymptotes, symmetries of even/odd functions, language of maximum/minimum/turning points, relevance of roots.

AF.4 Construct, use, and describe transformations and operations of functions.

To include: Operations of functions, vertical and horizontal shifts and stretches.

AF.5 Construct, use, and describe composition of functions.

To include: How composition of functions can be used to generate other important functions, how composition of functions transmits variation.

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AF.6 Construct, use, and describe inverses of functions.

To include: Roots (radicals) and logarithms.

Algebraic Reasoning

Outcome: Students will identify and apply algebraic reasoning to write equivalent expressions, solve equations, and interpret inequalities.

Students will be able to:

AR.1 Use factoring techniques to simplify expressions and locate roots.

To include: The distributive property, multiplication of polynomials, completing the square, and work with inequalities (as they arise from absolute value, distances, and other similar geometric interpretations).

AR.2 Use algebraic reasoning to simplify a variety of expressions and find roots of equations involving multiple function types.

To include: Facility with rules for exponents and logarithms; polynomial, power, radical, and rational functions; and asymptotic behavior of functions near roots of the denominator and as x increases/decreases without bound.

AR.3 Recognize, solve, and apply systems of linear equations using matrices.

To include: Setting up and solving systems of linear equations using simple substitution and Gaussian elimination.