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Chapter 3

Content Trends in Quantitative Reasoning Courses

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Abstract

This chapter traces the rise of quantitative reasoning (QR) course offerings, along with the factors that led to the wide variability in the content of those courses, the concerns that resulted from that variability, and the responses to those concerns. The advent of mathematics pathways has resulted in increasing numbers of students taking QR courses. This growth in QR enrollment has revealed issues with transfer between institutions and applicability to programs, resulting in increased costs to students in terms of both time and money. Research indicates that regions and states are moving toward standardization of learning outcomes to alleviate those issues. This chapter documents the trends in standardized mathematics outcomes in QR courses toward argumentation and communication, proportional reasoning, probability and statistics, and modeling as well as some common applications such as financial literacy and technology. Recommendations are presented for those looking to begin conversations about quantitative reasoning in their own region or state.

Introduction

The Quantitative Reasoning (QR) movement can trace its origins to the publication of the text, *Mathematics and Democracy* (National Council on Education and the Disciplines [NCED], 2001), which laid out the rationale for QR coursework in terms of requisite elements, expressions, and skills. The authors of this text point to an earlier document (Sons et al., 1996) for the list of skills that would help instructors plan curricula. Since that time, QR curricula have proliferated across the country with QR pathways introduced in 2009 and now available in many states and institutions. The goals and characteristics of these courses remain heavily influenced by the *Mathematics and Democracy* design team's admonition that "quantitative literacy is inseparable from its context. In this respect it is more like writing than like algebra, more like speaking than like history. Numeracy has no special content of its own, but inherits its content from its context" (NCED, 2001, p.17). This priority of developing the habits of mind required to be a numerate citizen over skill development has led to wide variability in the content and outcomes of QR courses. In an effort to help the mathematics community address the inconsistencies and issues of transferability of courses and applicability to programs, the authors of this chapter embarked on a research project to synthesize the extant learning outcomes in QR curricula at the state level.

In this chapter, the trends in both content and contexts among the states that have worked toward a standardized quantitative pathway are analyzed. In addition, recommendations for systems and states to develop consistent and coherent student learning outcomes that are relevant to students' programs of study and lives are presented.

Mathematics Pathways and Quantitative Reasoning Courses

The expansion of mathematics pathways is resulting in a proliferation of entry-level quantitative reasoning courses aimed at providing students with the mathematics needed to meet the quantitative demands of everyday life. The Mathematical Association of America (MAA) and other professional societies accelerated this process with a recommendation in 2015 for implementation of multiple mathematics pathways aligned to fields of study, some of which should include early exposure to statistics, modeling, and computation (Burdman, 2015; Saxe & Braddy, 2015). This recommendation grew out of the MAA's acknowledgment in 2004 that College Algebra was not an appropriate default gateway course for mathematics (MAA, 2004). In response, many institutions are redesigning College Algebra to serve solely as an entry point to the path to calculus for STEM and business majors, and aligning their remaining programs of study toward introductory statistics courses or a general education mathematics course to serve a quantitative pathway.

With today's highly mobile student population, transfer between institutions often leads to increased costs, lost credits, and decreased likelihood of completion (National Center for Public Policy and Higher Education, 2011). These issues have necessitated a closer inspection of the content of QR courses with the goal of standardizing learning outcomes and ensuring applicability. The last few years have seen movement toward standardization of the content among regional transfer partners and at the state level and across states.

A further complication has been that examples of the general education courses often required for liberal arts and fine arts and other similar programs are found under a variety of names such as quantitative reasoning, quantitative literacy, liberal arts math, contemporary math, and math in society, among others. Getz, Richardson, Hartzler, and Leahy (in press) noted the rise in enrollment of such courses in two-year colleges. According to the National Student Clearinghouse Research Center (Shapiro et al., 2017), 46 percent of all students who completed a degree at a four-year institution in 2013–2014 had been enrolled at a two-year institution at some point in the previous 10 years.

The content and intention of these QR courses have varied widely within and between systems and states. Examples abound of course descriptions that allow the instructor to choose the topics for the course from a list (e.g., Texas' former description stated, "Topics may include…."). The authors' analysis of 20 sets of "QR learning outcomes" from 18 states led to a differentiation between QR courses and Math for the Liberal Arts (MLA) courses grouped in the following categories (see table below).

The decision of which specific QR learning outcomes to include under MLA courses was partly due to four or fewer states choosing these for their QR courses and partly due to the authors' inspection of MLA textbooks' tables of contents. The five broad categories for QR courses listed in Table 1 will be broken down into specific learning outcomes and analyzed in more depth in a following section.

States and Their Courses

The content of these QR courses is starting to crystallize in several leading states (Gaze, 2014), but much variability remains across the country. Research during the summer and fall of 2017 uncovered 18 states that have common learning outcomes for courses that we will describe as being under the QR umbrella (see Table 4; note that there are 20 sets of outcomes due to two courses each in Georgia and Florida). Many of these outcomes are mandated from the state level but, in a few locations, they are simply an informal agreement across institutions. The outcomes vary from broad to highly prescriptive. Florida provides an example of a broad outcome, asking instructors to introduce students to "the beauty and utility of mathematics." On the other end of the spectrum, Georgia specifies in part

Table 1. Broad learning outcomes for QR courses vs. specific outcomes for MLA courses

QR Courses (broad categories)	MLA Courses (specific outcomes)
Argumentation/Communication	Math Appreciation
Proportional Reasoning	History of Math
Probability and Statistics	Sets
Modeling	Geometry
Applications	Graph Theory
	Art
	Across Disciplines

that students should "be able to distinguish among linear, quadratic and exponential growth models." Common QR student learning outcomes also range from very sparse (27 words in Arkansas) to highly detailed (six-plus pages in Ohio and Virginia).

Content and Context

Inspection of the 20 sets of QR learning outcomes reveals many commonalities in content. For example, 14 sets of outcomes stipulate instruction in statistics while 13 indicate probability content. However, the depth and breadth of the outcomes related to these two topics differ among the states, such as ranging from descriptive statistics to statistical inference. The term *modeling* is used in only five outcomes documents, but various modeling topics permeate almost all of the examples.

See Table on the next page.

Table 2. Number of states selecting learning outcomes using the authors' designation of QR vs. non-QR

QR Courses	# of States
Argumentation/Communication	
Critical thinking	5
Decision-making/prediction	5
Communication	10
Analyze arguments	6
Construct arguments	7
Logic	7
Proportional Reasoning	
Estimation/precision/reasonableness	7
Convert within/between different measurement scales	5
Rates/percentages/decimals	3
Number sense	7
Uses and abuses of percentages	3
Proportional reasoning	5
Absolute and relative change	4
Probability and Risk	
Probability, odds, risk	13
Statistics	14
Modeling	
Linear, non-linear, exponential growth	8
Modeling	5
Algebraic, symbolic reasoning	7
Multiple representations	10
Applications	
Use appropriate technology	8
Develop problem solving strategies	8
Real-world data applications	8
Consumer/financial math	12
Citizenship, social issues, voting, fair division	4

Non-QR Courses	# of States
Math for the Liberal Arts	
Math appreciation	3
History of math	3
Sets	4
Geometry	2
Graph theory	2
Art	1
Across disciplines	2
Miscellaneous	
Numbers and number systems	1
Elementary number theory	2
Systematic counting	1
Logarithmic functions	2
Optimization	1

Similar content may be described in a variety of ways. For example, probability content in Table 2 is sometimes stated as instruction related to odds or risk. Some outcomes refer to absolute and relative change; others require work with linear and exponential functions; and still others stipulate simple and compound interest. These related concepts may range from numerical work to symbolic work or both. Most outcomes contain little guidance as to the expected depth. More than half of the documents indicate the need to move between representations, thus spanning the range between numerical and symbolic forms.

The outcomes in Table 2 also illustrate that mathematical learning outcomes and application are often conflated. At least 12 outcomes reference financial math, with varying degrees of specificity. Ohio provides a robust example of separation of mathematics outcomes from their application as well as specificity of depth. Outcome 2.4 calls for the use of "basic logarithm properties" and then suggests sample tasks such as finding "the time required to achieve a personal savings goal."

Many state outcomes documents also contain ideas that may be more accurately described as pedagogical recommendations, cross-disciplinary goals, or expectations for use of tools. At least half specifically mention communication as a learning outcome or as a goal of the course, while others imply it (e.g.,"interpret solutions," "show an understanding…both orally and in writing"). A majority refer to critical thinking and/or problem solving, concepts that can be interpreted in many ways and difficult to measure. Seven are explicit about the use of technology to perform certain tasks or analyses.

These differences in structure could lead to a variety of organizational strategies. Table 2 shows how many states selected each specific outcome. For example, five states list "critical thinking" and five states list "decision making," but it is unclear if these are the same five states or 10 distinct states. To get a picture of how often states identified each broad category, the authors have broken out how many states have selected only one learning outcome from a category, only two learning outcomes, etc. Table 3 shows that 18 states have selected at least one learning outcome associated with the broad category of Argumentation/ Communication. Of these 18 states, seven have selected only one learning outcome from this category, and six have selected two, while only one state selected all six learning outcomes for the category.

Table 3. Broad categories for QR courses broken down by number of states selecting a given number of specific learning outcomes from each category

QR Courses (broad categories)	Number of States Selecting Various Number of Specific Outcomes							
	Any	1	2	3	4	5	6	7
Argumentation/Communication	18	7	6	2	2	0	1	NA
Proportional Reasoning	12	3	2	3	1	3	0	0
Probability and Statistics	14	13	14	NA	NA	NA	NA	NA
Modeling	14	5	6	2	1	NA	NA	NA
Applications	17	4	7	1	4	1	NA	NA

The outcomes from Arkansas, Michigan, and New Mexico do not align with the structure of the remaining states' outcomes. As noted earlier, the Arkansas outcomes are very brief and broad, calling for students to develop a basic understanding of and appreciation for mathematics; develop the ability to think and reason critically, quantitatively, and logically; and be able to analyze arguments. Michigan adapted its outcomes from the Association of American Colleges and Universities Quantitative Literacy Rubric, listing and describing the skills of interpretation, representation, calculation, application/analysis, and communication (Michigan Right Math & the Right Time Working Group, 2018). New Mexico requires students to construct and analyze graphs and/or data sets, use and solve various kinds of equations, understand and write mathematical explanations using appropriate definitions and symbols, and demonstrate problem-solving skills within the context of mathematical applications. It is worth noting that many traditional algebra courses could claim to satisfy these generic outcomes.

Call for Further Action

The effort to standardize the quantitative pathway outcomes and content is encouraging. In order to bring consistency across institutions and states, the following action items are recommended:

- Additional states or regions should undertake the work by forming a working group with broad representation from the field, collecting the recommendations of professional associations, and referring to work done in other states.
- States should follow the lead of North Carolina (see Todd & Wagaman, 2015), Indiana, Ohio, and others by including and codifying a focus on real data and source material as well as authentic applications that are relevant to all students' lives (i.e., financial, civic, risk literacy). Real data and sources are more engaging for students, demonstrate the life relevance of the mathematics, and can help prevent artificial problem constructs. *Mathematics and Democracy* calls for engaging our students with complex problems that are "anchored in data derived from and attached to the empirical world" (NCED, p. 5) and emphasizes that in a QR course, "content is inseparable from pedagogy and context is inseparable from content" (NCED, p. 18). Perhaps most important, analysis of authentic sources naturally leads to an understanding not only of uses of mathematics, but also of misuses, and can result in students who think for themselves rather than consume media reports. Steen (1999) asserted that "numbers have become the chief instruments through which we attempt to exercise control over nature, over risk, and over life itself" (p.10).
- A clear delineation in outcomes documents between mathematics content and the applications that must or may be used to demonstrate the relevance of those mathematics outcomes should be created.
- Within the classroom, the mathematics content should not be isolated from context; students must be able to apply their mathematical skills. Madison (2001) noted that applying mathematics is not easily learned, so instruction must contain the contextual use of the skills.
- Specificity in the mathematics outcomes and applications, which leads to clearer expectations of depth and breadth and therefore promotes transparency between transfer partners, should be encouraged.
- Lastly, much of the work in setting common outcomes impacts community colleges more than four-year institutions. Transfer among four-year institutions and reverse transfer from four-year to two-year institutions is also high (Shapiro et al., 2015). Consideration should be given to setting a common standard in two-year and four-year institutions in order to facilitate the attainment of relevant mathematics content and promote program completion.

In closing, the critical need for helping students gain the necessary skills to navigate the quantitative world we inhabit cannot be overstated. Robust quantitative reasoning curricula can empower students to fully participate in today's data-driven society. It is imperative that work continue on developing the fundamental skills outlined in this chapter that are required for informed decision making for all citizens.

Table 4. Courses in the Quantitative Reasoning umbrella

State	Course Number	Course Name
Arkansas	Math 1003	College Math
Florida	Math 106	Math for Liberal Arts I
Florida	Math 107	Math for Liberal Arts II
Georgia	Math 1001	Quantitative Reasoning
Georgia	Math 1101	Introduction to Math Modeling
Indiana Ivy Tech	Math 123	Quantitative Reasoning
Kansas	Mat 1040	Contemporary Math/Essential Math
Louisiana	CMAT 1103	Contemporary Mathematics
Maryland		Topics for Mathematical Literacy
Michigan		Quantitative Reasoning
Missouri		Mathematical Reasoning and Modeling
Montana	M105	Contemporary Mathematics
Nevada	Math 120	Fundamentals of College Math
New Mexico	Math 1110	Quantitative Reasoning
North Carolina	Mat 143	Quantitative Literacy
Ohio	TMM011	Quantitative Reasoning
Oregon	Math 105	Math in Society
Texas	Math 1332	Contemporary Mathematics (QR)
Virginia CCS	MTH 154	Quantitative Reasoning
Washington	Math &107	Finite Math in Society

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