

About *Quantitative Reasoning*

Quantitative Reasoning is designed for students who have completed *Foundations of Mathematical Reasoning* and the co-requisite *Frameworks for Mathematics and Collegiate Learning*.

Course structure and contact hours

Quantitative Reasoning is designed to be taught as a one-semester course with 3 student-contact hours per week or in a quarter system with an equivalent number of contact hours.

Active learning design principles are evident in each lesson. Students will be expected to actively *do* mathematics—such as analyzing data, constructing hypotheses, solving problems, reflecting on their work, and making connections.

The *Quantitative Reasoning* course is organized around big mathematical and statistical concepts. The course's nontraditional treatment of content will help students develop conceptual understanding by supporting them in making connections between concepts and applying previously learned material to new contexts. The course will help to prepare students for success in future courses, gain skills for the workplace, and participate as productive citizens in our society.

Structure of the curriculum

The curriculum is designed in 25-minute learning episodes, which may be paced to conform to any class length. These short bursts of active learning, combined with whole-class discussion and summary, can produce increased memory retention.¹

Readiness competencies

Students enrolling in *Quantitative Reasoning* should be able to do the following:

- Demonstrate procedural fluency with real number arithmetic operations and use those operations to represent real-world scenarios and to solve stated problems. Demonstrate number sense, including dimensional analysis and conversions between fractions, decimals, and percentages. Determine when approximations are appropriate and when exact calculations are necessary.

¹ Sources: Buzan, T. (1989). *Master your memory* (Birmingham: Typesetters); Buzan, T. (1989). *Use your head* (London: BBC Books); Sousa, D. (2011). *How the brain learns*, 4th ed. (Thousand Oaks, CA: Corwin); Gazzaniga, M., Ivry, R. B., & Mangun, G. R. (2002). *Cognitive neuroscience: The biology of the mind*, 2nd ed. (New York: W. W. Norton); Stephane, M., Ince, N., Kuskowski, M., Leuthold, A., Tewfik, A., Nelson, K., McClannahan, K., Fletcher, C., & Tadipatri, V. (2010). Neural oscillations associated with the primary and recency effects of verbal working memory. *Neuroscience Letters*, 473, 172–177; Thomas, E. (1972). The variation of memory with time for information appearing during a lecture. *Studies in Adult Education*, 57–62.

- Solve linear equations, graph and interpret linear models, and read and apply formulas.
- Demonstrate a basic understanding of displays of univariate data such as bar graphs, histograms, dotplots, and circle graphs, including appropriate labeling.
- Take charge of their own learning through good classroom habits, time management, and persistence. Participate in the classroom community through written and oral communication.

Learning goals

The following five learning goals apply to all DCMP mathematics courses, with the complexity of problem-solving skills and use of strategies increasing as students advance through the pathways.

For each course, we define the ways that the learning goals are applied and the expectations for mastery. The bullets below each of the five learning goals specify the ways each learning goal is applied in the *Quantitative Reasoning* course.

Each DCMP course is designed so that students meet the goals across the courses in a given pathway. Within a course, the learning goals are addressed across the course's content-based learning outcomes. These learning goals and the content-based learning outcomes were developed and informed by the individuals and resources acknowledged at the end of this document.

Communication Goal: Students will be able to interpret and communicate quantitative information and mathematical and statistical concepts using language appropriate to the context and intended audience.

In the *Quantitative Reasoning* course, students will...

- Use appropriate mathematical and statistical language in oral, written, and graphical forms.
- Read and interpret authentic texts such as advertisements, consumer information, government forms, and newspaper articles containing quantitative information, including graphical displays of quantitative information. These texts may be as long as a standard magazine article and will include comparison, analysis, and synthesis of multiple forms or sources of quantitative information.
- Write 1 to 2 pages using quantitative information to synthesize information from multiple sources or to make or critique an argument.

Problem Solving Goal: Students will be able to make sense of problems, develop strategies to find solutions, and persevere in solving them.

In the *Quantitative Reasoning* course, students will...

- Develop an answer to an open-ended question requiring analysis and synthesis of multiple calculations, data summaries, and/or models. Students will be expected to

develop their own process with support from peers and the instructor. This type of question would be expected to extend over time (beyond one or two class meetings) with work occurring both in and outside of class time with specific checkpoints to monitor progress.

Reasoning Goal: Students will be able to reason, model, and draw conclusions or make decisions with mathematical, statistical, and quantitative information.

In the *Quantitative Reasoning* course, students will...

- Draw conclusions or make decisions in quantitatively based situations that are dependent upon multiple factors. Students will analyze how different situations would affect the decisions.
- Present written or verbal justifications of decisions that include appropriate discussion of the mathematics involved.
- Recognize when additional information is needed.

Evaluation Goal: Students will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information.

In the *Quantitative Reasoning* course, students will...

- Evaluate the validity and possible biases in arguments presented in authentic contexts based on multiple sources of quantitative information (e.g., advertising, internet postings, consumer information, political arguments).

Technology Goal: Students will be able to use appropriate technology in a given context.

In the *Quantitative Reasoning* course, students will...

- Use a spreadsheet to organize quantitative information and make repeated calculations using simple formulas.
- Use the internet to find quantitative information on a given subject and evaluate the validity and possible bias of information based on the source.
- Use internet-based tools appropriate for a given context (e.g., an online tool to calculate credit card interest).

Content learning outcomes

Quantitative Reasoning serves students who are focused on developing quantitative literacy skills that will be meaningful for their professional, civic, and personal lives. Such reasoning is a habit of mind, seeking pattern and order when faced with unfamiliar contexts. In this course, an emphasis is placed on the need for data to make good decisions and an understanding of the dangers inherent in basing decisions on anecdotal evidence rather than data. The topics for the *Quantitative Reasoning* course are:

- Number, Ratio, and Proportional Reasoning
- Modeling

- Probability
- Statistics

Number, Ratio, and Proportional Reasoning

Outcome: Students will draw conclusions and/or make decisions based on analysis and critique of quantitative information using proportional reasoning. Students will also effectively justify and communicate their conclusions in ways appropriate to the audience.

Students will be able to:

N.1 Solve real-life problems requiring interpretation and comparison of complex numeric summaries which extend beyond simple measures of center.

For example: Interpret and/or compare weighted averages, indices, coding, and ranking. Evaluate claims based on complex numeric summaries.

N.2 Solve real-life problems requiring interpretation and comparison of various representations of ratios (i.e., fractions, decimals, rates, and percentages).

For example: Interpret nonstandard ratios used in media and risk reporting. Identify and contrast in written statements part-to-part versus part-to-whole ratios taken from meaningful context. Understand and communicate percentages as rates per 100. Identify uses and misuses of percentages related to a proper understanding of the base. Analyze growth and decay using absolute and relative change, and comparisons using absolute and relative difference.

N.3 Distinguish between proportional and nonproportional situations and, when appropriate, apply proportional reasoning.

For example: Solve for an unknown quantity in proportional situations. Determine the constant of proportionality in proportional situations, leading to a symbolic model for the situation (i.e., an equation based upon a rate of change, $y = kx$). Solve real-life problems requiring conversion of units using dimensional analysis. Apply scale factors to perform indirect measurements (e.g., maps, blueprints, concentrations, dosages, and densities). Recognize when proportional techniques do not apply.

Modeling

Outcome: Students will draw conclusions and/or make decisions by analyzing and/or critiquing mathematical models, including situations for which the student must recognize underlying assumptions and/or make reasonable assumptions for the model.

Students will be able to:

M.1 Analyze and critique mathematical models and be able to describe their limitations.

For example: Distinguish between correlation and causation. Determine whether interpolation and/or extrapolation are appropriate.

M.2 Use models, including models created with spreadsheets or other tools, to estimate solutions to contextual questions, identify patterns, and identify how changing parameters affect the results.

For example: Use functional models to estimate future population and spreadsheets to model financial applications (e.g., credit card debt, installment savings, amortization schedules, mortgage and other loan scenarios).

M.3 Choose and create models for bivariate data sets, and use the models to answer questions and draw conclusions or make decisions.

For example: Choose a linear, exponential, logistic, or periodic function to model a given set of data and justify the chosen model. Create models by hand or with technology. Use models appropriately. Demonstrate understanding of the limitations of chosen models and how the choice of model has long-term ramifications. Recognize when a change in model is needed.

Probability

Outcome: Students will apply probabilistic reasoning to draw conclusions, to make decisions, and to evaluate outcomes of decisions.

Students will be able to:

P.1 Evaluate claims based on empirical, theoretical, and subjective probabilities.

For example: Analyze outcomes and draw conclusions or make decisions related to risk, pay-off, expected value, and false negatives/positives in various probabilities contexts.

P.2 Use data displays and models to determine probabilities (including conditional probabilities) and use these probabilities to make informed decisions.

For example: Use and make decisions based on data from two-way tables, tree diagrams, Venn diagrams, and area models.

Statistics

Outcome: Students will draw conclusions or make decisions and communicate their rationale based on understanding, analysis, and critique of self-created or reported statistical information and statistical summaries.

Students will be able to:

S.1 Use statistical information from studies, surveys, and polls (including when reported in condensed form or as summary statistics) to make informed decisions.

For example: Identify limitations, strengths, or lack of information in studies, including data collection methods (e.g., sampling, experimental, observational) and possible sources of bias. Identify errors or misuses of statistics to justify particular conclusions. Interpret and compare the results of polls using margin of error.

S.2 Create and use visual displays of data.

For example: Create (with and without technology) visual representations of real-world data such as charts, tables and graphs. Interpret and analyze visual representations of data. Describe strengths, limitations, and fallacies of various graphical displays.

S.3 Summarize, represent, and interpret data sets on a single count or measurement variable.

For example: Use plots and statistics appropriate to the shape of the data distribution to represent a single data set. Compare center, shape, and spread of two or more data sets. Interpret the differences in context.

S.4 Use properties of distributions to analyze data and answer questions.

For example: Recognize when data are normally distributed and use the mean and standard deviation of the data to fit them to a normal distribution.