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We welcome your comments and suggestions for improvements. Please contact us at danaweb@austin.utexas.edu or at the mailing address above.

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About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We work with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—*Statistical Reasoning*, *Quantitative Reasoning*, and *Reasoning with Functions I* and *Reasoning with Functions II* (a two-course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point—a developmental math course that helps students develop foundational skills and conceptual understanding in the context of college-level course material.

In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies—and tenacity—necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.

In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

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Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
3.C	3.C	How Much Water Does Our Class Drink? (Optional) *Sample standard deviation*	30	23	47	3.C
4.A	4.A	What Are the Risks? *Theoretical probability of two or more independent events*	35	27	50	4.A
4.B	4.B	Calculating Risk *Conditional probability of two or more dependent events*	39	31	54	4.B
5.A	5.A	Cost of Living Comparisons *Conversion to create equivalent units; supplemental spreadsheet*	44	35	58	5.A
5.B	5.B	Index Numbers *Using indices such as Consumer Price Index; supplemental spreadsheet*	48	39	63	5.B
5.C	5.C	Polls, Polls, Polls! *Weighted averages*	52	43	67	5.C
5.D	5.D	Average Income *Weighted averages and expected value; supplemental spreadsheet*	56	47	72	5.D
6.A	6.A	How Can We Smooth the Data? (Optional) *Simple and weighted moving averages; supplemental spreadsheet*	59	49	76	6.A
6.B	-	Mini-Project: Income Disparities (Optional) *Written analysis of graphical display of weighted moving average*	63	51	81	-
7.A	7.A	U.S. Budget Priorities *Part-part vs. part-whole ratios*	68	53	86	7.A
7.B	7.B	Understanding U.S. Budget Priorities *Decimals, percentages, and part-whole ratios*	72	57	90	7.B
7.C	7.C	Changes to U.S. Budget Priorities *Absolute and relative change*	78	61	95	7.C
7.D	7.D	Percent of Total U.S. Budget *Dotplots used to introduce symmetry and skewness*	82	63	99	7.D

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
7.E	7.E	What's My Credit Score? *Application of ratios; Practice assignment can be mini-project. Collect data for Lesson 8, Part D; schedule lab for 8.D and 10.A.*	87	65	102	7.E
7.F	7.F	U.S. Incarceration Rates *Applications of ratios; comparison*	91	69	107	7.F

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8.C	8.C	How Money Makes Money *Non-linear models*	103	79	123	8.C
8.D	8.D	Have My Choices Affected My Learning? *Regression using student data. Computer lab day, if possible.*	109	83	129	8.D
8.E	8.E	Mini-Project: Progressive and Flat Income Tax Systems (Optional) *Informal piecewise linear function*	113	87	135	-
8.F	8.F	Mini-Project: Estimating the Number of People in a Crowd (Optional) *Using proportionality to estimate*	122	93	140	-
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9.B	9.B	Appreciating Depreciation *Linear interpolation via similar triangles*	135	103	153	9.B
9.C	9.C	How Much Should I Be Paid? *Correlation*	140	107	159	9.C
9.D	9.D	Why Are You Wearing the Same Old Socks? *Correlation vs. causation; strength*	148	113	164	9.D
10.A	10.A	Fibonacci's Rabbits *Exponential growth; limitations. Computer lab day, if possible.*	155	117	168	10.A

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
10.B	10.B	Is It Getting Crowded? *Exponential growth; limitations*	158	121	173	10.B
		You may wish to consider various configurations with the upcoming modeling lessons. For example, you may wish to consider having different groups complete and present the various logistic lessons or having some groups do logistic models while other groups do the periodic models. You may also choose to omit either logistic or periodic models.				
11.A	11.A	Population Growth (Optional) *Logistic models*	163	125	177	11.A
11.B	11.B	Oh, Deer! (Optional) *Time series model of logistic growth*	169	129	182	11.B
11.C	11.C	Can You Hear Me Now? (Optional) *Logistic models. Spreadsheet demonstration or computer lab day, if possible.*	173	133	187	11.C
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11.E	11.E	Reindeer and Lichens (Optional) *Effects of parameter choices on model predictions*	183	141	198	11.E
12.A	12.A	How Long Is the Longest Day? (Optional) *Cyclical data*	188	145	202	12.A
12.B	12.B	What's My Sine? (Optional) *Periodic functions*	196	151	206	12.B
12.C	12.C	SIR Disease (Optional) *Effect of parameters on a model (epidemics)*	201	155	211	12.C
12.D	-	SIR (Continued) (Optional) *Create a time-series model using a spreadsheet; Practice assignment could be a mini-project.*	206	159	216	-
Lessons 13-15: Statistical Studies						
13.A	13.A	Mind the Gap in Income Inequality *Introductory vocabulary for statistical studies*	209	161	224	13.A

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
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13.C	13.C	A Lesson Worth Weighting For *Sampling processes*	219	169	234	13.C
13.D	13.D	Weight . . . There's More! *Evaluate and design sampling processes*	225	175	241	13.D
14.A	14.A	Blood Pressure and Bias *Sampling and non-sampling error*	231	181	247	14.A
14.B	14.B	Taking Aim at Bias *Types of bias*	235	185	251	14.B
14.C	14.C	Conclusions in Observational Studies *Minimizing bias; appropriate conclusions*	240	189	255	14.C
15.A	15.A	The Video Game Diet *Designing experimental studies; cause and effect*	244	193	261	15.A
15.B	15.B	All Things in Moderation *Confounding variables*	248	197	266	15.B
15.C	15.C	The Power of the Pill *Blinding; placebo effect; placebos*	253	201	271	15.C
15.D	15.D	Designing an Experiment *Double blinding; blocking*	257	205	276	15.D
15.E	15.E	In Conclusion *Culminating lesson on conclusions from statistical studies*	262	209	284	15.E

You may wish to consider various configurations with the upcoming lessons on analyzing and writing about graphical displays. For example, you may wish to consider having different groups complete Lesson 16, Parts B, D, E, and F, and present to the class.

Lessons 16-18: Complex Quantitative Information and Graphical Displays						
16.A	16.A	Education Pays *Analyzing stacked column graphs*	266	213	289	16.A
16.B	16.B	Looking for Links *Analyzing comparative stacked columns graphs*	271	217	295	16.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
16.C	16.C	It's About Time! *Building stacked columns graphs from class data*	275	221	301	16.C
16.D	16.D	Connecting the Dots *Analyzing motion bubble charts*	280	225	207	16.D
16.E	16.E	Big Data (GIS) *Analysis problems associated with large, volatile data*	285	229	313	16.E
16.F	16.F	Big Brother – They're Watching! *Conclusions from heat maps*	292	237	319	16.F
17.A	17.A	Decisions, Decisions *Decision making based on multiple pieces of quantitative information*	296	241	324	17.A
17.B	17.B	The Write Approach to Data *Improving written analyses of graphical displays*	302	245	330	17.B
17.C	17.C	Numbers Never Lie *Misleading and erroneous graphical displays*	309	249	344	17.C
17.D	17.D	Can You Feel the Heat? *Using data to understand complex issues*	316	255	339	17.D
18.A	18.A	Mini-Project: Tornado Climatology *Choosing appropriate ways to represent data*	321	259	346	-
18.B	18.B	The Making of a Model *Various ways to present mathematical models*	325	263	352	18.B
18.C	18.C	What a Wonderful World! *Using multiple representations to choose a model*	329	267	356	18.C
18.D	18.D	Mathematical Models *Limitations of models*	335	271	361	18.D

Curriculum Overview

Contents

About Quantitative *Reasoning*

Quantitative Reasoning (*QR*) is designed for students who have completed *Foundations of Mathematical Reasoning* (*Foundations* or *FMR*) and the co-requisite *Frameworks for Mathematics and Collegiate Learning* (*Frameworks* or *FMCL*). Students who have not completed either or both of these classes may need extra support in acclimating to the structure and climate of the course.

Quantitative Reasoning serves students who are focused on developing quantitative literacy skills that will be meaningful for their professional, civic, and personal lives. Such reasoning is a habit of mind, seeking pattern and order when faced with unfamiliar contexts. In this course, an emphasis is placed on the need for data to make good decisions and to have an understanding of the dangers inherent in basing decisions on anecdotal evidence rather than on data.

Structure of the curriculum

The *QR* curriculum is designed in 25-minute learning episodes, which can be taught in one, two, three, or more lesson groups to conform to the desired class length. These short bursts of active learning, combined with whole class discussion and summary, produce increased memory retention.[1]

[1] Sources: Buzan, T. (1989). *Master your memory* (Typersetters); Buzan, T. (1989). *Use your head* (BBC Books); Sousa, D. (2011). *How the brain learns, 4th ed.* (Corwin); Gazzaniga, M., Ivry, R. B., & Mangun, G. R. (2002). *Cognitive neuroscience: The biology of the mind, 2nd ed.* (W.W. Norton); Stephane, M., Ince, N., Kuskowski, M., Leuthold, A., Tewfik, A., Nelson, K.,

QR continues the philosophy and structures of the *Foundations* course. The lesson structure includes a Preview Assignment (to be completed before class); the Lesson (Lesson Planning Suggestions and Student Pages); and a Practice Assignment (to be completed after class). Some lessons do not have a Preview or Practice, but all Lessons include Lesson Planning Suggestions and Student Pages. If you have been an instructor of *Foundations*, most items in this document will be familiar to you.

Structure of the Lesson Planning Suggestions for the lessons

The main features of the **Lesson Planning Suggestions** for the lessons are:

- **Overview and student objectives** – includes the constructive perseverance level of the lesson, the learning outcomes, and goals addressed.
- **Suggested resources and preparation** – includes technology needs, physical materials, and preparation needed for activities. A consolidated materials list is included in the last section.
- **Prerequisite assumptions** – lists the skills that students need to be prepared for the lesson. The same list is often given to the students in the Preview Assignment, in which students are asked to rate their confidence level on many skills. If students struggle with transference of these skills into the new context of a lesson, the instructor can refer back to the Preview questions to help students review, get help, and recognize that they have done similar problems.
- **Making connections** – details the main concepts of *QR* that are extensions of *FMR* objectives and earlier work in the course, and lists connections to future lessons in this course.
- **Background context** – includes the main points of any informational pieces that were given to students in Preview Assignments.
- **Suggested instructional plan** – includes instructional strategies for presenting and discussing the concepts and guiding questions to promote class discussion of problems on Student Pages. The instructional plan is comprised of the following components:
    - Frame the lesson – includes suggestions to elicit prior student knowledge, focus discussion, or ask for a prediction.
    - Lesson activities – provides detailed suggestions for probing questions for students or groups, guiding questions for class discussions, and literacy supports.

McClannahan, K., Fletcher, C., & Tadipatri, V. (2010). Neural oscillations associated with the primary and recency effects of verbal working memory. *Neuroscience Letters*, 473, 172–177); Thomas, E. (1972). The variation of memory with time for information appearing during a lecture. *Studies in Adult Education,* 57–62.

- Wrap-up/transition – offers concluding statements for the day, reflection on concepts, and/or transition to the next activity.

The Lesson Planning Suggestions do not summarize all ideas of the lesson; rather, they are intended to help instructors engage students and facilitate the inclusion of broader ideas. The goal in *QR* is to have students actively engaged in making connections themselves, rather than have the instructor lecture about the connections. Making connections is a challenging skill that students will develop throughout the course. Students who completed the *Foundations* course are likely to be familiar and comfortable with this format. However, students who did not take *FMR* may need help acclimating to this learning environment.

Early discussions are likely to be slow-starting and require a great deal of prompting by the instructor. Instructors can build on what students say and model how to express these abstract concepts. The facilitation prompts provide instructors with ideas on how to promote student discussion. As the explicit connections emerge, the instructor should record the ideas on the board and, especially early in the course, focusing on ensuring that students record the ideas in their notes.

Lesson Planning Suggestions also include:

- **Suggested assessment, assignments, and reflections** – offers references to the homework assignments that accompany the lessons. Occasionally, additional assessments, projects, or reflections are suggested. Assignments for the next day will usually include one or more Preview Assignments.

Constructive perseverance level

The levels of constructive perseverance are a way to help instructors think about scaffolding questions and productive struggle through the course. The levels should be viewed broadly as a continuum rather than as distinct, well-defined categories. In general, the level increases through the course, but this does not mean that all of the later lessons will be a Level 3. The level is based both on the development of students and the demands of the content. Some content requires greater structure and more direct instruction. The levels of constructive perseverance are as follows:

- **Level 1:** The problem is broken into sub-questions that help develop strategies. Students reflect on and discuss questions briefly and then are brought back together to discuss with the full class. This process moves back and forth between individual or small group discussion and class discussion in short intervals.

  Goal of the instructor: Develop the culture of discussion, establish norms of listening, and model the language used to discuss quantitative concepts. In addition, emphasize to students that struggling indicates learning. If struggle is not taking place, students are not being challenged and are not gaining new knowledge and skills.

- **Level 2:** The problem is broken into sub-questions that give students some direction but do not explicitly define or limit strategies and approaches. Students work in groups on multiple steps for longer periods, and the instructor facilitates with individual groups, as needed. The instructor brings the class together at strategic points to make important connections explicit or when breakdowns of understanding have occurred or are likely to occur.

  Goal of the instructor: Support students in working more independently and evaluating their own work so they feel confident about moving through multiple questions without constant reinforcement from the instructor. Ask student(s) or group(s) guiding questions to help them make progress.

- **Level 3:** The problem is not broken into steps or is broken into very few steps. Students are expected to identify strategies for themselves. Groups work independently on the problem with facilitation by the instructor, as necessary. Groups report on results, and class discussion focuses on reflecting on the problem as a whole.

  Goal of the instructor: Support students in persisting with challenging problems, including trying multiple strategies before asking for help.

Technology

It is assumed that all instructors will have access to and be able to utilize technology in each class. The most common example is displaying and manipulating a spreadsheet. Some in-class lessons could be taught with students working with a spreadsheet, if available. If students do not have access to computers, then the instructor will engage students in a whole class discussion and work with a spreadsheet or internet site. Check the Table of Contents for lessons that would be best accomplished in a computer lab or if students have access to their own computers during class.

Out-of-class activities assume that all students can access technology either in their homes or in a college computer lab.

Some out-of-class assignments ask students to create and/or extend a spreadsheet that is to be turned in and graded by the instructor. Instructors and students are provided templates, and instructors are also provided a sample grading rubric with helpful hints for grading work submitted by students.

Table of contents information

The table of contents contains the following information.

- Lesson number – Note that the first document in the Table of Contents (after this Overview) is "Suggestions for Prep Week," which consists of suggestions for the instructor on best practices to be incorporated into the syllabus.

- Preview Assignment, if any – These entries signal the presence of a Preview Assignment, which students complete before coming to class. The section below has more information about Preview Assignments.
- Lesson title and brief description – Occasionally, a lesson consists of the in-class introduction to an out-of-class mini-project that incorporates the concepts and objectives covered in earlier lessons. These projects are an excellent strategy to encourage students to work together in out-of-class study groups.
- Lesson Planning Suggestions page reference
- Student Page reference
- Practice Assignment, if any – These assignments consist of problems designed to assess student understanding of the concepts addressed in the topic.

The role of the Preview and Practice Assignments

One of the most important aspects of the *Quantitative Reasoning* curriculum is the role and design of the homework assignments (Preview Assignments before the lesson and Practice Assignments after the lesson). These assignments differ from traditional homework in several ways.

- The Preview Assignments are designed to prepare students for the next lesson and to review mathematical concepts that will be needed. Students are given a set of skills for the next lesson and asked to rate themselves on their readiness to apply those skills. Each of these skills is used in the lesson. Students are instructed to seek help before the next class meeting if they are unable to successfully complete these problems.

- The Preview Assignments occasionally contain information or activities (internet search or review, view a video, read an article, etc.) that are used in the next lesson. These assignments will be generally referenced in the **Lesson Planning Suggestions** under **Background context**.

- The Practice Assignments provide students with opportunities to develop, practice, and extend skills from the current lesson.

- Each question in the homework assignments has a specific purpose. While some questions are specifically skill based, repetition of a skill in a single form is never used. If repetition is deemed valuable, it is done with different contexts or formats that require students to think about each question rather than assume they can repeat the steps of the previous question.

The design of all assignments is based on the same principles of constructive perseverance as the rest of the curriculum. Ideally, each assignment should offer entry- level questions that all students should be able to complete successfully, followed by more challenging questions. One goal of the entire curriculum is that students will increasingly engage and become comfortable with productive struggle. The expectation is not that every student should be able to answer every question correctly or immediately, but that every student should make a valid attempt on each question. Therefore, there are questions in the assignments, especially in the later lessons, that some students may not answer correctly. These challenging questions raise issues about grading practices.

- In some cases, the assignments include actual instructional materials. It is expected that students will read these materials, as they will usually not be presented directly in class. This information distinguishes the material from traditional textbooks in which the text is often assigned by instructors, but sometimes only used by students as a reference.

- In general, the Preview and Practice questions are written in ways that facilitate the use of an online platform. There are occasional exceptions when students are instructed to complete a free-response question in their notebook and take it to class. It is up to the instructor to decide whether and how to read and grade these responses.

Strategies for supporting the Preview and Practice Assignments

The central role and unique design of the assignments in this curriculum require instructors to develop strategies and procedures for encouraging students to think independently and with their peers and for supporting students to use the assignments appropriately. The following section offers some suggestions to help plan instructional strategies.

Motivating students to complete the assignments – The design of the assignments supports students' motivation. As students come to realize that much of the material is actually useful to them in class, they should see the importance of completing the Preview and Practice Assignments. Instructors can support student motivation by doing the following:

- Discuss the role of the assignments with students.

- Set and maintain an expectation that students will use the Preview materials and/or skills for the lesson. Students may take this lightly at first. It is important that instructors do not use class time to review the prerequisite skills. Make it clear that students are responsible for being prepared. Keep in mind:

  - Students may have prepared, but may not recognize, that what they are being asked to do in class is the same skill they used in the Preview Assignment. Be prepared to refer back to specific Preview questions to help them make this connection.

  - If a student is truly unprepared, do not reprimand him or her in front of the class. Privately explain the expectation for preparation to the student and invite him or her to meet with you outside of class to review the material. If you do meet the student outside of class, take the opportunity to talk about the importance of preparation and inquire about how the student does the self-assessment. Help the student develop strategies for using this tool more effectively.

- Some Preview Assignments will require work to be printed out to be used in class; have some way for students who do the work to receive credit. The model of group work allows all students to participate in the lesson even if they did not do the Preview questions. Allowing all students to participate is important, but students who do come prepared should feel that their work is valued. You may choose to give students a quick completion grade by walking around the room and seeing who has their printed work complete while students are working with their groups.

- Notify students at the end of the first, second, and third weeks if they have failed to complete any of their work (Preview, in class, and/or Practice). This notification can be done by email or by handing out notes in class. It is important for students to know that the instructor is aware of students' individual work. Always include an offer of help and expressions of support in these notices. For example, "If there is something preventing you from completing your work, please come to see me. I want to help you be successful in this course." Keep in mind that there are many reasons that students fail to complete out-of-class work.

- Writing is an important component of the course (see the **Language and literacy skills** section). Some assignments and mini-projects contain writing tasks. Take the time to grade these written responses individually.

- **Grading assignments** – Effective grading strategies should be individualized depending on the grading time instructors have, the length of classes, and the student population. Instructors will design their own processes for grading Preview, in-class, and Practice Assignments. Since the assignments are designed to challenge students and promote constructive perseverance, grading only on correct answers may not always be appropriate and may discourage students. On the other hand, grading on completion has drawbacks as well. Some ideas follow.

- Use a scoring method that gives points for both completion and correctness.

- The curriculum is intended to build language and literacy skills over the course of the semester. *QR* Writing Grading Rubrics are provided in lessons with more complex writing tasks. Especially early in the course, weight the grade toward the quality of the attempt versus the correct answer. Additionally, the expectation of what constitutes quality work should build throughout the course and be communicated clearly.

- Grade on correctness but occasionally allow students to turn in written explanations for problems they missed and to earn back points. Instructors can manage this work by limiting the opportunity to one or two problems each week or to certain assignments.

Some students may struggle with organization of course materials. Students who have completed the *Foundations* course will be familiar with how to organize course materials. *QR* instructors may consider talking with *Foundations* instructors for ideas and advice on practices used in that course. Instructors should provide some guidelines for structure to help students get started. It is suggested that this organization of course materials be a requirement of the course. Strategies include the following:

- Explain to students why it is important to organize their materials. Give specific examples of the ways in which they will use the materials in this course.

- Require that students keep materials in a three-ring binder.

  - **High structure:** Give students guidelines on how to order and label materials.

  - **Moderate structure:** Give students guidelines but also give them the option to create their own method of organization.

- Any structure that is required should be graded in some way (or students will not do it). Checks should be done in the first few weeks of the course to establish a routine.

  - Check in class on a regular basis. Tell students to find a specific document within a specified amount of time (e.g., two minutes). Students get a grade for showing the instructor the document.

  - Start with a quick check for having the system (e.g., binder, folder) set up. Then occasionally have students turn in their materials and do a spot-check for certain documents.

- Give timed quizzes in which students are referred to certain documents and must respond to some quick question about the materials.

Resource materials for students

The student resource packet is designed to be the starting point for course reference materials. Encourage students to keep a section of their class binder dedicated to the provided resources as well as any additional resources they may collect.

Language and literacy skills

Quantitative literacy has unique language demands that are distinct from other subjects, including other math courses. Even skilled readers and writers often struggle with understanding and using mathematical language and interpreting quantitative information. One of the greatest challenges of the *Foundations of Mathematical Reasoning* and *Quantitative Reasoning* courses is that they seek to teach quantitative literacy to a high proportion of students who are not college-level readers or writers.

The purpose of writing in the *Quantitative Reasoning* course is to:

- Make sense of quantitative information and processes, especially in relationship to a context.
- Build skills in communicating about quantitative information.
- Provide a form of assessment by which students demonstrate their understanding of the course material in writing. (Note: Other assessment methods will be used including verbal responses [in class], short answer, fill in the blank, multiple choice, true–false, presentations, etc.)

By the end of the *Quantitative Reasoning* course, students should be able to write contextual papers that make appropriate and accurate use and analysis of quantitative information. This skill will be achieved by stressing reading and writing skills along with the mathematics skills throughout the course.

Curriculum design standards

The Dana Center Mathematics Pathways (DCMP) model is made up of individual courses that form *pathways* for students to and through college-level mathematics. The concept of the pathway as a yearlong experience is critical to the DCMP because these courses are designed to articulate in a way that provides students with the experience of learning mathematics and/or statistics through coherent, consistent practices and structures. The design standards outlined in this section set the guidelines for how the curricular materials for individual DCMP courses are designed to support that coherent experience for students.

Note: The numbering in the description of the design standards does not indicate level of importance.

Standard I: Structure and Organization of Curricular Materials

The DCMP model is organized around big mathematical and statistical ideas and concepts as opposed to skills and topics.

Standard II: Active Learning

The DCMP model is designed to actively involve students in doing mathematics and statistics, analyzing data, constructing hypotheses, solving problems, reflecting on their work, and learning and making connections.

Class activities provide regular opportunities for students to actively engage in discussions and tasks using a variety of different instructional strategies (e.g., small groups, class discussions, interactive lectures).

Standard III: Constructive Perseverance

The DCMP model supports students in developing the tenacity, persistence, and perseverance necessary for learning mathematics.

Standard IV: Problem Solving

The DCMP model supports students in developing problem-solving skills, and students apply previously learned skills to solve nonroutine and unfamiliar problems.

Standard V: Context and Interdisciplinary Connections

The DCMP model presents mathematics and statistics in context and connects mathematics and statistics to various disciplines.

Standard VI: Use of Terminology

The DCMP model uses discipline-specific terminology, language constructs, and symbols to intentionally build mathematical and statistical understanding and to ensure that terminology is not an obstacle to understanding.

Standard VII: Reading and Writing

The DCMP model develops students' ability to communicate about and with mathematics and statistics in contextual situations appropriate to the pathway.

Standard VIII: Technology

The DCMP model uses technology to facilitate active learning by enabling students to directly engage with and use mathematical concepts. Technology should support the learning objectives of the lesson. In some cases, the use of technology may be a learning objective in itself, as in learning to use a statistical package in a statistics course.

Note: A more detailed description of the design standards is available on the Dana Center website at: https://dcmathpathways.org/resources/dcmp-curriculum-design-standards.

Readiness competencies

Students enrolling in *Quantitative Reasoning* should be able to do the following:

- Demonstrate procedural fluency with real number arithmetic operations and use those operations to represent real-world scenarios and to solve stated problems. Demonstrate number sense, including dimensional analysis and conversions between fractions, decimals, and percentages. Determine when approximations are appropriate and when exact calculations are necessary.
- Solve linear equations, graph and interpret linear models, and read and apply formulas.
- Demonstrate a basic understanding of displays of univariate data such as bar graphs, histograms, dotplots, and circle graphs, including appropriate labeling.
- Take charge of their own learning through good classroom habits, time management, and persistence. Participate in the classroom community through written and oral communication.

DCMP learning goals with applications for the *QR* course

The following five learning goals apply to all DCMP mathematics courses, with the complexity of problem-solving skills and use of strategies increasing as students advance through the pathways.

For each DCMP course, we define the ways that the learning goals are applied and the expectations for mastery. The bullets below each of the five learning goals specify the ways in which each learning goal is applied in the *Quantitative Reasoning* course.

Each DCMP course is designed so that students meet the goals across the courses in a given pathway. Within a course, the learning goals are addressed across the course's content-based learning outcomes.

Communication Goal: Students will be able to interpret and communicate

quantitative information and mathematical and statistical concepts using language appropriate to the context and intended audience.

In the *Quantitative Reasoning* course, students will...

- Use appropriate mathematical and statistical language in oral, written, and graphical forms.
- Read and interpret authentic texts such as advertisements, consumer information, government forms, and newspaper articles containing quantitative information, including graphical displays of quantitative information. These texts may be as long as a standard magazine article and will include comparisons, analysis, and synthesis of multiple forms or sources of quantitative information.
- Write 1 to 2 pages using quantitative information to synthesize information from multiple sources or to make or critique an argument.

Problem Solving Goal: Students will be able to make sense of problems, develop strategies to find solutions, and persevere in solving them.

In the *Quantitative Reasoning* course, students will...

- Develop an answer to an open-ended question requiring analysis and synthesis of multiple calculations, data summaries, and/or models. Students will be expected to develop their own process with support from peers and the instructor. This type of question would be expected to extend over time (beyond one or two class meetings) with work occurring both in class and outside of class with specific checkpoints to monitor progress.

Reasoning Goal: Students will be able to reason, model, and draw conclusions or make decisions with mathematical, statistical, and quantitative information.

In the *Quantitative Reasoning* course, students will...

- Draw conclusions or make decisions in quantitatively based situations that are dependent upon multiple factors. Students will analyze how different situations would affect the decisions.
- Present written or verbal justifications of decisions that include appropriate discussion of the mathematics involved.
- Recognize when additional information is needed.

Evaluation Goal: Students will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information.

In the *Quantitative Reasoning* course, students will...

- Evaluate the validity and possible biases in arguments presented in authentic contexts based on multiple sources of quantitative information (e.g., advertising,

internet postings, consumer information, political arguments).

Technology Goal: Students will be able to use appropriate technology in a given context.

In the *Quantitative Reasoning* course, students will...

- Use a spreadsheet to organize quantitative information and make repeated calculations using simple formulas.
- Use the internet to find quantitative information on a given subject and evaluate the validity and possible bias of information based on the source.
- Use internet-based tools appropriate for a given context (e.g., an online tool to calculate credit card interest).

Content learning outcomes for *Quantitative Reasoning*

The topics for the *Quantitative Reasoning* course are:

- Number, Ratio, and Proportional Reasoning
- Modeling
- Probability
- Statistics

Number, Ratio, and Proportional Reasoning

Outcome: Students will draw conclusions and/or make decisions based on analysis and critique of quantitative information using proportional reasoning. Students will also effectively justify conclusions and communicate about their conclusions in ways appropriate to the audience.

Students will be able to:

N.1 Solve real-life problems requiring interpretation and comparison of complex numeric summaries which extend beyond simple measures of center.

For example: Interpret and/or compare weighted averages, indices, coding, ranking; evaluate claims based on complex numeric summaries.

N.2 Solve real-life problems requiring interpretation and comparison of various representations of ratios, (i.e. fractions, decimals, rate, and percentages).

For example: Interpret non-standard ratios used in media and risk reporting; identify and contrast in written statements part-to-part versus part-to-whole ratios taken from meaningful context; understand and communicate percentages as rates per 100; identify uses and misuses of percentages related to a proper

understanding of the base; analyze growth and decay using absolute and relative change and comparisons using absolute and relative difference.

N.3 Distinguish between proportional and non-proportional situations, and, when appropriate, apply proportional reasoning.

For example: Solve for an unknown quantity in proportional situations; determine the constant of proportionality in proportional situations, leading to a symbolic model for the situation (i.e., an equation based upon a rate of change, $(y = kx)$; solve real-life problems requiring conversion of units using dimensional analysis; apply scale factors to perform indirect measurements (e.g., maps, blueprints, concentrations, dosages, densities); recognize when proportional techniques do not apply.

Modeling

Outcome: Students will draw conclusions and/or make decisions by analyzing and/or critiquing mathematical models, including situations for which the student must recognize underlying assumptions and/or make reasonable assumptions for the model.

Students will be able to:

M.1 Analyze and critique mathematical models and be able to describe their limitations.

For example: Distinguish between correlation and causation; determine whether interpolation and/or extrapolation are appropriate.

M.2 Use models, including models created with spreadsheets or other tools, to estimate solutions to contextual questions, identify patterns and identify how changing parameters affect the results.

For example: Functional models to estimate future population; spreadsheets to model financial applications (e.g., credit card debt, installment savings, amortization schedules, mortgage and other loan scenarios).

M.3 Choose and create models for bivariate data sets, and use the models to answer questions and make decisions.

For example: Determine whether data can best be modeled by a linear, exponential, logistic, or periodic function; create models by hand or with technology; use models appropriately; demonstrate understanding of the limitations of chosen models.

Probability

Outcome: Students will apply probabilistic reasoning to draw conclusions, to make decisions, and to evaluate outcomes of decisions.

Students will be able to:

P.1 Evaluate claims based on empirical, theoretical, and subjective probabilities.

For example: Analyze outcomes and make decisions related to risk, pay-off, expected value, and false negatives/positives in various probabilities contexts.

P.2 Use data displays and models to determine probabilities (including conditional probabilities) and use these probabilities to make informed decisions.

For example: Two-way tables, tree diagrams, Venn diagrams, and area models.

Statistics

Outcome: Students will draw conclusions or make decisions and communicate their rationale based on understanding, analysis, and critique of self-created or reported statistical information and statistical summaries.

Students will be able to:

S.1 Use statistical information from studies, surveys, and polls (including when reported in condensed form or summary statistics) to make informed decisions.

For example: Identify limitations, strengths, or lack of information in studies, including data collection methods (e.g., sampling, experimental, observational) and possible sources of bias; identify errors or misuses of statistics to justify particular conclusions; interpret and compare the results of polls using margin of error.

S.2 Create and use visual displays of data.

For example: Create (with and without technology) visual representations of real-world data, such as charts, tables and graphs; interpret and analyze visual representations of data; describe strengths, limitations, and fallacies of various graphical displays.

S.3 Summarize, represent, and interpret data sets on a single count or measurement variable.

For example: Use plots and statistics appropriate to the shape of the data distribution to represent a single data set; compare center, shape, and spread of two or more data sets; interpret the differences in context.

S.4 Use properties of distributions to analyze data and answer questions.

For example: Recognize when data are normally distributed and use the mean and standard deviation of the data to fit to a normal distribution.

Materials list

- Index cards
- Grouping tools such as cards, colored dots, etc.
- Painter's tape
- Poster paper or flip-chart paper for group presentations
- Probability tools such as colored balls, buttons, cards, coins, dice
- Rulers or straightedges for individual work
- Tape measures or meter/yard sticks for group work