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## About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We work with our nation’s education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at [www.utdanacenter.org](http://www.utdanacenter.org).

## About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—*Statistical Reasoning*, *Quantitative Reasoning*, and *Reasoning with Functions I* and *Reasoning with Functions II* (a two-course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point—a developmental math course that helps students develop foundational skills and conceptual understanding in the context of college-level course material.

In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies—and tenacity—necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.

In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

## Acknowledgments

The development of the Dana Center Mathematics Pathways curricular materials began with the formation of the **DCMP Curricular Design Team**, who set the design standards for the curricular materials for individual DCMP courses. The team members are:

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The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The **Usability Advisory Group** members are:

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## About the *Statistical Reasoning* Course

*Statistical Reasoning*, a four-credit course with the option of cutting topics to create a three-credit course, is for students who have completed *Foundations of Mathematical Reasoning* and the co-requisite *Frameworks for Mathematics and Collegiate Learning*. The *Statistical Reasoning* course is designed for students in business, nursing, allied health, and the social and behavioral sciences, and for any student whose college and career path requires knowledge of the fundamentals of the collection, analysis, and interpretation of data.

The *Statistical Reasoning* student learning outcomes were developed as an extension of the earlier work in Statway™<sup>1</sup> and are the result of careful review and revision by the DCMP **Statistical Reasoning Course**

**Design Team**, whose members are:

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This *Statistical Reasoning* course was produced in Microsoft Word 2008 and 2011 for the Mac. Version 2.0 is supplied in PDF. The following are some issues to be aware of:

- PDF files need to be viewed with Adobe Acrobat for full functionality. If viewed through Preview, which is the default on some computers, URLs in the content may not render accurately.
- The files are named and organized into folders so that when sorted by name, they are listed in the order listed in the Table of Contents. We recommend you print the Table of Contents as a guide to the electronic files.
- The file names indicate the lesson number and whether the document is Instructor Notes, Student Pages, Resources, or some other component of the course.

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<sup>1</sup> Some of the content for *Statistical Reasoning* is derived from the Statway™ course. The original versions of the Statway™ (and Quantway™) courses were created by The Charles A. Dana Center at The University of Texas at Austin under sponsorship of the Carnegie Foundation for the Advancement of Teaching, and are copyright © 2011 by the Carnegie Foundation for the Advancement of Teaching and the Charles A. Dana Center at The University of Texas at Austin. STATWAY™/Statway™ and Quantway™ are trademarks of the Carnegie Foundation for the Advancement of Teaching.

## Acknowledgments for Statistical Reasoning Version 2.0

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*Statistical Reasoning version 2.0 is adapted from Statway™ and Statistical Reasoning Version 1.0.*

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### About *Statistical Reasoning*

*Statistical Reasoning* is designed for students who have completed *Foundations of Mathematical Reasoning* and the co-requisite *Frameworks for Mathematics and Collegiate Learning* course.

#### Course structure and contact hours

*Statistical Reasoning* is designed to be taught as a one-semester course with 4 student contact hours per week or in a quarter system with an equivalent number of contact hours. Some colleges may offer the course as a 3-credit course by removing topics.

Colleges may choose to offer *Statistical Reasoning* as a 4-credit course or as a combination of course credits and lab credits. Note that the curriculum is not designed for instruction in separate and distinct lab meetings. Rather, the intent is for the instructor to use all 4 contact hours for classroom instruction with embedded activities.

Active-learning design principles are evident in each lesson. Students will be expected to actively *do* statistics—analyzing data, constructing and testing hypotheses, solving problems, reflecting on their work, and making connections.

*Statistical Reasoning* is organized around broad statistical concepts. Statistical literacy and statistical thinking are primary themes. The course’s nontraditional treatment of content will help students develop a conceptual understanding by supporting them in making connections between concepts and applying previously learned material to new contexts. It will prepare students for success in future courses, help them gain skills for the workplace, and help prepare them for participation as well-informed, productive citizens in our society.

## Structure of the curriculum

The curriculum is designed in 25-minute learning episodes, which may be paced to conform to any class length. These short bursts of active learning, combined with whole-class discussion and summary, can produce increased memory retention.<sup>1</sup>

## Structure of the Suggested Instructor Notes for the lessons

The main features of the Suggested Instructor’s Notes for the lessons are:

- **Overview and (student) objectives**—includes the lesson’s constructive perseverance level (described later in this section), the content learning outcomes, and the learning goals addressed.
- **Suggested resources and preparation**—includes technology needs, physical materials, and any additional preparation (such as materials to be printed or for students to bring to class) needed for activities.
- **Prerequisite assumptions**—lists the skills that students need to have already so they are prepared for the lesson. The same list is given to the students in the Preview Assignment. At the end of each Preview Assignment, students are asked to rate their confidence level on each prerequisite skill. If they struggle with transferring these skills into the new context of the lessons, the instructor can refer back to the preview questions to help students recognize that they have done similar problems.
- **Making connections**—details the main concepts that are extensions of earlier work in the course as well as connections forward to new concepts in this and later courses.
- **Background context**—includes the main points of any informational pieces that were given to students in the Preview Assignments. For example, the Preview Assignment for Lesson 2, Part B introduces new vocabulary. The main points are included in the Background context section of the Suggested Instructor Notes, so looking through the Preview Assignment (homework) to determine what students have read is optional, though encouraged.
- **Suggested instructional plan**—includes excerpts from the Student Pages and the following:

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<sup>1</sup>Sources: Buzan, T. (1989). *Master your memory* (Birmingham: Typesetters); Buzan, T. (1989). *Use your head* (London: BBC Books); Sousa, D. (2011). *How the brain learns*, 4th ed. (Thousand Oaks, CA: Corwin); Gazzaniga, M., Ivry, R. B., & Mangun, G. R. (2002). *Cognitive neuroscience: The biology of the mind*, 2nd ed. (New York: W. W. Norton); Stephane, M., Ince, N., Kuskowski, M., Leuthold, A., Tewfik, A., Nelson, K., McClannahan, K., Fletcher, C., & Tadipatri, V. (2010). Neural oscillations associated with the primary and recency effects of verbal working memory. *Neuroscience Letters*, 473, 172–177; Thomas, E. (1972). The variation of memory with time for information appearing during a lecture. *Studies in Adult Education*, 57–62.

- **Frame the lesson**—suggestions for eliciting prior student knowledge, focusing discussion, or asking for a prediction.
- **Lesson activities**—detailed suggestions for probing questions for students or groups, guiding questions for class discussions, and literacy supports.
- **Wrap-up/transition**—wrap-up for the day or transition to the next activity.

The instructor’s notes do not summarize all the ideas for a lesson; rather, they are intended to facilitate the inclusion of broader ideas. Rather than having the instructor inform students of the concept and content connections, the goal is to have students actively engaged in making those connections. Learning to make these connections is a challenging skill that students will develop throughout the course.

Early discussions are likely to be slow-starting and require a great deal of prompting. Instructors can build on what students say and model how to express these abstract concepts. The facilitation prompts provide instructors with ideas on how to promote student discussion. As the explicit connections emerge, the instructor should record the ideas on the board and, particularly early in the semester, make sure students record the ideas in their notes.

- **Suggested assessment, assignments, and reflections**—includes recommended assignments of the Preview and Practice Assignments that accompany the lessons. Occasionally, additional assessments, projects, or reflections are suggested.
- **Instructor version of the Student Pages**—includes answers and/or sample answers where appropriate. Additional space is provided for the instructor to add notes or to incorporate facilitation tips and guiding questions from the **Suggested instructor notes**.

## Constructive perseverance level

The levels of constructive perseverance help instructors think about scaffolding productive struggle through the course. The levels, outlined below, should be viewed as a broad continuum, rather than as distinct, well-defined categories.

Some content requires greater structure and more direct instruction. In general, the level of constructive perseverance increases through specific sections. For example, although inference appears late in the course, early lessons in inference will generally have more scaffolding than do later lessons. The level of constructive perseverance is based both on the development of the students and the demands of the content.

The levels of constructive perseverance are as follows:

- **Level 1:** The problem is broken into subquestions that help students develop strategies for addressing the problem. Students reflect on and discuss questions

briefly and then are brought back together to discuss with the whole class. This process moves back and forth in short intervals between individual or small-group discussion and class discussion.

**Goal of the instructor:** Develop a culture of discussion, establish norms of listening, and model the language used to discuss quantitative concepts. In addition, emphasize to students that struggling indicates learning. If struggle is not taking place, students are not being challenged and are not gaining new knowledge and skills.

- **Level 2:** The problem is broken into subquestions that give students some direction but do not explicitly define or limit strategies and approaches. Students work in groups on multiple steps for longer periods, and the instructor facilitates individual groups, as needed. The instructor brings the class together at strategic points at which important connections need to be made explicit, or when breakdowns of understanding are likely to occur.

**Goal of the instructor:** Support students in working more independently and evaluating their own work so they feel confident about moving through multiple questions without regular reinforcement from the instructor.

- **Level 3:** The problem is not broken into steps or is broken into very few steps. Students are expected to identify strategies for themselves. Groups work independently, with facilitation by the instructor as necessary. Groups report on results, and class discussion focuses on reflection about the problem as a whole.

**Goal of the instructor:** Support students in persisting with challenging problems, including trying multiple strategies before asking for help.

## Table of contents information

The table of contents contains the following information:

- **Lesson number**—Note that “Suggestions for Prep Week” offers ideas on best practices you might want to incorporate into the syllabus.
- **Preview Assignment** (if any)—Preview Assignments consist of problems designed to assess student readiness for the prerequisite assumptions of the lesson. Students are instructed to seek help before the next class meeting if they are unable to complete these problems successfully.
- **Lesson title**
- **Instructor Notes** pages
- **Student Pages**
- **Practice Assignment**, if any—Practice Assignments consist of problems designed to assess student understanding of the concepts addressed in the topic.

## The role of the Preview and Practice Assignments

One of the most important aspects of the *Statistical Reasoning* curriculum is the role and design of the homework assignments. These assignments differ from traditional homework in several ways:

- The Preview Assignments are designed to prepare students explicitly for the next lesson. In a Preview Assignment, students are given a prerequisite set of skills that will be used in the next lesson and are asked to rate themselves. Each of these prerequisite skills is used in the Preview Assignment.
- The Preview Assignments occasionally contain information or questions that are used in the next lesson. Such information will generally be referenced in the **Suggested Instructor Notes** under **Background context**.
- The Practice Assignments enable students to develop and practice skills from the current lesson. Practice Assignments may include similar problems in a new context or an extension of the learning within the same context.
- One goal of the course is for students to engage more and more in productive struggle. Therefore, the assignments are based on the same principle of constructive perseverance as the rest of the curriculum. Ideally, each assignment should offer entry-level questions that all students should be able to complete successfully and also more challenging questions that will engage students in constructive perseverance. The expectation is not that every student should be able to answer every question correctly, but that every student should make a valid attempt at responding to each question. Therefore, there are questions in the assignments, especially in the later units, that many students may not answer correctly. These challenging questions may raise issues about grading practices, which are discussed later in this section.
- In some cases, the assignments include informational materials. It is expected that students will read these materials, as this information is not usually presented directly in class. This strategy for conveying the information distinguishes the material from a traditional use of textbooks, in which the text is assigned by instructors, but often used by students only as a reference.

### Strategies for supporting the assignments

The central role and unique design of the assignments in the curriculum requires instructors to develop strategies and procedures for supporting students in using the assignments appropriately. Some considerations and suggestions for strategies include the following:

**Motivating students to complete the assignments**—The design of the assignments supports motivation, as students come to realize that much of the assigned material will help them get more out of class time. You can support student motivation by doing the following:

- Discuss the role of the assignments with students.
- Set and maintain an expectation that students be able to use the prerequisite skills for a lesson. At first, students may take this expectation lightly, so it is important that you not use class time to review these skills. Rather, you should make it clear that students are responsible for being prepared. Keep in mind the following:
  - Students may have prepared, but may not recognize that what they are being asked to do in class is the same skill they used in the preview questions. Be prepared to refer back to specific preview questions to help them make this connection.
  - If a student is truly unprepared, do not reprimand him or her in front of the class. Privately explain the expectation for preparation to the student and invite him or her to meet with you outside of class to review the material. If you do meet the student outside of class, take the opportunity to talk about the importance of preparation and inquire about how the student does the self-assessment. Help the student develop strategies for using this tool more effectively.
- Occasionally, students are directed to bring a separate copy of their work to class for discussion. In this case, have some way for students who do the work to receive credit so students feel their work is valued. The group work model enables all students to participate in the lesson even if they did not complete the preview questions. You can assign a quick completion grade by walking around the room and seeing who has their printed work complete while students are working in their groups.
- Notify students at the end of the first, second, and third weeks if they have failed to complete any of their work. This notification can be done by email or by handing out notes in class. It is important for students to know that the instructor is aware of their individual work. Always include an offer of help and expressions of support in these notices. For example, “If there is something preventing you from completing your work, please come to see me. I want to help you be successful in this course.” Keep in mind that there are many reasons that students may fail to complete out-of-class work.
- Writing is an important component of the course (see the **Language and literacy skills** section later in this overview). Many of the in-class pages and assignments contain writing prompts. Early in the course, take time to give feedback on some of these paragraph responses individually.

**Grading assignments**—Since the assignments are designed to challenge students and promote constructive perseverance, giving feedback only on correct answers may not always be appropriate and may discourage students. On the other hand, grading on completion has drawbacks as well. Effective grading strategies need to be individualized based upon the grading time instructors have, the length of classes, and the student population. Some ideas follow:

- Use a scoring method that gives points for completion and for correctness.
- The curriculum is intended to build language and literacy skills over the course of the semester. Early in the course, weight the grade more toward the quality of the attempt than toward the correct answer. In addition, as the course progresses, the expectation of what constitutes quality work should also progress.
- Grade on correctness, but occasionally ask students to turn in written explanations for how they would reattempt problems they originally missed, and allow them to earn back points. This work can be managed by limiting the opportunity to one or two problems each week or to certain assignments.

**Helping students organize their work**—Many students struggle with organization. Instructors should provide some sort of structure to support students. Strategies include the following:

- Explain to students why it is important to organize their materials. Give specific examples of the ways in which they will use the materials in this course.
- Require that students keep materials in a three-ring binder. Depending on your sense of the need, you may provide students with a high level of structure or a moderate level.
  - **High structure:** Give students guidelines on how to order and label materials.
  - **Moderate structure:** Give students guidelines but also give them the option to create their own method of organization.
- Any structure you require should be graded in some way in order to encourage students to complete it. Checks of students' organization of their work should be done in the first few weeks of the course to establish a routine.
  - During class, regularly check students' organization of their work. Tell students to find a specific document within a specified amount of time (e.g., 2 minutes). Students get a grade for showing the instructor the document.
  - Start with a quick check for having the system (e.g., binder, folder) set up. Then occasionally have students turn their materials in and do a spot-check for certain documents.
- Give timed quizzes in which students are referred to certain documents and must respond to a quick question about the materials.

## Resource materials for students

The student resource packet is the starting point for course reference materials. Some of the resources are taken directly from an assignment because they contain material that may be useful to students later in the course. Other resources act as supplementary material. Encourage students to keep a section of their class binders dedicated to these resources.

## Language and literacy skills

Statistical literacy has unique language demands that differ from those of other subjects—even those of other math courses. Even skilled readers and writers may struggle with using and interpreting statistical information in conjunction with new terminology. Thus, one challenging aspect of the *Statistical Reasoning* course is teaching statistical literacy skills to students who are not yet college-ready.

The learning outcomes for language and literacy skills include the following:

- Reading and interpreting statistical information from a variety of real-world sources.
- Communicating statistical results in writing and orally, using appropriate language, symbols, data, and graphs.

The *Statistical Reasoning* course designers have further defined the expectations and purpose of reading and writing in the course. Students will read and use authentic text, defined as text that comes from a real-world source or that has been written by an author to simulate a real-life source. Using authentic text improves engagement and thus the development of skills in reading quantitative information in a variety of real-world situations.

The purpose of writing in *Statistical Reasoning* is to:

- Make sense of statistical information and processes, especially in relationship to a context.
- Develop skills in communicating statistical information.
- Provide a form of assessment by which students may demonstrate their understanding of the course material. Note: Other assessment methods will be used as well, including verbal responses (in class), short answer, fill-in-the-blank, multiple-choice, true–false, and presentations.

Students who completed the feeder course *Foundations of Mathematical Reasoning* have had substantial practice in mathematical literacy. Students should be able to write two to three paragraphs that make appropriate use of quantitative information. *Statistical Reasoning* will continue to expand on these skills. The Resource **Writing Principles** supports this work.

Due to time constraints, it may not be reasonable to have students write complete statements in response to every prompt in the classroom, but instructors can select some prompts—such as those that are more summary in nature—to use for this purpose. Early in the course, it may be useful to prompt students to remember their contextual writing



from the *Foundations* course and note that specificity is especially important in statistics. For example, in Lesson 1, Part D, students are asked to identify the steps in a statistical investigation for a given study. One student may say, “Do they want to choose the sex of the baby?” Ask another student to clarify or extend:

Instructor: “Does who?”

Student 2: “Women.”

Instructor: “What women?”

Student 3: “Women who visit fertility clinics.”

Continue soliciting feedback from students in this manner until a complete response is crafted.

## DCMP curriculum design standards

The Dana Center Mathematics Pathways (DCMP) is made up of individual courses that form *pathways* for students to and through college-level mathematics. The concept of the pathway as a yearlong experience is critical to the DCMP because these courses are designed to articulate with each other to provide students with the experience of learning mathematics and/or statistics through coherent, consistent practices and structures.

The design standards outlined in this section set the guidelines for how the curricular materials for individual DCMP courses are designed to support that coherent experience for students.

**Note:** The numbering in the description of the design standards does not indicate level of importance.

### Standard I: Structure and Organization of Curricular Materials

The DCMP is organized around big mathematical and statistical ideas and concepts as opposed to skills and topics.

### Standard II: Active Learning

The DCMP is designed to actively involve you in doing mathematics and statistics, analyzing data, constructing hypotheses, solving problems, reflecting on your work, and learning and making connections.

Class activities provide regular opportunities for you to actively engage in discussions and tasks using a variety of different instructional strategies (e.g., small groups, class discussions, interactive lectures).

### **Standard III: Constructive Perseverance**

The DCMP supports students in developing the tenacity, persistence, and perseverance necessary for learning mathematics.

### **Standard IV: Problem Solving**

The DCMP supports you in developing problem-solving skills and in applying previously learned skills to solve nonroutine and unfamiliar problems.

### **Standard V: Context and Interdisciplinary Connections**

The DCMP presents mathematics and statistics in context and connects mathematics and statistics to various disciplines.

### **Standard VI: Use of Terminology**

The DCMP uses discipline-specific terminology, language constructs, and symbols to intentionally build mathematical and statistical understanding and to ensure that terminology is not an obstacle to understanding.

### **Standard VII: Reading and Writing**

The DCMP develops your ability to communicate about and with mathematics and statistics in contextual situations appropriate to the pathway.

### **Standard VIII: Technology**

The DCMP uses technology to facilitate active learning by enabling you to directly engage with and use mathematical concepts. Technology should support the learning objectives of the lesson. In some cases, the use of technology may be a learning objective in itself, as in learning to use a statistical package in a statistics course.

## **Readiness competencies**

Students enrolling in *Statistical Reasoning* should be able to do the following:

- Demonstrate number sense, including dimensional analysis and conversions between fractions, decimals, and percentages. Determine when approximations are appropriate and when exact calculations are necessary.
- Solve linear equations, graph and interpret linear models, and read and apply formulas.
- Demonstrate a basic understanding of displays of univariate data—such as bar graphs, histograms, dotplots, and circle graphs, including appropriate labeling.

- Take charge of their own learning through good classroom habits, time management, and persistence. Participate in the class community through written and oral communication.

## Learning goals

The following five learning goals apply to all DCMP mathematics courses, with the complexity and diversity of problem-solving skills and strategies increasing as students advance through the pathways.

For each course, we define the ways that the learning goals are applied and the expectations for mastery. The bullets below each of the five learning goals specify the ways in which each learning goal is addressed in the *Statistical Reasoning* course.

Each DCMP course is designed so that students meet the goals across the courses in a given pathway. Within a course, the learning goals are addressed across the course's content-based learning outcomes.

**Communication Goal: You will be able to interpret and communicate quantitative information and mathematical and statistical concepts using language appropriate to the context and intended audience.**

In the *Statistical Reasoning* course, you will...

- Use appropriate statistical language in oral, written, and graphical forms.
- Read and interpret graphs and descriptive statistics.
- Read short, authentic texts, such as graphical displays and journal and newspaper articles describing statistical studies. Evaluate the design, analysis, and conclusion of a given study both orally and in written form.

**Problem-Solving Goal: You will be able to make sense of problems, develop strategies to find solutions, and persevere in solving them.**

In the *Statistical Reasoning* course, you will...

- Understand what statistical question is being addressed, use appropriate strategies to answer the question of interest, and state conclusions using appropriate statistical language.

**Reasoning Goal: You will be able to reason, model, and make decisions with mathematical, statistical, and quantitative information.**

In the *Statistical Reasoning* course, you will...

- Use probability, graphical and numerical summaries of data, confidence intervals, and hypothesis testing methods to make decisions.
- Support conclusions by providing appropriate statistical justifications.
- Present short written or verbal justifications of decisions that include appropriate discussion of the mathematics involved.

**Evaluation Goal: You will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information.**

In the *Statistical Reasoning* course, you will...

- Identify errors—such as inappropriate sampling methods, sources of bias, and potentially confounding variables—in both observational and experimental studies.
- Identify mathematical or statistical errors, inconsistencies, or missing information in arguments.

**Technology Goal: You will be able to use appropriate technology in a given context.**

In the *Statistical Reasoning* course, you will...

- Use some form of spreadsheet application to organize information and make repeated calculations using simple formulas and statistical functions.
- Use the internet to find statistical information. Topics should be limited to those that can be researched with a simple search.
- Use internet-based tools appropriate for a given context (e.g., an online tool to calculate p-values).
- Use technology to calculate descriptive statistics and to test hypotheses.
- Interpret and apply output from a statistical software package.

## Content learning outcomes

*Statistical Reasoning* is a first course in statistics and is intended for students in business, nursing, allied health, and the social and behavioral sciences, and for any student whose college and career paths require knowledge of the fundamentals of the collection, analysis, and interpretation of data.

Course topics include the presentation and interpretation of univariate data using graphical and numerical methods, probability, discrete and continuous probability distributions, linear regression, an understanding of good practice in study design, statistical inference, confidence intervals, and hypothesis testing. Emphasis is placed on the

development of statistical thinking, the use of simulations, and the application of statistical software.

Students should develop an appreciation of the need for data to make good decisions and an understanding of the dangers inherent in basing decisions on anecdotal evidence rather than on data. To that end, students will use appropriate data-collection methods and statistical techniques to support reasonable conclusions through the following content learning outcomes:

- Data Exploration
- Statistical Design
- Probability and Simulation
- Statistical Inference

## Data Exploration

**Outcome: You will analyze data using graphical and numerical methods to study patterns and departures from patterns, using appropriate technology as needed.**

You will be able to:

### **DE.1 Construct and interpret graphical displays of distributions of univariate data.**

To include: Create and interpret dotplots, boxplots, stemplots, and histograms. Analyze center, shape, and spread, as well as clusters, gaps, outliers, and other unusual features.

### **DE.2 Summarize distributions of univariate data and compare multiple distributions.**

To include: Compute measures of center (median, mean), measures of spread (range, interquartile range, standard deviation), and measures of position (quartiles, other percentiles, and standardized scores). Compare groups using back-to-back stemplots, parallel boxplots, and dotplots.

### **DE.3 Explore bivariate data.**

To include: Analyze scatterplots for patterns, linearity, outliers, and influential points. Determine the equation of the least-squares regression line and interpret its slope and intercept in context. Calculate and interpret the correlation coefficient and the coefficient of determination. Construct and interpret residual plots.

### **DE.4 Explore categorical data.**

To include: Create and interpret frequency tables and bar charts, compare distributions of categorical data, use two-way tables to analyze and interpret

marginal, joint, and conditional relative frequencies, and estimate the probability of events defined in context.

## Statistical Design

**Outcome: You will develop an appropriate data-collection plan to answer a given research question.**

You will be able to:

**SD.1 Identify characteristics of good study designs. Understand what conclusions are appropriate for a given design and whether conclusions can be generalized to a larger population.**

To include: Identify the population of interest. Determine whether an observational or an experimental study is appropriate and feasible. Explain the difference between, and importance of, random selection and random assignment in study design.

**SD.2 Plan and conduct an observational study when appropriate.**

To include: Determine when a census or a sample survey is appropriate. Develop and implement an appropriate sampling plan (including the use of simple random, stratified, and cluster sampling). Identify potential sources of bias in sampling and surveys.

**SD.3 Plan and conduct an experimental study when appropriate.**

To include: Design and implement an appropriate plan for carrying out an experiment. Explain the purpose of including a control group and blinding in an experiment. Identify potential sources of confounding in an experiment.

## Probability and Simulation

**Outcome: You will use probability concepts and simulation.**

You will be able to:

**PS.1 Calculate and interpret probabilities.**

To include: Interpret a probability as a long-run relative frequency of occurrence. Calculate the probability of a specified event in a chance experiment with equally likely outcomes. Determine probabilities using the complement rule, the addition rule for disjoint events, and the multiplication rule for independent events.

Analyze and conduct simulations to estimate the probability of an event.

**PS.2 Use probability distributions to describe the behavior of discrete and continuous random variables.**

To include: Distinguish between discrete random variables and continuous random variables. Compute and interpret the mean and standard deviation of the probability distribution of a discrete random variable. Demonstrate an understanding of the mean, standard deviation, and shape of continuous probability distributions (uniform, normal, and skewed).

**PS.3 Understand distributions.**

To include: Distinguish between the distribution of a sample and a sampling distribution. Describe the sampling distributions of a sample mean and a sample proportion in terms of center, shape, and spread. Explain how these relate to sample size. Identify when the use of the normal distribution is appropriate. Use the normal distribution in modeling.

**Statistical Inference**

**Outcome: You will use statistical models to draw conclusions from data.**

You will be able to:

**SI.1 Estimate population parameters using confidence intervals when appropriate.**

To include: Verify that the appropriate conditions have been met. Construct one- and two-sample confidence intervals for means and for proportions. Interpret confidence intervals in context and interpret the confidence level associated with a confidence interval estimate. Analyze authentic statistical studies that report confidence intervals and evaluate whether the conclusions are reasonable.

**SI.2 Conduct tests of significance when appropriate.**

To include: Verify that the appropriate conditions have been met. Carry out one- and two-sample hypothesis tests for means and proportions, and for chi-squared tests. Identify appropriate hypotheses. Describe type I and type II errors in context. Interpret the meanings of rejection of the null hypothesis and of failure to reject the null hypothesis, in context. Demonstrate an understanding of the use of a  $p$ -value to reach a conclusion, and of the difference between practical significance and statistical significance. Analyze authentic statistical studies that report the results of a hypothesis test and evaluate whether the conclusions are reasonable.