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We welcome your comments and suggestions for improvements. Please contact us at danaweb@austin.utexas.edu or at the mailing address above.

About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We work with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at www.utdanacenter.org.

About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—*Statistical Reasoning*, *Quantitative Reasoning*, and *Reasoning with Functions I* and *II* (a two- course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point—a developmental math course that helps students develop foundational skills and conceptual understanding in the context of college-level course material.

In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies—and tenacity—necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.

In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

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The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The **Usability Advisory Group** members are:

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Curriculum Overview

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About *Foundations of Mathematical Reasoning*

Foundations of Mathematical Reasoning is a semester-long, quantitative literacy-based course designed to provide students with the skills and conceptual understanding to succeed in a college-level statistics, quantitative literacy, or STEM path algebraic reasoning course.

Foundations of Mathematical Reasoning is organized around big mathematical and statistical ideas. The course will help students develop conceptual understanding and acquire multiple strategies for solving problems (as described in the DCMP Curriculum Design Standards as described on page xxx). The course will prepare students for success in future courses and will help them develop skills for the workplace and as productive citizens.

Foundations of Mathematical Reasoning is typically taught as a developmental-level course that is designed to accelerate students to college-level mathematics.

Course structure and contact hours

Foundations is designed to be taught in a one-semester course with 4 student contact hours per week or in a quarter system with an equivalent number of contact hours. Colleges may choose to offer this as a 4-credit course or as a combination of course credits and lab credits. Regarding lab credits, it is important to note that the curriculum is not designed for instruction in separate and distinct lab meetings. Rather, the intent is for the instructor to use all 4 contact hours for classroom instruction.

Active learning design standards are evident in each lesson. Students will be expected to actively do mathematics—such as analyzing data, constructing hypotheses, solving problems, reflecting on their work, and making connections among and between mathematical concepts.

Structure of the curriculum

The curriculum is designed in 25-minute learning episodes, which can be pieced together to conform to any class length. These short bursts of active learning, combined with whole class discussion and summary, produce increased memory retention.[1]

Prerequisite assumptions

Students enrolling in *Foundations of Mathematical Reasoning* should be able to do the following:

- Demonstrate procedural fluency with real number arithmetic operations (e.g., basic operations, comparing, contrasting), use arithmetic operations to represent real-world scenarios, and use those operations to solve stated problems.
- Use graphical representations on a real number line to demonstrate fluency when ordering numbers, to represent operations (e.g., addition, subtraction, doubling, halving, averaging), and to represent fractions and decimals.
- Demonstrate a basic understanding and familiarity with fractions, decimals, and percentages. Procedural competency for representing each number form and moving from one to the other is desired upon enrollment in this course, but the student may also need to work with materials outside of class to review basic concepts and build basic skills.

[1] Sources: Buzan, T. (1989). *Master your memory* (Birmingham: Typersetters); Buzan, T. (1989). *Use your head* (London: BBC Books); Sousa, D. (2011). *How the brain learns, 4th ed.* (Thousand Oaks, CA: Corwin); Gazzaniga, M., Ivry, R. B., & Mangun, G. R. (2002). *Cognitive neuroscience: The biology of the mind, 2nd ed.* (New York: W.W. Norton); Stephane, M., Ince, N., Kuskowski, M., Leuthold, A., Tewfik, A., Nelson, K., McClannahan, K., Fletcher, C., & Tadipatri, V. (2010). Neural oscillations associated with the primary and recency effects of verbal working memory. *Neuroscience Letters*, 473, 172–177.); Thomas, E. (1972). The variation of memory with time for information appearing during a lecture. *Studies in Adult Education,* 57–62.

Structure of the Suggested Instructor Notes for the lessons

The main features of the **Suggested Instructor Notes** for the lessons are:

- **Overview and student objectives –** includes the constructive perseverance level of the lesson, the learning outcomes, and goals addressed.
- **Suggested resources and preparation –** includes technology needs, physical materials, and preparation needed for activities. A consolidated materials list is included in the next section.
- **Prerequisite assumptions –** lists the skills that students need to be prepared for the lesson. The same list is given to the students in the preview assignment. Students are asked to rate their confidence level on each skill. If they struggle with transference of these skills into the new context of the lessons, the instructor can refer back to the preview questions to help students recognize that they have done similar problems.
- **Making connections –** details the main concepts that are extensions of earlier work in the course as well as connections forward in this course and later courses.
- **Background context –** includes the main points of any informational pieces that were given to students in preview assignments. For example, the preview assignment for Lesson 13, Part B contains information about the grade of a road. The main points of that information are included in the **Background context** section of the **Suggested Instructor Notes** for that lesson so that the instructor does not have to look through the homework to determine what students have read.
- **Suggested instructional plan –** includes excerpts of **Student Pages** and the following:
    - Frame the lesson – suggestions to elicit prior student knowledge, focus discussion, or ask for a prediction.
    - Lesson activities – detailed suggestions for probing questions for students or groups, guiding questions for class discussions, and literacy supports.
    - Wrap-up for the day or transition to the next activity.

The **Suggested Instructor Notes** do not summarize all ideas of the lesson; rather, they are intended to facilitate the inclusion of broader ideas. Instead of having the instructor inform them of the connections, the goal is to have students actively engaged in making those connections. This is a challenging skill that will be developed throughout the course. Early discussions are likely to be slow-starting and require a great deal of prompting. Instructors can build on what students say and can model how to express these abstract concepts. The facilitation prompts provide instructors with ideas on how to promote student discussion.

As the explicit connections emerge, the instructor should record the ideas on the board and, especially early in the course, make sure students record the ideas in their notes.

- **Suggested assessment, assignments, and reflections** – includes references to the homework assignments that accompany the lessons. Occasionally, additional assessments, projects, or reflections are suggested. In addition, instructors are reminded to assign any preview assignments for upcoming lessons.

- **Instructor version of student pages** – includes answers and/or sample answers where appropriate. Additional space is provided for the instructor to add notes or to incorporate facilitation tips and guiding questions from the **Suggested Instructor Notes**.

Constructive perseverance level

The levels of constructive perseverance are a way to help instructors think about scaffolding productive struggle through the course. The levels should be viewed broadly as a continuum rather than as distinct, well-defined categories. In general, the level increases through the course, but this does not mean that every lesson later in the course will be a Level 3. The level is based both on the development of students and the demands of the content. Some content requires greater structure and more direct instruction. The levels of constructive perseverance are as follows:

- **Level 1:** The problem is broken into sub-questions that help develop strategies. Students reflect on and discuss questions briefly and then are brought back together to discuss with the full class. This process moves back and forth between individual or small-group discussion and class discussion in short intervals.

  Goal of the instructor: Develop the culture of discussion, establish norms of listening, and model the language used to discuss quantitative concepts. In addition, emphasize to students that struggling indicates learning. If struggle is not taking place, students are not being challenged and are not gaining new knowledge and skills.

- **Level 2:** The problem is broken into sub-questions that give students some direction but do not explicitly define or limit strategies and approaches. Students work in groups on multiple steps for longer periods, and the instructor facilitates individual groups, as needed. The instructor brings the class together at strategic points at which important connections need to be made explicit or when breakdowns of understanding are likely to occur.

  Goal of the instructor: Support students in working more independently and evaluating their own work so they feel confident about moving through multiple questions without constant reinforcement from the instructor.

- **Level 3:** The problem is not broken into steps or is broken into very few steps. Students are expected to identify strategies for themselves. Groups work independently on the problem with facilitation by the instructor, as necessary. Groups report on results, and class discussion focuses on reflecting on the problem as a whole.

  Goal of the instructor: Support students in persisting with challenging problems, including trying multiple strategies before asking for help.

Materials list

The following materials are recommended for use in the *Foundations* course. In most cases, replacements are possible. For example, estimation strategies may be used in place of a measuring tape or meter stick, folded paper instead of rulers or straightedges, etc.

- Picture of campus logo or mascot (optional for Lesson 15)
- Campus map – 1 copy per group (optional for Lesson 10)
- Chart paper for displaying group work simultaneously (notebook paper can be used if a document camera is available and time is available for several presentations)
- Dry spaghetti for trend lines (Lesson 17)
- Markers – at least 2 colors per group
- Measuring tapes – 1 for every 4 students (meter sticks could be used instead)
- Meter sticks – 3
- Name tag for each student for the first day
- Painter's tape
- Rulers – at least 1 per group
- Transparency pages – for overlaying graphs from different groups on document camera (Lessons 17 and 19)

Table of contents information

The table of contents contains the following information:

- Lesson number – Note that the first "lesson" actually consists of suggestions for the instructor on best practices to be incorporated into the syllabus.
- Preview assignment, if any – Preview assignments consist of problems designed to assess student readiness for the prerequisite assumptions of the lesson. Students are instructed to seek help before the next class meeting if they are unable to successfully complete these problems.
- Lesson title and brief description
- Instructor's notes page reference
- Student page reference

The role of the preview and practice assignments

One of the most important aspects of the *Foundations of Mathematical Reasoning* curriculum is the role and design of the homework assignments. These assignments differ from traditional homework in several ways:

- Each question has a specific purpose. While some questions are specifically skill based, repetition of a skill in a single form is never used. If repetition is deemed valuable, it is done with different contexts or formats that require students to think about each question rather than assume they can repeat the steps of the previous question.

- The preview assignments are designed to prepare students for the next lesson. This preparation is done explicitly. Students are given a prerequisite set of skills for the next lesson and asked to rate themselves. Each of these prerequisite skills is used in the assignment.

- The preview assignments occasionally contain information or questions that are directly used in the next lesson. These will generally be referenced in the **Suggested Instructor Notes** under **Background context.**

- The practice assignments review previous material and allow students to practice and develop skills from the current lesson.

- The design of the practice assignments is based on the same principle of constructive perseverance as the rest of the curriculum. Ideally, each assignment should offer entry-level questions that all students should be able to complete successfully and also more challenging questions. One goal of the entire curriculum is that students will increasingly engage in productive struggle. The expectation is not that every student should be able to answer every question correctly, but that every student should make a valid attempt on each question. Therefore, there are questions in the assignments, especially later in the course, that many students may not answer correctly. These challenging questions raise issues about grading practices, as discussed in the following paragraphs.

- In some cases, the assignments include actual instructional materials. It is expected that students will read these materials, as they are usually not presented directly in class. This information distinguishes the material from traditional textbooks in which the text is often assigned by instructors, but often only used by students as a reference.

- The assignments purposefully refer students back to previous lessons. This is done to support students in making connections across the course, encourage students to review previous material, and support good organizational habits.

Strategies for supporting the assignments

The central role and unique design of the assignments in the curriculum require instructors to develop strategies and procedures for supporting students to use the assignments appropriately. The following are some areas instructors should consider and some suggestions for strategies.

Motivating students to complete the assignments – The design of the assignments supports motivation as students come to realize that much of the material is actually useful to them in class. Instructors can support student motivation by doing the following:

- Discuss the role of the assignments with students.

- Set and maintain an expectation that students should be able to use the prerequisite skills for a lesson. Students may take this lightly at first, so it is important that instructors do not use class time to review these skills but make it clear that students are responsible for being prepared. Keep in mind the following:

  - Students may have prepared but may not recognize that what they are being asked to do in class is the same skill they used in the preview questions. Be prepared to refer back to specific questions to help them make this connection.
  - If a student is truly unprepared, do not reprimand him or her in front of the class. Privately explain the expectation for preparation to the student and invite him or her to meet with you outside of class to review the material. If you do meet the student outside of class, take the opportunity to talk about the importance of preparation and inquire about how the student does the self-assessment. Help the student develop strategies for using this tool more effectively.

- Occasionally students are directed to bring a separate copy of their work to class for discussion. In this case, have some way for students who do the work to receive credit. The group work model allows all students to participate in the lesson even if they did not do the preview questions. This is important, but students who come prepared should feel that their work is valued. You can give students a quick completion grade by walking around the room and seeing who has their printed work complete while students are working with their groups.

- Notify students at the end of the first, second, and third weeks if they have failed to complete any of their work. This notification can be done by email or by handing out notes in class. It is important for students to know that the instructor is aware of their individual work. Always include an offer of help and expressions of support in these notices. For example, "If there is something preventing you from completing your work, please come to see me. I want to help you be successful in this course." Keep in mind that there are many reasons that students fail to complete out-of-class work.

- Writing is an important component of the course (see the **Language and literacy skills** section in this Overview). Many of the in-class pages and assignments contain some writing prompts. Early in the course, take the time to grade some of these paragraph responses individually.

Grading assignments – Since the assignments are designed to challenge students and promote constructive perseverance, grading only on correct answers may not always be appropriate and may discourage students. On the other hand, grading on completion has drawbacks as well. Effective grading strategies have to be individualized depending on the grading time instructors have, the length of classes, and the student population. Some ideas follow.

- Use a scoring method that gives points for both completion and correctness.

- The curriculum is intended to build language and literacy skills over the course of the semester. Especially early in the course, weight the grade toward the quality of the attempt versus the correct answer. Additionally, the expectation of what constitutes quality work should build.

- Grade on correctness but occasionally allow students to turn in written explanations for problems they missed and earn back points. This work can be managed by limiting the opportunity to one or two problems each week or to certain assignments.

Many students struggle with organization. Instructors should provide some sort of structure to support students. Strategies include the following:

- Explain to students why it is important to organize their materials. Give specific examples of the ways in which they will use the materials in this course.

- Require that students keep materials in a three-ring binder.
    - **High structure:** Give students guidelines on how to order and label materials.
    - **Moderate structure:** Give students guidelines but also give them the option to create their own method of organization.

- Any structure that is required should be graded in some way in order to encourage students to complete it. Checks should be done in the first few weeks of the course to establish a routine.
    - Check in class on a regular basis: Tell students to find a specific document within a specified amount of time (e.g., 2 minutes). Students get a grade for showing the instructor the document.
    - Start with a quick check for having the system (e.g., binder, folder) set up. Then occasionally have students turn their materials in and do a spot-check for certain documents.
    - Give timed quizzes in which students are referred to certain documents and must respond to some quick question about the materials.

Resource materials for students

The student resource packet is designed to be the starting point for course reference materials. Some of the resources are directly lifted from an assignment because they contain material that may be useful to students later in the course. Other resources were created as supplementary material. Encourage students to keep a section of their class binder dedicated to the provided resources as well as any additional resources they may collect.

Language and literacy skills

Quantitative literacy has unique language demands that are different from other subjects, even other math courses. Even skilled readers and writers often struggle with using and interpreting quantitative information in conjunction with language. One of the greatest challenges of the *Foundations* course is that it seeks to teach quantitative literacy to a population that has a high proportion of students who are not college-level readers or writers for a variety of reasons.

The learning outcomes of the course include the following:

- Reading and interpreting quantitative information from a variety of real-world sources.
- Communicating quantitative results both in writing and orally using appropriate language, symbolism, data, and graphs.

The designers of the course have further defined the expectations and purpose of reading and writing in the *Foundations* course. Students will read and use *authentic text*, which is defined as text that come from a real-life source or, in rare occasions, has been written by a lesson author to replicate a real-life source. The purpose of using authentic text is to support engagement and the development of skills in reading quantitative information in real-life situations.

The purpose of writing in *Foundations* is to:

- Make sense of quantitative information and processes, especially in relationship to a context.
- Build skills in communicating about quantitative information.
- Provide one form of assessment by which students may demonstrate their understanding of the course material. (Note: Other assessment methods will be used including verbal responses [in class], short answer, fill-in-the-blank, multiple choice, true–false, presentations.)

As much as possible, writing assignments are framed with a specific context, purpose, and audience (e.g., write a letter to your congressman supporting your views on…).

By the end of the *Foundations* course, students should be able to write two to three paragraphs that make appropriate and accurate use of quantitative information. This skill will be achieved by stressing reading and writing skills along with the mathematics skills throughout the course. Two of the Resource materials support this work: **Writing Principles** and **Understanding Visual Displays of Information**. In addition, points of direct instruction and practice are built into the first half of the course as indicated in the table beginning on the next page). The first half of the course provides these opportunities for instruction and highly structured skill building to prepare students for the second half of the course when students are expected to use their reading and writing skills more independently. Instructors should pay special attention to these points in the lessons and give ample time to discuss them in class.

Due to time constraints in class, it may not be reasonable to have students write complete statements in response to every prompt, but instructors can select prompts that are more summary in nature to use for this purpose. See the **Suggested Instructor Notes** for Lesson 2, Part A for ideas about modeling and giving feedback on writing.

Lesson	Literacy Task/Goal
1.C	In the practice assignment, students write about issues of interest to them related to the course themes of civic life, personal finance, and risk assessment.
2.A	Students see examples of well-written and poorly written statements, which set the stage for the use of language in the course. Students write a statement with quantitative information. *Instructors should use and refer students to Resource* ***Writing Principles.*** In the practice assignment, students are presented with two sample answers and give at least two reasons why Statement 2 is better than Statement 1.
2.C	Students are exposed to slightly more extensive reading. Students practice writing contextual sentences. This type of writing continues throughout the course. In the practice assignment, students are given a multiple-choice question on selecting the best statement based on authentic text. This type of question occurs multiple times throughout the course.
2.D	In the practice assignment, students are given web resources. They choose information from the sources, calculate ratios, and write comparative statements.
3.A	Students are introduced to the complexity of communicating quantitative concepts through a context in which quantitative information has been misrepresented.
3.C	The preview assignment provides instruction about words used with estimation to ensure that students read it. The information is repeated in Resource **Rounding and Estimation** so that students have it for future use. Similar introduction and follow-up with Resource documents occurs throughout the course. In the lesson and in the assignment, students write estimation strategies and statements with quantitative information.
4.A	In the preview assignment and in the lesson, students read and interpret pie charts. *Instructors should refer students to Resource* ***Understanding Visual Displays of Information.*** In the lesson, students record their sequence of steps. In the practice assignment, students are given a very sophisticated open-response pie chart question that sets the stage for Lesson 4, Part D.

Lesson	Literacy Task/Goal
4.B	Students read and interpret spreadsheets and spreadsheet formulas. They work with increasingly complex spreadsheets throughout the course.
4.C, 4.D, 5.A	In the preview assignments, students read labels and values from line graphs and bar graphs. In the lessons and practice assignments, students read and interpret line graphs, pie charts, bar graphs, and stem-and-leaf plots.
5.B, 5.C	Students communicate information through the creation and interpretation of frequency tables and histograms. Lesson 5, Part C contains an optional "Displaying Data" project.
5.D	Students read and interpret dotplots.
6.C	Students read and use information presented in the form of advertising. In the practice assignment, students read and use information from press releases and from a website.
6.D	Students communicate information through the creation and interpretation of box-and-whisker plots.
7.A	Students are introduced to reading strategies for an authentic, complex document. Students read complex information with instructor support and discuss strategies about picking out important information.
7.C and 7.D	Students use IRS forms with instructor support.
8 and 9	Students interpret nuanced quantitative statements related to percentages.

Reading and writing about quantitative information continues to be embedded in the second half of the course, although there is less emphasis on instruction. Instructors should continue to grade and give feedback on occasional writing assignments. Several lessons contain writing prompts or have topics that lend themselves to instructor-created prompts that can be used for this purpose.

Content outline of the curriculum

The following outline gives an overview of the curriculum. Because of the spiraling nature of the curriculum, it can be difficult for an instructor to know what level of mastery to expect at different points. This outline shows where topics are introduced and what should be mastered by the end of each set of lessons.

	Lessons	Concepts	Expectations for Mastery at the End of Each Set of Lessons
Course Introduction, Toolkit Building, Data Displays, Literacy Approximately 2.5 weeks	1.A, 1.C, 2.A	Large numbers, relative magnitude, place value, estimating, rounding, building learning community, doubling, introduction to ratios	**Mastered:** All skills listed.
	2.B, 2.C, 2.D, 3.A, 3.B	Scientific notation, ratios, percentages, reading and writing, misrepresented information, help seeking and giving	**Mastered:** Basic concept of ratio, creating and interpreting ratios with units, using technology to calculate with large numbers, writing large numbers in scientific notation. Students should master the basics such as: • Order of operations for expressions such as 4 + 3(5) • a x ½ is the same as a ÷ 2 • Addition and multiplication can be reordered to make calculations easier.
	3.C, 3.D, 3.E, 4.A, 4.B	Percentages and equivalent forms, estimation, calculation fluency, properties (informal), order of operations Throughout the course, students will generally use technology for calculations, other than when estimating. The emphasis is on the types of skills that will be helpful in using technology, using formulas, and solving equations.	**Mastered:** Interpreting percentages; estimating simple percentages based on benchmarks, including explaining estimation strategies.
	4.C, 4.D, 5.A, 5.B	Data, data displays, interpretation, introduction to absolute and relative change, recognize misleading graphs, preparing for an exam	**Mastered:** Reading and interpreting line graphs, bar charts, pie charts; calculating percentages.
	Lessons	**Concepts**	**Expectations for Mastery at the**

			End of Each Set of Lessons
Data Analysis, Proportional Reasoning, Complex Use of Percentages, Probability Approximately 4 weeks	5.C, 5.D	Histograms, cumulative frequency, distribution shapes, dotplots, optional mini-project, how the brain learns and building brain power	**Mastered:** Reading and interpreting histograms and dotplots, shapes of distributions.
	6	Calculate and interpret measures of central tendency, create data to match a given statistic, calculate the 5- number summary, create and interpret boxplots, resetting goals	The lessons introduce concepts about differences in the measures of central tendency and deciding which is most appropriate, but mastery is not expected. **Mastered:** Calculating and interpreting measures of central tendency; creating data to match a given statistic; reading and interpreting boxplots.
	7	Multi-step instruction, spreadsheets/formulas, order of operations, properties, expressions	Students should be gaining confidence with reading and interpreting instructions for calculations. **Mastered**: Able to perform a three-step calculation based on simple instructions (less complex than those given in the lessons).
	8	Ratios, risk, absolute and relative change, critical reading	**Mastered:** Recognizing problems in comparing absolute and relative change and percentages with different base values (in pie charts and data).
	9	Two-way tables, conditional probabilities, shifting base values, time management, creating a study plan and study guide	Students should recognize the complexities of situations with percentages. While they may not have mastery of these situations, they should know to take care in thinking about the base value. **Mastered:** Calculating a percentage of a number and what percent one number is of another; using information in a two-way table.

	Lessons	Concepts	Expectations for Mastery at the End of Each Set of Lessons
Data Analysis, Concept of Variable, Geometric Reasoning, Solving Equations Approximately 4 weeks	10	More practice with ratios and using proportional reasoning to calculate a new value based on a ratio; absolute and relative change	**Mastered:** Calculating absolute and relative change; recognizing problems in comparing absolute and relative change and percentages with different base values.
	11	Understanding the use of variables, including importance of units; evaluating formulas; geometric concepts of linear measure, area, and volume.	**Mastered:** All skills listed.
	12	Evaluating formulas; multiplication of fractions; using units to set up and check for errors in dimensional analysis problems; using dimensional analysis to make calculations using multiple conversion factors.	While multiplication of fractions is assumed in the dimensional analysis, it is not separately assessed. **Mastered:** Using dimensional analysis to make calculations using multiple conversion factors.
	13	More complex use of variables; reading to understand and apply unfamiliar formulas; analyze effect of changing values of one variable while holding others fixed.	**Mastered:** All skills listed.
	14 and 15	Solving equations, including: • Basic two-step equations • Equations written as proportions • Literal equations • Equations with multiple variable terms that need to be simplified	**Mastered:** All skills listed.

	Lessons	Concepts	Expectations for Mastery at the End of Each Set of Lessons
Algebraic Reasoning and Modeling Approximately 3 weeks	16	Linear models: • Four representations • Rate of change • Contextual interpretation of slope and intercepts	**Mastered:** All skills listed.
	17	Approximate linear models, scatterplots and trendlines, interpolation/extrapolation, estimate solution to a system	**Mastered:** All skills listed.
	18	Relative increase/decrease, shifting base values, expressions and equations, repeated multiplication, basic understanding of the difference between linear and exponential relationships, understanding annual compound interest and creating/using a formula to find values. Optional mini-project.	**Mastered:** All skills listed.
	19	Other compounding periods, writing/using exponential equations given a starting value and rate, creating exponential graphs, comparing models. Optional mini-project.	**Mastered:** All skills listed.

Curriculum design standards

The Dana Center Mathematics Pathways (DCMP) model is made up of individual courses that form *pathways* for students to and through college-level mathematics. The concept of the pathway as a yearlong experience is critical to the DCMP model because these courses are designed to articulate in a way that provides students with the experience of learning mathematics and/or statistics through coherent, consistent practices and structures.

The design standards outlined in this section set the guidelines for how the curricular materials for individual DCMP courses are designed to support that coherent experience for students.

Note: The numbering in the description of the design standards does not indicate level of importance.

Standard I: Structure and Organization of Curricular Materials

The DCMP is organized around big mathematical and statistical ideas and concepts as opposed to skills and topics.

Standard II: Active Learning

The DCMP is designed to actively involve students in doing mathematics and statistics, analyzing data, constructing hypotheses, solving problems, reflecting on their work, and learning and making connections.

Class activities provide regular opportunities for students to actively engage in discussions and tasks using a variety of different instructional strategies (e.g., small groups, class discussions, interactive lectures).

Standard III: Constructive Perseverance

The DCMP supports students in developing the tenacity, persistence, and perseverance necessary for learning mathematics.

Standard IV: Problem Solving

The DCMP supports students in developing problem-solving skills, and students apply previously learned skills to solve nonroutine and unfamiliar problems.

Standard V: Context and Interdisciplinary Connections

The DCMP presents mathematics and statistics in context and connects mathematics and statistics to various disciplines.

Standard VI: Use of Terminology

The DCMP uses discipline-specific terminology, language constructs, and symbols to intentionally build mathematical and statistical understanding and to ensure that terminology is not an obstacle to understanding.

Standard VII: Reading and Writing

The DCMP develops students' ability to communicate about and with mathematics and statistics in contextual situations appropriate to the pathway.

Standard VIII: Technology

The DCMP uses technology to facilitate active learning by enabling students to directly engage with and use mathematical concepts. Technology should support the learning objectives of the lesson. In some cases, the use of technology may be a learning objective in itself, as in learning to use a statistical package in a statistics course.

Note: A more detailed description of the design standards is available on the Dana Center website at https://dcmathpathways.org/resources/nmp-curriculum-design-standards.

Learning goals

The following five learning goals apply to all DCMP mathematics courses, with the complexity of problem-solving skills and use of strategies increasing as students advance through the pathways.

For each course, we define the ways that the learning goals are applied and the expectations for mastery. The bullets below each of the five learning goals specify the ways in which each learning goal is applied in the *Foundations of Mathematical Reasoning* course.

Each DCMP course is designed so that students meet the goals across the courses in a given pathway. Within a course, the learning goals are addressed across the course's content-based learning outcomes.

Communication Goal: Students will be able to interpret and communicate quantitative information and mathematical and statistical concepts using language appropriate to the context and intended audience.

In the *Foundations* course, students will…

- Use appropriate mathematical language.
- Read and interpret short, authentic texts such as advertisements, consumer information, government forms, and newspaper articles containing quantitative information, including graphical displays of quantitative information.
- Write 1 to 2 paragraphs using quantitative information to make or critique an argument or to summarize information from multiple sources.

Problem Solving Goal: Students will be able to make sense of problems, develop strategies to find solutions, and persevere in solving them.

In the *Foundations* course, students will...

- Solve multi-step problems by applying strategies in new contexts or by extending strategies to related problems within a context.

Reasoning Goal: Students will be able to reason, model, and make decisions with mathematical, statistical, and quantitative information.

In the *Foundations* course, students will...

- Make decisions in quantitatively based situations that offer a small number of defined options. The situations will not be limited to contexts in which there is a single correct answer based on the mathematics (e.g., which buying plan costs less over time), but will include situations in which the quantitative information must be considered along with other factors.
- Present short written or verbal justifications of decisions that include appropriate discussion of the mathematics involved. **Evaluation Goal: Students will be able to**

critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information.

In the *Foundations* course, students will...

- Identify mathematical or statistical errors, inconsistencies, or missing information in arguments.

Technology Goal: Students will be able to use appropriate technology in a given context.

In the *Foundations* course, students will...

- Use a spreadsheet to organize quantitative information and make repeated calculations using simple formulas.
- Use the internet to find quantitative information on a given subject. The topics should be limited to those that can be researched with a relatively simple search.
- Use internet-based tools appropriate for a given context (e.g., an online tool to calculate credit card interest).

Content learning outcomes

The content learning outcomes include both mathematical and contextual topics in keeping with the role of this course as a quantitative literacy course. The topics for the *Foundations of Mathematical Reasoning* course are:

- Numeracy
- Proportional Reasoning
- Algebraic Competence, Reasoning, and Modeling
- Probabilistic Reasoning to Assess Risk
- Quantitative Reasoning in Personal Finance
- Quantitative Reasoning in Civic Life

Numeracy

Outcome: Students will develop number sense and the ability to apply concepts of numeracy to investigate and describe quantitative relationships and solve real-world problems in a variety of contexts.

Students will be able to:

N.1 Demonstrate operation sense and communicate verbally and symbolically with real numbers.

For example: Know when and how to perform arithmetic operations with the use of technology. Use order of operations to identify an error in a spreadsheet formula. Predict the effects of multiplying a number by a number between 0 and 1.

N.2 Demonstrate an understanding of fractions, decimals, and percentages by representing quantities in equivalent forms, comparing the size of numbers in different forms and interpreting the meaning of numbers in different forms.

For example: Write a fraction in equivalent decimal form and vice versa. Compare growth expressed as a fraction versus as a percentage. Interpret the meaning of a fraction. Interpret the meaning of percentages greater than 100% and state if such a percentage is possible in a given context.

N.3 Solve problems involving calculations with percentages and interpret the results.

For example: Calculate a percentage rate. Explain the difference between a discount of 30% and two consecutive discounts of 15%. Calculate relative change and explain how it differs from absolute change.

N.4 Demonstrate an understanding of large and small numbers by interpreting and communicating with different forms (including words, fractions, decimals, standard notation, and scientific notation) and compare magnitudes.

For example: Compare large numbers in context, such as the population of the US compared to the population of the world. Calculate ratios with large numbers such as water use per capita for a large population. Interpret a growth rate less than 1%.

N.5 Use estimation skills, and know why, how, and when to estimate results.

For example: Identify and use numeric benchmarks for estimating calculations (e.g., using 25% as an estimate for 23%). Identify and use contextual benchmarks for comparison to other numbers (e.g., using US population as a benchmark to evaluate reasonableness of statistical claims or giving context to numbers). Check for reasonableness using both types of benchmarks.

N.6 Solve problems involving measurement including the correct use of units.

For example: Identify the appropriate units for perimeter, area, and volume. Calculate the amount of paint needed to paint a non-rectangular surface. Interpret measurements expressed in graphical form.

N.7 Use dimensional analysis to convert between units of measurements and to solve problems involving multiple units of measurement.

For example: Convert between currencies. Calculate the cost of gasoline to drive a given car a given distance. Calculate dosages of medicine.

N.8 Read, interpret, and make decisions about data summarized numerically (e.g., measures of central tendency and spread), in tables, and in graphical displays (e.g., line graphs, bar graphs, scatterplots, and histograms).

For example: Critique a graphical display by recognizing that the choice of scale can distort information. Explain the difference between bar graphs and histograms. Explain why the mean may not represent a typical salary.

Proportional Reasoning

Outcome: Students will use proportional reasoning to solve problems that require ratios, rates, proportions, and scaling.

Students will be able to:

PR.1 Represent, and use ratios in a variety of forms (including percentages) and contexts.

For example: Interpret a rate of change within a context using appropriate units. Interpret a percentage as a number out of 1,000. Compare risks expressed in ratios with unequal denominators (e.g., 1 in 8 people will have side effects versus 2 in 14).

PR.2 Determine whether a proportional relationship exists based on how one value influences another.

For example: Simple versus compound interest. Analyze whether an estimated percentage is reasonable based on proportions. Analyze the effects of scaling and shrinking that are proportional and non-proportional (e.g., the impact of changing various dimensions on perimeter, area, and volume).

PR.3 Analyze, represent, and solve real-world problems involving proportional relationships, with attention to appropriate use of units.

For example: Use individual water-use rates to predict the water used by a population. Use the Consumer Price Index to compare prices over time. Use a scale to calculate measurements in a graphic.

Algebraic Competence, Reasoning, and Modeling

Outcome: Students will transition from specific and numeric reasoning to general and abstract reasoning using the language and structure of algebra to investigate, represent, and solve problems.

NOTE: The goal is to capture the broad concept of a function but not in a formal mathematical manner. The focus is on solving problems and modeling, not on defining or exploring the concept of function in detail (e.g., no domain or range concepts). Students will represent relationships between quantities using multiple strategies to solve problems.

Students will be able to:

A.1 Demonstrate understanding of the meaning and uses of variables as unknowns, in equations, in simplifying expressions, and as quantities that vary, and use that understanding to represent quantitative situations symbolically.

For example: Understand the different uses of variables and the difference between a variable and a constant. Be able to use variables in context and use variables as placeholders, as in formulas. Write an algebraic expression to represent a quantity in a problem. Combine simple expressions. Use notation with variables (e.g., exponents, subscripts) in simple and moderately complex expressions.

A.2 Describe, identify, compare, and contrast the effect of multiplicative or additive change.

For example: Compare and contrast the rate of change and/or behavior of a linear and an exponential relationship in context. Recognize that a multiplicative change is different from an additive change. Explain how the rate of change of a linear relationship differs from an exponential rate of change, as well as the ramifications of exponential change (growth can be very slow for a time but then increase rapidly).

A.3 Analyze real-world problem situations, and use variables to construct and solve equations involving one or more unknown or variable quantities.

For example: Demonstrate understanding of the meaning of a solution. Write a spreadsheet formula to calculate prices based on percentage mark-up. Solve a formula for a given value. Identify when there is insufficient information given to solve a problem.

A.4 Express and interpret relationships using inequality symbols.

For example: Use inequalities to express the relationship between the probabilities of two events or the size of two groups. Interpret a histogram based on intervals expressed with inequality symbols.

A.5 Construct and use mathematical models to solve problems from a variety of contexts and to make predictions/decisions.

Representations will include linear and exponential contexts.

For example: Given a statement of how the balance in a savings account grows because of monthly interest, construct a table of months and balances and then write a mathematical model that provides the balance for a given month.

A.6 Represent mathematical models in verbal, algebraic, graphical, and tabular form.

For example: Be able to move from any one representation to another. Given an initial value and information about change, create a table, graph, and/or algebraic model. Given an algebraic model, create a table of values.

A.7 Recognize when a linear model is appropriate and, if appropriate, use a linear model to represent the relationship between two quantitative variables.

For example: Given a set of data, make an informal, intuitive evaluation of the applicability of a linear or exponential model with a focus on recognizing the limitations of the model and identifying an appropriate domain (do not need to use this term with students) for which the model might be used to make accurate predictions. Describe the rate of change using appropriate units: slope for linear relationships, or average rate of change over an interval for nonlinear relationships.

Probabilistic Reasoning to Assess Risk

Outcome: Students will understand and critically evaluate statements that appear in the popular media (especially in presenting medical information) involving risk and arguments based on probability.

Students will be able to:

R.1 Interpret statements about chance, risk, and probability that appear in everyday media (including terms like unlikely, rare, impossible).

For example: Interpret statements such as "for a certain population the risk of a particular disease is 0.005". Compare incidences of side effects in unequal group sizes.

R.2 Identify common pitfalls in reasoning about risk and probability.

For example: Identify inappropriate risk statements, such as when the size of reference groups is unknown (e.g., California, 2009, 88% of motorcycle accident fatalities were helmeted, 12% unhelmeted).

R.3 Interpret in context marginal, joint, and conditional relative frequencies in context for data summarized in a two-way table and identify which relative frequency is appropriate to answer a contextual question.

For example: Distinguish between reported relative frequencies that are marginal, joint, or conditional. Choose the relative frequency that is the most informative for a given purpose. Choose the appropriate direction of conditioning for a given context (the chance of cancer given a positive test result is not the same as the chance of a positive test result given cancer).

R.4 Demonstrate understanding of absolute risk and relative risk (percentage change in risk) by describing how each provides different information about risk.

For example: Interpret the different information conveyed when comparing the magnitude of the absolute risks and percentage change in risk (e.g., an 80% increase in risk associated with taking a particular medication could mean a change in risk from 0.001 to 0.0018 or from 0.1 to 0.18).

Quantitative Reasoning in Personal Finance

Outcome: Students will understand, interpret, and make decisions based on financial information commonly presented to consumers.

Students will be able to:

PF.1 Demonstrate understanding of common types of consumer debt and explain how different factors affect the amount that the consumer pays.

For example: Calculate the interest paid on credit card debt based on a credit score; explain how the length of the pay-off period affects the total interest paid; demonstrate the relationship between a percentage rate and the amount of interest paid; define basic terminology such as principal, interest rate, balance, minimum payment, etc.

PF.2 Demonstrate understanding of compound interest and how it relates to saving money.

For example: Demonstrate the different impacts of the saving period and the amount saved on the accumulated balance; use a given formula to calculate a balance; demonstrate an understanding of the meaning of a compounding period and use the appropriate terminology for different periods (e.g., quarterly, annually, etc.).

PF.3 Identify erroneous or misleading information in advertising or consumer information.

For example: Explain why statements about “average” benefits of a product such as a weight loss plan are misleading; identify misleading graphs that create an appearance of greater impact than is warranted.

Quantitative Reasoning in Civic Life

Outcome: Students will understand that quantitative information presented in the media and by other entities can sometimes be useful and sometimes be misleading.

Students will be able to:

CL.1 Use quantitative information to explore the impact of policies or behaviors on a population. This might include issues with social, economic, or environmental impacts.

For example: Calculate the effects of a small decrease in individual water use on the amount of water needed by a large population over time; determine if the minimum wage has kept pace with inflation over time.

CL.2 Identify erroneous, misleading, or conflicting information presented by individuals or groups regarding social, economic, or environmental issues.

For example: Explain how two statements can be both contradictory and true (e.g., the "average" amount of a tax cut expressed in terms of the mean and the median); identify how two pie charts representing different populations can be misleading.