Massachusetts Department of Higher Education

Designing Math Pathways: Supporting Students and Increasing Opportunities for Success

Final Report from the Task Force on Transforming Developmental Math Education – Mathematics Pathways Subcommittee

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The group would like to thank the Charles A. Dana Center at the University of Texas at Austin for their partnership. The Math Pathways Subcommittee would also like to thank members of the Task Force on Transforming Developmental Math Education for their work in building a solid foundation for future developmental math reform.

Background and Context

The Department of Higher Education (DHE), in partnership with public higher education institutions in Massachusetts, is focusing its efforts on three strategies to increase the numbers of students graduating with degrees and certificates.[1] The "Big Three" Completion Plan goals include: Boost College Completion Rates, Close Achievement Gaps, and Attract and Graduate More Students from Underserved Populations.

The Board of Higher Education (BHE), DHE, and our public institutions of higher education are working together to implement the "Big Three" Completion Plan. One of the most pressing issues that affects both college retention and graduation rates is the significant number of students who are required to enroll in developmental education courses despite the fact that they have successfully met all high school graduation requirements.

In March 2012, the Task Force on Transforming Developmental Math Education was formed and focused on four major areas: Research and Education, Developmental Math Assessment and Placement, Developmental Math Structure, and Developmental Math Content. The Task Force met several times and engaged with national organizations in order to develop recommendations that would "systematically improve the percentage of students that complete developmental math education and pass the first college-level math course required for their program of study" (2013, page 6).

According to the final report issued by the Task Force on Transforming Developmental Math Education, approximately 38 percent of students enrolled at Massachusetts' public institutions of higher education are required to enroll in developmental education courses. Approximately two-thirds of community college students are enrolled in these courses, in addition to approximately 20 percent and 10 percent at our state universities and UMass undergraduate campuses, respectively. African American, Latinx, and Pell-eligible students are required to enroll in these courses at significantly higher rates than their peers.

In 2013, the Task Force released a report that contained four comprehensive recommendations:

1. The BHE shall set a period of experimentation for campuses to experiment with the use of GPA as an alternative placement measure;
2. The BHE shall encourage institutions to revise the content, sequencing, and timeframe of their developmental math offerings;

[1] Additional information about the "Big Three" Completion Plan is available at http://www.mass.edu/visionproject/bigthree.asp.

3. The BHE shall urge campuses to design general "academic pathways" for all students, including math sequences consistent with the academic requirements of each pathway or "meta-major," such as social sciences, liberal arts, and STEM (science, technology, engineering and math); and
4. The DHE will provide ongoing support for the implementation of these recommendations.

Once the report and recommendations were adopted, the BHE set a period of experimentation and campuses were allowed to pilot new placement criteria, specifically high school GPA or limited variation. Prior to this period, students were required to take Accuplacer and score 40 on the College-Level Math test or 82 on the Accuplacer Elementary Algebra test in order to place in college-level mathematics. Initially, the DHE focused its attention on the first recommendation and the majority of institutions implemented pilots that use GPA and other criteria to assess student readiness for college-level mathematics.

In 2015, the Department began carefully examining the adoption of the co-requisite model for developmental education and multiple mathematics pathways. With the co-requisite course model, "students enroll directly into college-level courses and receive academic support alongside their classes. Rather than facing a long sequence of prerequisite, non-credit courses, students get up to speed while working toward their degree" (Complete College America, 2016, p. 2). These courses allow students to complete college-level gateway math and/or English courses within one academic year. The DHE organized two statewide conferences in 2017 in order to convene campus practitioners and national content experts to bring these promising practices to scale.

Several states, including Georgia and West Virginia, have seen significant positive results after the adoption of the co-requisite model. In Tennessee, "success in college-level math improved from 12 percent to 61 percent... [and] the state's data suggest that this approach works for virtually all students" (Complete College America, p. 3). Massachusetts is one of twelve states that has partnered with Complete College America to bring the co-requisite model to scale statewide.

Massachusetts has adopted a three-pronged approach to reduce remediation and increase student success:

1. Assess students properly for credit-bearing courses using multiple measures;
2. Ensure that students are taking and completing the appropriate math for their major;

3. Give students who require remediation access to co-requisite courses in math, reading, and writing.

The Charge

The DHE wanted to pay special attention to two areas: co-requisite education and multiple math pathways. As a result, two separate groups were formed and each was tasked with further exploring its respective area.

Commissioner Carlos E. Santiago charged the Math Pathways Subcommittee of the Task Force on Transforming Developmental Math Education with exploring the benefits and implications of multiple math pathways and with developing recommendations for increasing success in mathematics that will:

1. Ensure students take the appropriate math for their major;
2. Increase alignment between college-level and developmental mathematics across the three segments of public higher education to promote on-time completion; and
3. Increase transferability of math courses and applicability across the three segments of public higher education.

The Math Pathways Subcommittee of the Task Force on Transforming Developmental Math Education built upon the work that began during the MassTransfer Pathways process, when mathematics faculty from all 28 undergraduate-serving institutions came together to build a statewide transfer pathway in Mathematics.

The DHE partnered with the Charles A. Dana Center at the University of Texas at Austin and joined other states that were committed to bringing multiple math pathways to scale under the Dana Center's Mathematics Pathways to Completion project. By partnering with the Dana Center, Massachusetts has also adopted the four principles[2] of the Dana Center Mathematics Pathways Model (DCMP). The first two principles focus on the pathways themselves and posit that math pathways should be structured so that:

- Principle 1: All students, regardless of college readiness, enter directly into mathematics pathways aligned to their programs of study.

[2] The DCMP's Four Guiding Principles: Selected Supporting Research: https://dcmathpathways.org/resources/dcmps-four-guiding-principles-selected-supporting-research-annotated-bibliography

- Principle 2: Students complete their first college-level mathematics requirement in their first year of college.

The DCMP Model also ensures that "students engage in a high-quality learning experience" and the following principles influence the design of mathematics pathways (University of Texas at Austin, Dana Center):

- Principle 3: Strategies to support students as learners are integrated into courses and are aligned across the institution.
- Principle 4: Instruction incorporates evidence-based curriculum and pedagogy.

The work of the Math Pathways Subcommittee of the Task Force on Transforming Developmental Math Education embodied these principles, which underpin the proposed recommendations.

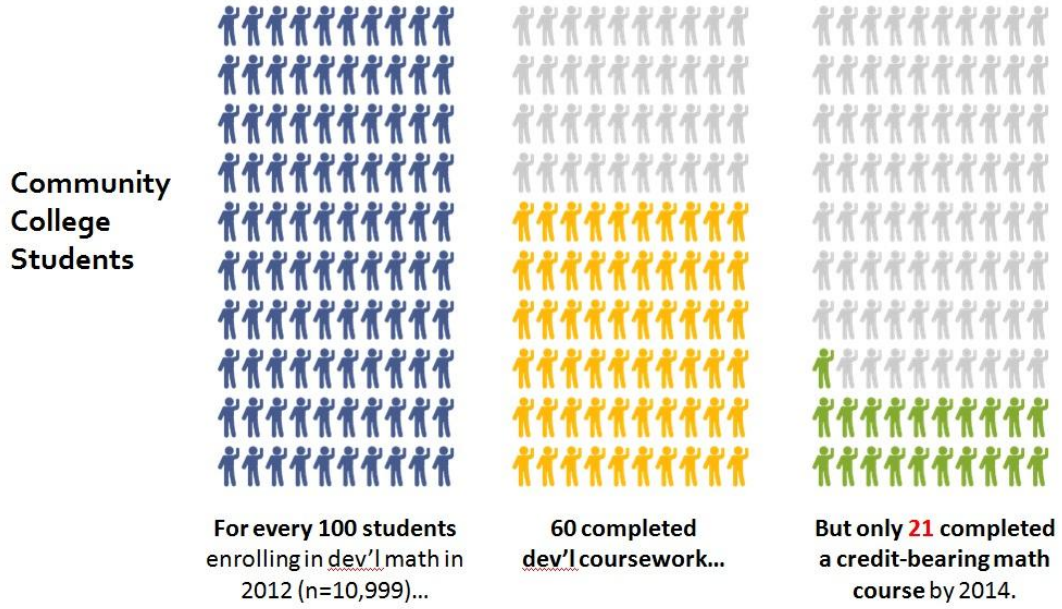
Obstacles to Implementing Mathematics Pathways—and Opportunities for Improvement

Enrollment in at least one stand-alone developmental math course is required for a majority of Massachusetts community college students and one-fifth of state university students. Community college students who are enrolled into developmental courses have only a little over 20 percent chance of continuing on to complete a credit-bearing math course. In Massachusetts, 20.6 percent of full-time first-time students who entered community college in Fall 2004 and did not take developmental coursework finished in three years compared to 10.3 percent of those who did take developmental coursework. For the four-year institutions, the figures are 59.1 percent for the University of Massachusetts and 51.2 percent for the state universities. The figures below show that progress has certainly been made, but more work is needed in order to dramatically improve success rates.

Despite several advances around building a unified system of transfer, the lack of statewide agreement on the appropriate math for each major or a general consensus on what constitutes a college-level math course complicates the transfer of credits and courses from one institution to another. Courses do not always transfer as equivalent and students are sometimes required to retake both developmental and credit-bearing math courses. Faculty and transfer professionals have worked hard to develop statewide transfer pathways, known as A2B (Associate to Baccalaureate) Mapped Pathways. These

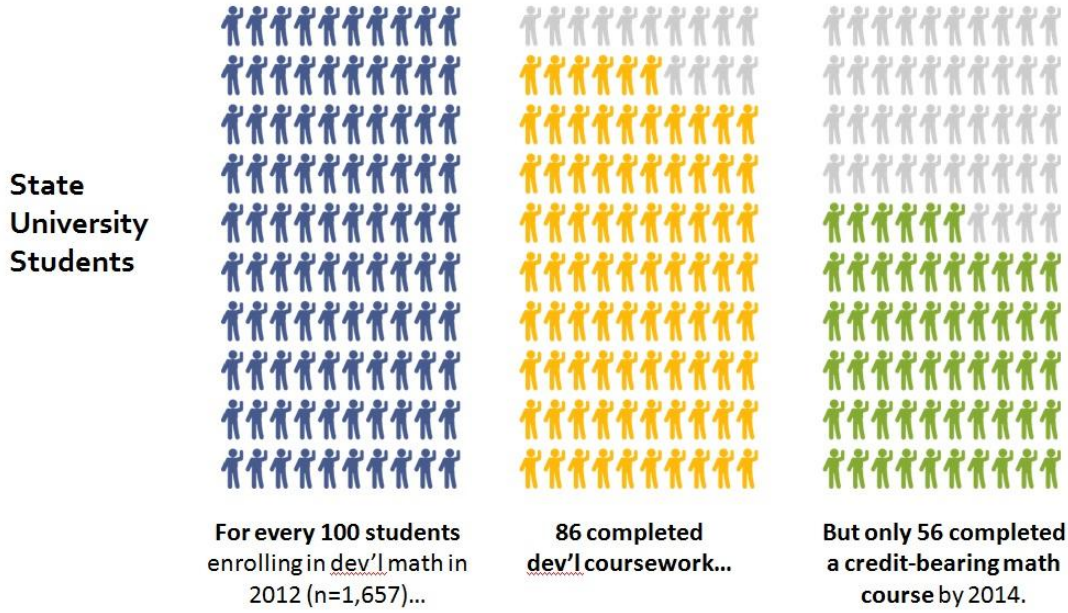
pathways can be strengthened by identifying the appropriate math course and sequence for each discipline.

Figure 1. Community College Students Progress to Credit-Bearing Coursework (Fall 2012 Cohort)



Source: Massachusetts Department of Higher Education

Figure 2. State University Students Progress to Credit-Bearing Coursework (Fall 2012 Cohort)



Source: Massachusetts Department of Higher Education

Other barriers also stand in the way of improving math performance in the state's colleges and universities. The dominance of algebra-based math courses, particularly when required or recommended for students not pursuing calculus- or statistics-based majors, poses an unnecessary obstacle for many students.

Additionally, DHE efforts now include scaling the co-requisite model for both English and mathematics. Working collaboratively with representatives from the Commonwealth's fifteen community colleges, nine state universities, and four undergraduate-serving UMass campuses, the DHE will develop recommendations around scaling the co-requisite model to improve student success and release a report in Spring 2018.

Significant progress has also been made in the area of using multiple placement measures to determine college readiness. A recent analysis[3] conducted by DHE staff of the GPA pilots from academic year 2014-2015 found that campuses that implemented the GPA pilots saw an increase in the percent of students enrolling and completing a gateway-level math course. At the same time, the percent of students enrolling in developmental math decreased. This is true for both the community colleges and state universities. Initially, institutions could choose either one or both of the pilot standards described below:

- Pilot A: The use of a 2.7 GPA or above. In many cases, campuses chose to use additional measures, including high school math GPA and SAT scores.
- Pilot B: The use of a GPA between 2.4 and 2.69. Like the Pilot A standard, campuses had the option to use additional measure, including high school math GPA and SAT scores.

DHE's preliminary analysis found that at the community colleges, 77.3 percent of students placed into college-level math using Accuplacer completed the course compared to 75.9 percent of students placed using Pilot A standard. Interestingly, 67.4 percent of students assessed as not college ready also completed a college-level math course. Simultaneously, at the state universities 84.3 percent of students placed into college-level math using Accuplacer completed the course compared to 91.0 percent of students placed using Pilot A standard. At the University of Massachusetts, 83.4 percent of students placed using Accuplacer completed a college-level math course, while 81.1

[3] The preliminary analysis can be found by reviewing AAC 16-19: Extending Campus Work in the Area of Developmental Mathematics: http://www.mass.edu/bhe/lib/documents/AAC/10_AAC%2016-19%20Developmental%20Math%20Pilots%20and%20Qualitative%20Study.pdf.

percent of students placed under Pilot A standard completed a college-level math course.

This preliminary analysis indicates that students placed using Pilot Standard A had a similar likelihood of success compared to those students using Accuplacer. At the same time, if the GPA standards were adopted at scale, more students would enter directly into credit-bearing mathematics courses, allowing more students to continue their postsecondary studies and institutions would see an increase in retention and completion rates.

Recommendations and Strategy for Implementation

The Math Pathways Subcommittee of the Task Force on Transforming Developmental Math Education came together over the course of Spring 2017 to develop recommendations and strategies for improving math pathways across the state. The five recommendations and strategies for implementing them are described below. In the coming months, the DHE will begin convening the Math Pathways Working Group with representatives from each institution to promote implementation and advance these recommendations.

Recommendation I: In an effort to ensure students complete the right mathematics course for their major, Massachusetts institutions of public higher education should develop at least four math pathways: Calculus, Elementary Education, Quantitative Reasoning, and Statistics.

Traditionally, students have been funneled into a one-size fits all developmental algebra course sequence designed to prepare them for Calculus, despite the fact that most students' majors do not require Calculus (Burdman, 2015, Chen & Soldner, 2013). These four pathways and their respective gateway courses can fit a variety of programs, from the behavioral and social sciences to humanities and fine arts to sciences and technology. In some cases, campuses might be able to offer more than four pathways. Institutions that are able to offer more than four pathways in order to further contextualize mathematics courses to meet the needs of students and ensure they are sufficiently prepared for success in their majors are encouraged to do so.

College Algebra is not an appropriate gateway course for Quantitative Reasoning and Statistics. In 2014, the American Mathematical Association of Two-year Colleges released its position on the appropriate use of intermediate algebra and stated that "the equivalent content in intermediate algebra courses is not required to master the content for most college-level mathematics courses that do not lead to calculus" and that "prerequisite courses other than

intermediate algebra can adequately prepare students for courses of study that do not lead to calculus." The Math Pathways Subcommittee adopts this position and believes that only students entering into the Calculus and Elementary Education pathways need to complete College Algebra.

Students in Calculus and Elementary Education should receive the same mathematical preparation before entering into their respective gateway mathematics courses. Massachusetts is known for formalizing course-taking requirements for students interested in becoming elementary school teachers. "The state guidelines specify that [elementary education] students should take at least 9 hours of coursework that spend proportional amounts of time on the following core topics: number and operations (45 percent), functions and algebra (25 percent), geometry and measurement (20 percent), and statistics and probability (10 percent) (Massachusetts Department of Education, 2007, p. 4) (Charles A. Dana Center, 2016, p. 4).

The appropriate mathematics for a specific program of study, no matter which institution a student attends, should be clearer to students and academic advisors as a result of this work. Furthermore, the Math Pathways Subcommittee believes these pathways are important for ease of transfer between two- and four-year institutions and also among four-year institutions in Massachusetts.

Next Steps

The Math Pathways Subcommittee of the Task Force on Transforming Developmental Math Education has developed suggested content lists for Calculus I, Calculus II, Calculus III, Introduction to Statistics, as well as a course description for Quantitative Reasoning courses taught within the mathematics department. These will be vetted by each institution and feedback submitted to the Department for review by the Math Pathways Working Group. The purpose of developing the content lists and sample course description is not to force institutions to adopt these, but rather to provide guidance for the development of statewide equivalencies in these areas. The primary goal is to ensure that students have similar experiences in these courses so that, if they transfer, they are prepared to do well in upper-level courses.

In order to build the math pathway for students interested in Elementary Education, members of the newly formed Math Pathways Working Group will work with elementary education faculty, as well as the Department of Elementary and Secondary Education (DESE). This math pathway should be consistent with the guidelines for the mathematical preparation of elementary teachers as developed by DESE. It will be developed using the MassTransfer Pathways process.

This group should also work on developing a preliminary common definition of "College Algebra" that would then be vetted by the Math Pathways Working Group and eventually public institutions.

The Math Pathways Working Group should also work with a broad group of faculty from the community colleges, state university, and University of Massachusetts and institutional administrators to develop suggested content for courses that satisfy a program's quantitative reasoning requirement but are taught outside the mathematics department (e.g. Drug Calculations or Logic).

The DHE and the Dana Center will host a two-day workshop at the end of Fall 2017 to support college and university teams to begin and/or deepen work to develop and implement math pathways aligned with the recommendations of the Massachusetts Math Pathways Subcommittee.

Recommendation II: MassTransfer Associate-to-Bachelor (A2B) mapped pathway disciplines should identify the appropriate default or recommended mathematics for their major.

One of the major barriers to transfer continues to be lack of agreement on foundational courses for particular disciplines. Through the MassTransfer Pathways project, faculty have been brought together to identify the courses a student needs to complete during the first two years of their major. To enhance the transfer process, disciplines with statewide transfer pathways should also agree to adopt a single math pathway, so that the transition from a two- to four-year institution becomes as seamless as possible.

<u>Next Steps</u>
The DHE will use the MassTransfer Pathways process to convene segmental leaders from disciplines with A2B mapped pathways to ensure alignment across segments and disciplines. Segmental leaders will be asked to work with other faculty representatives and come to consensus on the math pathway that works best for their major. As with foundational courses in A2B mapped disciplines, courses should be considered equivalent across institutions of public higher education if agreed upon topics are covered. This will help further efforts to improve the process of transfer for students.

Several majors require students to satisfy a Quantitative Reasoning requirement. Oftentimes, courses that satisfy this requirement are taught outside of the mathematics department. The new Math Pathways Working Group will create a subcommittee to help identify potential shared

learning outcomes for Quantitative Reasoning courses taught outside of the mathematics department. This work will occur in partnership with statewide groups that focus on advancing a culture of assessment in Massachusetts.

Finally, transcripts will be revised to indicate whether a student who requires remediation has completed developmental coursework. This will ensure that students do not have to repeat completed developmental math courses because receiving institutions will understand that a student is prepared for college-level coursework.

Recommendation III: Students who require remediation should have the opportunity to complete their college-level mathematics course within one year of enrollment, preferably within a co-requisite model.

As previously mentioned, one of many obstacles to the timely completion of an academic degree program is lack of preparedness of students, particularly in the area of mathematics education. In Massachusetts, 60 percent of students entering our community colleges require developmental (remedial) work; state universities are in the 22–23 percent range; and University of Massachusetts campuses are approximately 10 percent (Task Force on Transforming Developmental Math Education, 2013). Differences are also apparent by race/ethnicity: African-American and Latinx students in Massachusetts require developmental education coursework at a considerably higher rate than white students (20 percentage point differential), in large part because of the relatively underfunded and underperforming school districts which many students of color attend.

Massachusetts and its institutions of public higher education are dedicated to transforming developmental education and redesigning mathematics and English pathways so that students can enter into credit-bearing courses faster with the ultimate goal of increasing the number of students participating and succeeding in college. Co-requisite courses allow students to complete college-level gateway math and/or English courses within one academic year. In some cases, programs may require students to take a gateway, college-level mathematics course during the second year rather than the first. The four-year course plans at some Baccalaureate granting institutions recommend that students take their gateway course during their second year of study. While it is important to ensure flexibility given varying curriculum and program requirements, the main objective remains: students should not have to complete multiple developmental courses before enrolling in a college-level mathematics course. Once again, the co-requisite model will allow students to remediate while completing a credit-bearing course and students can complete these courses when it makes the most sense based on their program's curriculum.

Next Steps

Two- and four-year institutions that offer developmental mathematics courses should develop sequences that allow students to complete college-level gateway mathematics courses within one academic year. Through a partnership with Complete College America, Massachusetts can provide technical assistance resources to institutions that are moving toward scaling the co-requisite model. A subcommittee of the Math Pathways Working Group will be charged with advancing and monitoring this work.

The Co-requisite at Scale Initiative builds on existing efforts related to alternative math pathways, the Big Three Completion Plan, campus-based co-requisite programs, college readiness, GPA placement pilots, MassTransfer Pathways, student learning outcomes assessment, and the Task Force on Transforming Developmental Math Education. The DHE will continue to move this initiative forward and work with campuses to find ways to systematically improve the percentage of students that complete a gateway (college-level) mathematics course during their first year. Moving forward, the Co-requisite at Scale Initiative and Designing Math Pathways Initiative will be more carefully integrated to ensure alignment of both projects.

Recommendation IV: The DHE should develop a course completion indicator ("flag") for all courses that satisfy Quantitative Reasoning requirements. This will improve the DHE's data collection and the ability to track student progress.

Current DHE collections do not capture data for students who have finished a developmental mathematics sequence and have gone on to satisfy their institutional quantitative reasoning requirements by completing a course outside of the mathematics department. In order to conduct more thorough and stronger analyses on students who progress through a developmental sequence and go on to be successful, a more effective and efficient way to capture students who meet institutional requirements is needed.

Next Steps

DHE staff, in collaboration with campus institutional research directors, will develop a course flag in the Higher Education Information Resource System (HEIRS) that allows institutions to show that a student has satisfied a quantitative reasoning requirement, even if the Quantitative Reasoning course is taught outside of the mathematics department.

Recommendation V: The Department of Higher Education should work with the Department of Elementary and Secondary Education to ensure alignment between K-12 and postsecondary course expectations and requirements.

While the alignment of mathematics courses and pathways within the system of public higher education is very important, so too is access of high school students to effective on-ramps to college-level courses after graduation. By ensuring that math pathways are aligned between high schools and postsecondary institutions, students will be better prepared to succeed in college mathematics.

Several policies have been promoted in the Commonwealth in order to ensure high school students succeed when they transition to postsecondary. One recent effort includes requiring four years of high school math as an admissions requirement for state universities and University of Massachusetts campuses, a policy adopted by the Board of Higher Education in 2011 which went into effect for the entering Fall 2016 freshmen class. Requiring four years of high school math, including during the final year of high school, was implemented to align with the Massachusetts High School Program of Students (MassCore), which was jointly developed by DESE and DHE to better prepare student for college and career. The policy was adopted by the Board of Education in November 2007 in an effort to reduce the number of students requiring remediation.

Next Steps

Postsecondary institutions should identify their top feeder schools and, with those institutions, form regional working groups to: 1) look at the outcomes and requirements for the classes that students take as their last math class in high school and 2) compare these to the first college-level math class at receiving institutions. Faculty from institutions of public higher education are encouraged to share syllabi for gateway level mathematics courses and work with local high school mathematics teachers to ensure that key concepts required for success in college-level coursework are covered in high school.

A subcommittee of the Math Pathways Working Group will be charged with moving this aspect of the math pathways work forward. This subcommittee will include representatives from DESE, as well as high school mathematics teachers. DHE, in collaboration with the Dana Center and DESE, will host a K-12 Math Pathways Summit in 2018 for K-12 and higher education administrators, high school teachers and postsecondary faculty, and system-level staff. The goal of the Summit will be the development of a statewide implementation plan for advancing this work.

If these five recommendations are implemented and the next steps are successful, the Commonwealth should expect to see promising results. In order to maximize potential impact, it is recommended that all three approaches to remediation reduction (using multiple measures

for placement, developing multiple math pathways, and implementing the co-requisite model) be adopted to see dramatic increases in student success. The DHE, in partnership with institutions of public higher education in the Commonwealth, will routinely collect data and revise policies as needed based on research and new learnings both within and outside of Massachusetts.

Campus Narratives

Many institutions of public higher education in the Commonwealth have already adopted several of the approaches the DHE is promoting. Below are select examples of institutions that have implemented multiple mathematics pathways and are using innovative practices to better support students.

Fitchburg State University

Development of Pathways for STEM and Non-STEM Students and Implementation Co-requisite Courses in Introduction to Functions and Applied Statistics

Fitchburg State University offers Basic Math I and Basic Math II as its developmental mathematics courses. In Fall 2014, Basic Math II was offered with two tracks for the first time. Students work through a series of modules on the computer at their own pace and must earn a Mastery Grade (80%) on all tests during the semester and a 70% on the final exam to pass the course. Students in the non-STEM track solve and graph linear equations; use radical expressions and equations, basic statistics including tables, graphs and technology; and work with compound inequalities and the midpoint and distance formulas. The STEM track focuses less on statistics and has a greater focus on exponents, polynomials, factoring, solving and working with rational expressions and equations.

Students who take the non-STEM track are qualified to take Math 1700 Applied Statistics, Math 1800 Business Statistics, Math 1200 Finite Math, or Math 2000 Informal Geometry. Students who complete the non-STEM track and decide later to take a mathematics course not listed above, must complete the remaining modules for the STEM track and pass the STEM Basic Math II final exam before taking the college-level course.

In addition to the two Basic Math II tracks, Fitchburg State University has instituted 50 minute co-requisite sessions for Applied Statistics (first offered in Fall 2015) and Math 1250 Introduction to Functions (first offered in Fall 2016), and plans to expand them to Math 1800 in Fall 2017. The co-requisite sessions meet one time per week and cover prerequisite material just before it is

discussed in the course. The modules can also serve as review and extra help for the course. The first day of the co-requisite section is devoted to a discussion of study skills, a topic that is revisited throughout the semester. All students in Applied Statistics are required to attend the sessions, regardless of their mathematics background. The current policy is that if a student misses more than three sessions, they must make them up in order to pass the course. In Fall 2017, the policy changed so that students are required to make up all missed co-requisite sessions.

Framingham State University

Development of Multiple Math Pathways and Implementation of the Co-requisite Model

In Fall 2017, the mathematics department at Framingham State University implemented a co-remediation model to supplement the 100-level mathematics courses. Any Framingham State University student who does not meet the minimum standards to enroll in a credit-bearing mathematics courses will be placed in a 100-level math course, appropriate to his/her major, with an additional 2-hour lab session each week. The intent of the 2-hour lab session is to provide "just-in-time" remediation to help students successfully complete their 100-level mathematics courses. The 2-hour lab sessions will be taught using an emporium approach. The sessions will be held in a computer lab, a.k.a. "Math Emporium," where students will work at their own pace to complete computer-based modules designed to fill gaps in their mathematical knowledge. The emporium will be staffed by a full-time instructor plus tutors to help students as they work through each lesson. Students seeking additional help beyond the regularly-scheduled 2-hour lab will be free to stop by the emporium at other times during the week.

The decision to implement a co-requisite model at Framingham State comes from different sources. Up until this past fall, Framingham had been offering a non-credit math course, MATH 095: General Math, for students who had not met the standards for a college-level math course. While the intent of the General Math course was to prepare students to be successful in a 100-level mathematics course, the evidence has not proven it to be successful. For example, of the 346 First-Time, Full-Time students who enrolled in MATH 095 in Fall 2014 and Fall 2015, only 240, (69 percent) returned for their second year. Of the 176 First-Time, Full-Time students who enrolled in Math 095 in Fall of 2013, only 115 (65 percent) had completed a credit bearing math course within two years of completing Math 095 (December 2015). In addition, data presented at the 2016 Transforming Developmental Math Education Conference indicated the lack of success of remedial math courses in preparing students to pass a college-level math course. After attending the conference, the team at Framingham State felt confident in its decision to try something new and implement the co-requisite model.

In anticipation of implementing the co-requisite remediation model, the mathematics department offered two 100-level courses during the 2016-2017 academic year using a ``just-in-time" remediation approach. In the fall of 2016, one section of College Algebra was used, and in the spring of 2017, one section of Introduction to Statistics was added. In each of those sections, students were required to meet for an additional two hours of class time each week, allowing them to review topics considered prerequisite material. While this pilot only involved two sections, the results were encouraging as the drop/fail/withdrawal rates for those sections were lower than average for 100-level math courses. Framingham State University is hopeful that the co-requisite model will help students be more successful in their introductory, college-level mathematics courses.

Middlesex Community College

Multiple Math Pathways and Developmental Mathematics Redesign

Middlesex Community College has used computer-based modules to deliver the developmental math sequence for several years. Modules 1-4 cover the fundamentals of mathematics, and modules 4-8 cover algebra I. Previously all students were then required to complete modules 9-12 (algebra II). Upon completion of module 12 a student could go into Math for Liberal Arts or Statistics. If, however, they needed to take pre-calculus for their major, they were required to take one more developmental math class (Intermediate Algebra).

In the Fall 2015 Middlesex Community College created two math pathways in the developmental math sequence. Modules 1-8 stayed as they were, however now once a student completes module 8 they select one of two paths based on their major. One path (modules 80-85) leads to the calculus sequence and is algebra focused. The other path (modules 70-73) leads to Statistics or Math for Liberal Arts (quantitative reasoning) and is number sense and skills/computation focused.

Modules 80-85 now incorporate Algebra II and Intermediate Algebra, thus eliminating another developmental class a student would need to take prior to entering pre-calculus. Middlesex is also piloting a co-requisite model with both Statistics and Math modeling which gives students the opportunity to complete the Math Literacy Path (modules 70-73) while taking Statistics or Math Modeling. The class meets 4 hours/week and saves the students 2 credits (that would not count toward their degree) and time. This pilot began Fall 2017.

Quinsigamond Community College

Development of Multiple Math Pathways and Use of Multiple Placement Measures

Quinsigamond Community College's Math Department's Academic Planning and Review (APR) process, which took place in Spring 2017, has resulted in the mathematics department proposing many changes that are grounded in research and data. Three pathways towards the completion of a 2-year degree have been proposed: STEM, non-STEM, and Education. All college-level mathematics courses require a prerequisite of a specific high school GPA, Accuplacer score, or successful completion of developmental mathematics. All students have the opportunity to attend one of many summer Math Boot Camps that are offered and focus on rapid remediation and a chance to retake the Accuplacer exam. Currently, developmental mathematics is offered in different formats (traditional, online, emporium, and 7-week sessions) to accommodate each student's learning style and to accelerate completion.

The STEM pathway will keep the traditional developmental, algebra-based math sequence to college-level mathematics. For college-level mathematics, all prerequisite courses leading to Calculus are offered in 7-week sessions to allow for acceleration. To increase success rates and overall understanding for students taking summer Calculus I, QCC offers a 10-week summer course, rather than the traditional 5-week summer course, which provides for an in-depth focus on Calculus I topics and ensures students are well-prepared for the rest of the Calculus sequence.

For the non-STEM pathways, the QCC Math Department has removed MAT 099 Intermediate Algebra as the prerequisite for MAT 121 Topics in Mathematics and MAT 122 Statistics in order to expedite student's math requirements. Non-STEM students will now move from MAT 095 Beginning Algebra into both MAT 121 Topics in Mathematics and MAT 122 Statistics. One additional non-STEM math course will be reviewed in Fall 2017 for the removal of MAT 099 Intermediate Algebra as the prerequisite.

The Education Pathway will remain the same with the traditional developmental, algebra-based math sequence as the prerequisite to provide the base for the Mathematics for Educators series. Quinsigamond Community College will collect and analyze student-level data on all of the aforementioned changes in order to make informed decisions on how to proceed.

University of Massachusetts Boston

Innovative Approach to Calculus Sequence

The third course in the calculus sequence at the University of Massachusetts Boston is a four-credit course (Math 242) that covers vectors, parameterized curves, differential and integral calculus of functions of two- or three-dimensional variables, line and surface integrals, and vector field analysis, with applications. The University of Massachusetts Boston also offers two split versions of Math 242: a three-credit version (Math 240) that covers the first three quarters of Math 242 (up to and including multiple integrals), and a one-credit version (Math 242R) that covers the last quarter (line and surface integrals, vector field analysis). The distribution of topics allows us to teach all three versions as combined sections, requiring no additional resources, other than careful advising: Math 242 students take the course for the entire semester, Math 240 students stop after three quarters, and Math 242R students take only the last quarter of the semester.

This model allows for a flexible approach with transfer credits: other institutions offer a third semester calculus course that does not cover line and surface integrals and vector field analysis, or that covers only line integrals. Students who have taken such a course and then transfer to UMass Boston are given transfer credit for Math 240. If their major at the University of Massachusetts Boston requires Math 242, then they can take the one-credit Math 242R to fulfill that requirement and do not have to re-take the entire Math 242 course. In addition, this approach also provides for flexibility in degree requirements: while Mathematics and Engineering majors are required to take Math 242, for a minor in Mathematics, only Math 240 is required. This approach makes the Mathematics minor more attractive to students in other majors, for example Computer Science or Economics, who are seeking to strengthen their math foundation in areas relevant to their main interests.

Westfield State University

A History of Using Multiple Measures, Co-requisite Courses, and Math Pathways

Over the past 25 – 30 years, Westfield State University's Department of Mathematics has developed and implemented the following five pathways:

- Calculus,
- Elementary Education,
- Statistics,
- Quantitative Reasoning, and
- Mathematics for Liberal Arts (MLA).

The Calculus Pathway introductory courses are MATH 104-Pre-Calculus, MATH 105-Calculus I and MATH 106-Calculus II. MATH 104, 105 and 106 are primarily populated with mathematics, computer science, biology and chemistry majors. Westfield State University also has movement science majors in MATH 104, as it is required for their major. Students are placed in MATH 104, 105 or 106 based on their high school transcripts and AP scores. In Fall 2017, Westfield State University is piloting a 1-credit co-requisite course for MATH 105 and MATH 106 students who need additional support with the content. Students are placed in this section based on a placement exam administered on the first day of class.

The Elementary Education Pathway consists of four courses: MATH 153-Foundations: Number Systems, MATH 250-Foundations: Patterns, Reasoning and Algebra, MATH 251-Foundations: Geometry, and MATH 252-Foundations: Probability and Statistics. MATH 153 and MATH 250 are required courses for all elementary education majors and are prerequisites for MATH 251 and 252. While they are not required, it is recommended that elementary education majors take both MATH 251 and 252 in order to pass the MTEL exam. Although there are students that come into this pathway with some gaps in their understanding of number systems, algebra and mathematical reasoning, studying the content in these areas at a level that is higher than the level at which they will be teaching allows them to fill in these gaps in their knowledge and gain an understanding deep enough to be an effective teacher.

The Statistics Pathway consists of MATH 108-Elementary Statistics and several sections are offered in an extended time format. Students are placed into extended time sections (1 extra hour per week for which students do not receive extra credit) based on their Accuplacer scores and high school GPA as suggested by the BHE pilot recommendations. Online and hybrid versions of this course are also offered.

The Quantitative Reasoning Pathway consists of MATH 115-Mathematics for Business and Social Sciences and MATH 123-Mathematical Models in the Natural Sciences. MATH 115 is primarily populated by business, economics, management, and accounting majors. Westfield State University has an in-house curriculum for this course that uses an active-learning approach to modeling that supports students at different levels. They also have extended time sections and again, students are placed into these sections based on their Accuplacer scores and high school GPA. MATH 123 is a required course for environmental science. Westfield also uses an in-house self-paced curriculum for this course; with remediation on an as-needed basis, in the sense that students have the choice to spend more time on topics unfamiliar to them, and less time on those with which they may already be comfortable.

The fifth pathway is the Mathematics for Liberal Arts Pathway. It consists of the courses MATH 110-Mathematical Explorations and MATH 111-Mathematical Applications. MATH 110 is the most popular and, arguably, most successful course for liberal arts students. It is designed for students whose majors are primarily in the Humanities (English, history, social work, sociology, art, music, communication, ethnic and gender studies). MATH 111 is an analogous course with a focus on applications. The Department uses in-house curriculums in both courses. Sections of MATH 110 are taught in an inquiry based manner, as are some sections of MATH 111. The MATH 110 curricula was developed through the NSF sponsored Discovering the Art of Mathematics project (http://artofmathematics.org). The topics in this curriculum are strongly tied to the majors for liberal arts students and encourage them to think about mathematics as an art and a creative, and humanistic endeavor. It also provides students an authentic experience doing mathematics where they are allowed to bring their creativity and strengths to a mathematics course in a manner that is different from what they have experienced in the past.

Worcester State University

Implementing Multiple Math Pathways and Piloting Co-requisite Education and Multiple Measures

There are four distinct pathways in Mathematics at Worcester State University: STEM (first college-level course is Introduction to Functions (MA 180), Pre-Calculus (MA 190), or Calculus I (MA 200), depending on placement), Statistics (first college-level course is Statistics I (MA 150)), Education (first college-level course is Number and Operations for Teachers (MA 130)) and Liberal Arts (first college-level course is Survey of Mathematics (MA 105)). Worcester State University's Developmental Math Director, Dr. Eileen Perez, has matched each WSU major to the most suitable pathway for that major. For students that do not place into a college-level course, Worcester State University offers two developmental courses: Arithmetic (MA 098) and Elementary Algebra (MA 099).

Most students, regardless of pathway, are placed into their first mathematics course based on their Accuplacer score (exceptions include first-year students in our pilot programs, which are discussed below). Students in the STEM pathway require a code of 5* (see end of narrative for code guide) to be placed into MA 180 (MA 190 requires a code of 6 and MA 200 requires a code of 7). STEM students that do not reach a code of 5 have different options based on their code. A code of 1 places them into MA 098, from which they will take MA 099. A code of 2 places them directly into MA 099. Students with a code of 3 or 4 are encouraged to enter a 'self-study' program in an attempt to retake Accuplacer and improve their code to 5; this entails working on practice materials with tutors in our Math Center. Note that the final exam for MA 099 is the Elementary Algebra Accuplacer exam; ideally STEM students in the developmental

sequence will reach a code 5 at this point. Students that reach a code of 3 or 4 are encouraged to self-study instead of retaking MA 099.

Students in the Statistics pathway require an Accuplacer code of 4 to register for MA 150. Those that do not receive a code of 4 have similar options as STEM students; students with a code of 1 take MA 098 followed by MA 099, students with a code of 2 will take MA 099, and students with a code of 3 are encouraged to self-study. However, we will be working on a co-requisite pilot program during AY 2017-2018 to be implemented in AY 2018-2019; Professors Eileen Perez and Mary Fowler (course coordinator for MA 150) will be creating a new developmental course for the Statistics pathway, and incoming first-year students in the fall of 2018 that do not pass Accuplacer with a code of at least 4 will be allowed to register for this new developmental course along with MA 150 (details to come).

Students in the Education pathway require a code of 5 to register for MA 130; those that fall short have the same options as STEM students. However, this year we are implementing a co-requisite option for first-year students who receive a code of 1 (and would otherwise be placed into MA 098). We are running a MA 098/MA 130 co-requisite option, created and taught by Professors Eileen Perez and Jason Hardin (Prof. Perez will be teaching the MA 098 portion, Prof. Hardin will be teaching the MA 130 portion). The details for this program are fully fleshed out and can be provided upon request.

Students in the Liberal Arts pathway require a code of 3 to register for MA 105. However, we are in the second year of a pilot that allows first-year students with a HS GPA of at least 2.7 (and who need no specific math course for their major) to register for MA 105.

*Please note 'AR' stands for arithmetic, 'EA' stands for elementary algebra, and 'CLM' stands for college-level math.

Code 1: AR less than 74 and EA less than 81
Code 2: AR between 75 and 120, EA less than 71
Code 3: AR between 75 and 81, EA between 72 and 81
Code 4: AR between 82 and 120, EA between 72 and 81
Code 5: EA between 82 and 120
Code 6: CLM between 52 and 88
Code 7: CLM between 89 and 120

Appendix A: Glossary

A2B Pathways: Associate to Baccalaureate transfer pathways from community colleges to public four-year institutions.

A2B Mapped Pathways: A subset of A2B Pathways, mapped associate degree programs are referred to as A2B Mapped Pathways. These refer to specific, statewide agreements that allow a student to start at any community college and transfer to any four-year institution while following a discipline-specific map. Students following these pathways will have graduated with a minimum of 60 credit hours and will have completed the 34-credit General Education Foundation or the 28-credit STEM General Education Foundation, exclusive of developmental coursework.

Co-Requisite Education: Students enroll directly into college-level courses and receive academic support alongside their classes. Rather than facing a long sequence of prerequisite, non-credit courses, students get up to speed while working toward their degree" (Complete College America, 2016, p. 2). This accelerated model allows students to move more efficiently and effectively into credit-bearing courses.

Mathematics Pathways: These pathways "refer to developmental and college-level course sequences that align to a student's academic and career goals, and that accelerate student completion of a gateway college-level math course" (Getz and Ortiz, 2016, p. 1).

Multiple Measures: The use of measures beyond standardized tests to assess a student's ability to succeed in college-level English and mathematics courses. In Massachusetts, this refers to using measures other than Accuplacer to determine students' preparedness for credit-bearing coursework in English and mathematics.

Appendix B: Proposed Content for Calculus Sequence for STEM

Calculus I

- Limits and continuity (understanding and working knowledge), including limit laws, one-sided limits, and limits at infinity.
- Differentiation
- Understanding of differentiation as rate of change
    - Max/min
    - Graph sketching
- Rule of differentiation
    - Power
    - Quotient and product
    - Transcendentals
    - Chain Rule
    - Implicit Differentiation
- Integration (introduction- through the Fundamental Theorem of Calculus)
    - Understanding of a definite integral as a summation
    - The Fundamental Theorem of Calculus

Calculus II

- Use and application of definite/indefinite integrals. Understand application with sequences and series particularly the Taylor series.
- Techniques of integration and when to use
    - Parts
    - Substitution
    - partial fractions
- Improper integrals
- Application of integrals
    - Area
    - Volume
    - Arc Length and Surface Area
- Basic Differential equations
- Sequence and numerical series (differences)
    - Definition
    - Properties
    - Use of notation
    - Tests for convergence and when to use
        - Ratio and Root Test
        - Alternating Series/Absolute Convergence
- Power series

    - Radius of convergence
    - Summing power series; recognizing the function
    - Operations with power series
    - Taylor Polynomials

Calculus III

- Coordinate systems
    - Rectangular
    - Polar
    - Cylindrical and/or Spherical
- Vectors
    - Operations
        - Linear Combinations
        - Linear Decomposition
        - Dot Product
        - Cross Product
        - Triple scalar product ((u X v) dot w)
    - Equations of Lines and Planes
- Parameterized Curves
    - Motion in Space
    - Derivatives and Integrals of Vector Functions
    - Arc length and curvature
- Functions of Several Variables
    - Graphical Representations
    - Limits and Continuity
    - Partial derivatives
    - Tangent Planes and Linear Approximation
    - Chain Rule
    - Implicit Functions
    - Directional Derivatives, Gradient Vector
    - Vector Fields
    - Maximum and Minimum Values
    - Lagrange Multipliers
- Integrals
    - Double (over rectangles, general regions, and polar coordinates)
    - Triple
    - Integrals in Polar, Cylindrical, and Spherical Coordinates

<u>Optional</u>

- Line Integrals
    - Computations of Line Integrals
    - Gradient Fields/Conservative Fields

    - Fundamental Theorem for Line Integrals
    - Green's Theorem
- Surface Integrals
    - Parametric Surfaces
    - Surface Integrals
    - Curl and Divergence
    - Stokes' Theorem
    - Divergence Theorem

Appendix C: Proposed Content for Quantitative Reasoning*

Quantitative Reasoning

The Quantitative Reasoning (QR) courses are intended to support students who are in non-STEM pathways. QR courses should help students attain quantitative literacy skills, pose problems that involve quantitative relationships in the real-world and/or experience mathematics as an art and a creative, humanistic endeavor. Students should be able to use data to observe patterns, make conjectures/predictions and prove or disprove them, make good decisions, and formulate alternatives. Students should improve their problem-solving skills, be able to communicate and critique quantitative arguments orally and in writing, and see themselves as lifelong learners.

Examples: Voting methods, financial management, probability theory, logic/set theory, numeration, the infinite, number theory, knot theory, music, dance, art and sculpture, games and puzzles.

Recommendation: Use of technology,

*For Quantitative Reasoning courses taught by the Department of Mathematics.

Appendix D: Proposed Content for Statistics

Introduction to Statistics

- Descriptive Statistics
    - Types of Data
    - Measures of Center
    - Measures of Variability
    - Graphs
        - Boxplots
        - Histograms
        - Bar and Line Graphs
    - Frequency Tables
    - Sampling/Designing Studies
- Probability
    - Theoretical
    - Formal Rules
    - Probability Distributions – Discrete and Continuous
- Inferential Statistics
    - Distribution of Means/Central Limit Theorem
    - Confidence Intervals
    - One sample Hypothesis Tests
        - Means and proportions
    - Regression/Correlation

Recommendation: Use of statistical software.

Note: Originally developed by CONNECT: A Southeastern Massachusetts Public Higher Education Partnership.

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