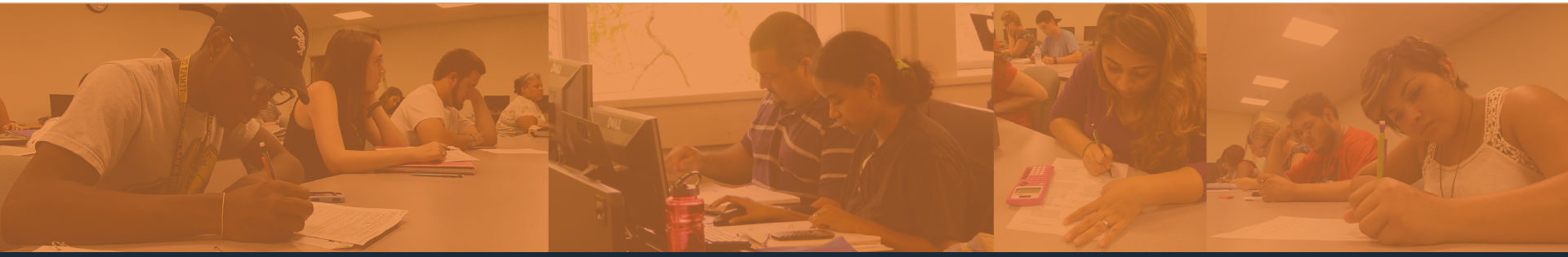
Dana Center
Mathematics
PATHWAYS

Mathematics Pathways:
Overview of co-requisite models

Dr. Jeff Shaver, The Charles A. Dana Center, The University of Texas at Austin

Southeast Texas Regional Mathematics Pathways Meeting

May 23, 2017



DCMP Vision

Dana Center Mathematics PATHWAYS
dcmathpathways.org

The DCMP seeks to ensure that **ALL** students in higher education will be:

- **Prepared** to use mathematical and quantitative reasoning skills in their careers and personal lives,
- **Enabled** to make timely progress towards completion of a certificate or degree, and
- **Supported** and **Empowered** as mathematical learners.

Session objectives

Participants will:

- Gain a common understanding of mathematics pathways
- Discuss the shifting purpose of developmental and college-level mathematics
- Backmap support objectives from student materials

Definition of *Math Pathway*

Noun | math · path · way | \ math \ ‘path-,wā \

Definition:

1: A mathematics course or sequence of courses that students take to meet the requirements of their program of study.

The concept of math pathways applies to both college-ready and underprepared students.

Dana Center Principles for Pathways

Mathematics pathways are structured so that:

1) All students, regardless of college readiness, enter directly into mathematics pathways aligned to their programs of study.
2) Students complete their first college-level math requirement in their first year of college.

Students engage in a high-quality learning experience in math pathways designed so that:

3) Strategies to support students as learners are integrated into courses and are aligned across the institution.
4) Instruction incorporates evidence-based curriculum and pedagogy.

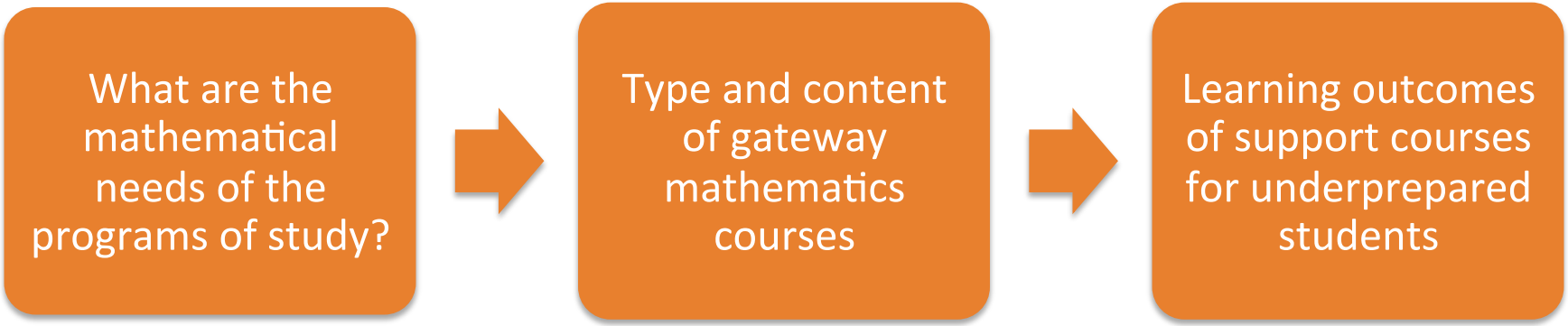
Defining Content

Supporting the desired student experience

Defining the content of prerequisite and co-requisite courses:

- How do we take underprepared students from where they are to a level of preparedness for the college-level course?

Backward mapping to define content



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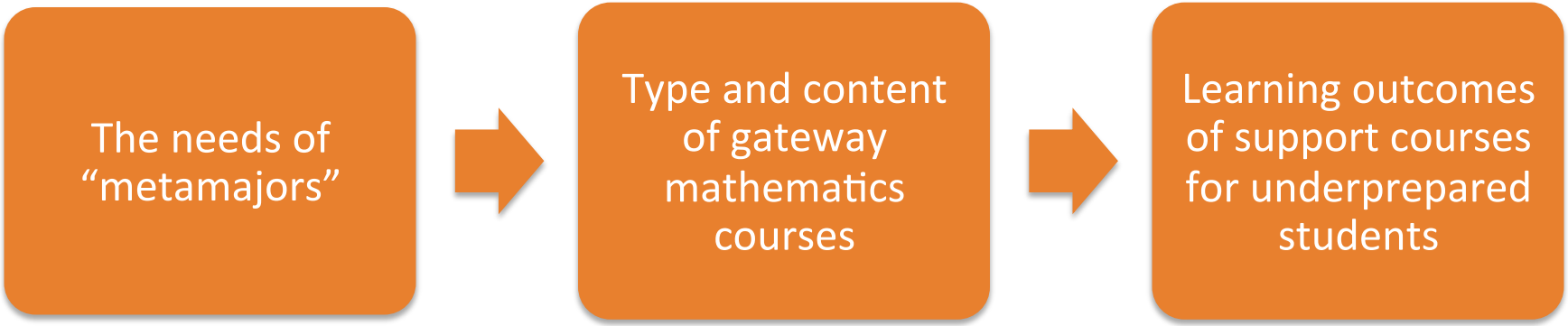
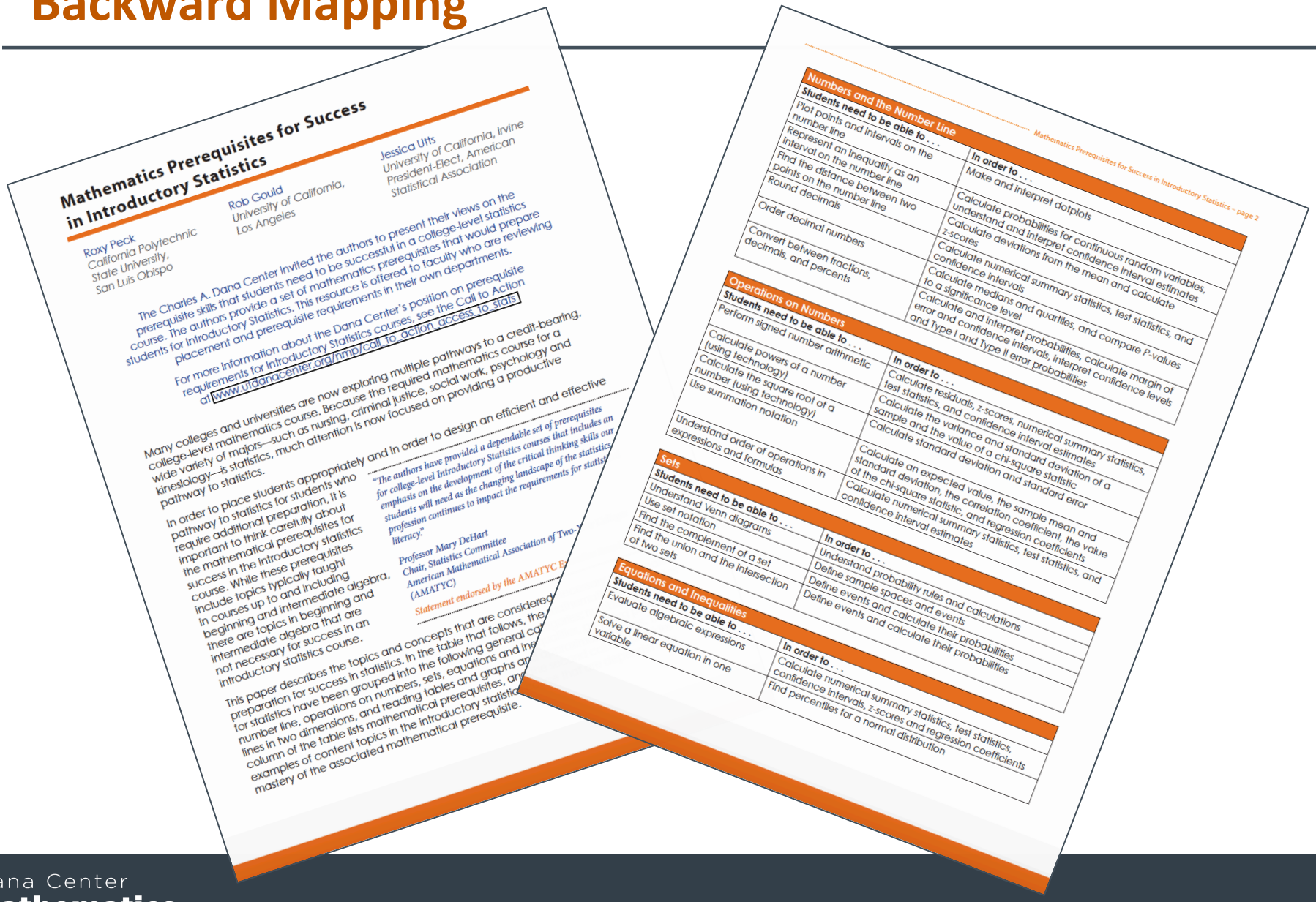


Backward Mapping



Mathematics Prerequisites for Success in Introductory Statistics

Roxy Peck, California Polytechnic State University, San Luis Obispo
Rob Gould, University of California, Los Angeles
Jessica Utts, University of California, Irvine, President-Elect, American Statistical Association

The Charles A. Dana Center invited the authors to present their views on the prerequisite skills that students need to be successful in a college-level statistics course. The authors provide a set of mathematics prerequisites that would prepare students for Introductory Statistics. This resource is offered to faculty who are reviewing placement and prerequisite requirements in their own departments.

For more information about the Dana Center's position on prerequisite requirements for Introductory Statistics courses, see the Call to Action at www.utdanacenter.org/nmp/call_to_action_access_to_stats

Many colleges and universities are now exploring multiple pathways to a credit-bearing, college-level mathematics course. Because the required mathematics course for a wide variety of majors—such as nursing, criminal justice, social work, psychology and kinesiology—is statistics, much attention is now focused on providing a productive pathway to statistics.

In order to place students appropriately and in order to design an efficient and effective pathway to statistics for students who require additional preparation, it is important to think carefully about the mathematical prerequisites for success in the Introductory Statistics course. While these prerequisites typically taught in courses up to and including beginning and intermediate algebra, include topics there are topics in beginning and intermediate algebra that are not necessary for success in an introductory statistics course.

"The authors have provided a dependable set of prerequisites for college-level Introductory Statistics courses that includes an emphasis on the development of the critical thinking skills our students will need as the changing landscape of the statistics profession continues to impact the requirements for statistical literacy."

Professor Mary DeHart
Chair, Statistics Committee
American Mathematical Association of Two-Year Colleges (AMATYC)

Statement endorsed by the AMATYC B

This paper describes the topics and concepts that are considered preparation for success in statistics. In the table that follows, the for statistics have been grouped into the following general co number line, operations on numbers, sets, equations and ine lines in two dimensions, and reading tables and graphs ar column of the table lists mathematical prerequisites, and examples of content topics in the introductory statistics mastery of the associated mathematical prerequisite.

Numbers and the Number Line	
Students need to be able to . . .	**In order to . . .**
Plot points and intervals on the number line	Make and interpret dotplots
Represent an inequality as an interval on the number line	Calculate probabilities for continuous random variables, understand and interpret confidence interval estimates
Find the distance between two points on the number line	Calculate deviations from the mean and calculate z-scores
Round decimals	Calculate numerical summary statistics, test statistics, and confidence intervals
Order decimal numbers	Calculate medians and quartiles, and compare *P*-values to a significance level
Convert between fractions, decimals, and percents	Calculate and interpret probabilities, calculate margin of error and confidence intervals, interpret confidence levels and Type I and Type II error probabilities

Operations on Numbers	
Students need to be able to . . .	**In order to . . .**
Perform signed number arithmetic	Calculate residuals, z-scores, numerical summary statistics, test statistics, and confidence interval estimates
Calculate powers of a number (using technology)	Calculate the variance and standard deviation of a sample and the value of a chi-square statistic
Calculate the square root of a number (using technology)	Calculate standard deviation and standard error
Use summation notation	Calculate an expected value, the sample mean and standard deviation, the correlation coefficient, the value of the chi-square statistic, and regression coefficients
Understand order of operations in expressions and formulas	Calculate numerical summary statistics, test statistics, and confidence interval estimates

Sets	
Students need to be able to . . .	**In order to . . .**
Understand Venn diagrams	Understand probability rules and calculations
Use set notation	Define sample spaces and events
Find the complement of a set	Define events and calculate their probabilities
Find the union and the intersection of two sets	Define events and calculate their probabilities

Equations and Inequalities	
Students need to be able to . . .	**In order to . . .**
Evaluate algebraic expressions	Calculate numerical summary statistics, test statistics, confidence intervals, z-scores and regression coefficients
Solve a linear equation in one variable	Find percentiles for a normal distribution

Activity: Discussion/reflection

Take a few minutes to discuss with your colleagues or reflect individually:

What information did you find that could be used in replicating this process for other courses on your campus?

Mathematics Prerequisites for Success in Intro. Statistics

- Mathematics content linked to content in the introductory statistics course that are dependent on mastery of the mathematics content.
- Grouped mathematics prerequisites into six general categories
  - Numbers and the number line
  - Operations on numbers
  - Sets
  - Equations and inequalities
  - Graphing points and lines in two dimensions
  - Reading tables and graphs and approximating areas

Justifying Mathematics Prerequisites

Example:

Represent an inequality as an interval on the number line.

Is this needed for statistics?

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Why?

Calculate probabilities for continuous variables

Understand and interpret confidence interval estimates

Justifying Mathematics Prerequisites

Example

Order decimal numbers

Is this needed for statistics?

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Why?

Calculate median and quartiles

Compare *P*-value to a significance level

Backward mapping to define content

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Mathematics pathways content:

- What learning outcomes does each gateway math course need to serve the appropriate pathway?
- What are the readiness outcomes for each gateway course?
- What will help underprepared students achieve readiness for the college-level course?

Backward mapping to define content

Defining the Content:
Content Backmapping Template

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The content of support courses (prerequisite or co-requisite) should be selected based on the skills that students need to be successful in the college-level course. This tool is designed to facilitate the process of backmapping learning outcomes for the support course from the readiness competencies of the college-level course.

To identify learning outcomes for support courses, list the specific skills from the learning outcomes of the college-level course in the first column. In the second column, identify the competencies needed in order to successfully engage in activities that develop the skills in the first column. Those competencies become the descriptors of the learning outcomes of the pre/co-requisite course.

For prerequisite course structures, consider carefully which skills may need to be reinforced in the college-level course or may even be best saved for initial introduction in the college-level course.

An example from a Quantitative Reasoning course is shown below.

Demonstrate procedural fluency with real number arithmetic operations.				
In the college-level course, students will:	**Therefore, they need the ability to:**	**These skills should be:**		
		Taught in support course	**Reinforced in college level**	**Taught in college level**

Backward mapping to define content

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An example from a Quantitative Reasoning course is shown below.

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In the college-level course, students will:	**Therefore, they need the ability to:**	**These skills should be:**		
		Taught in support course	**Reinforced in college level**	**Taught in college level**
Calculate absolute change.	Select and perform the four basic operations.	X		
Calculate relative change.	Calculate a percentage.	X		
	Interpret a percentage.	X	X	
Compare two budget categories over time.	Calculate absolute and relative change.			X

05

Activity: Practice

Tailor this example to fit your course (e.g., algebraic, statistical, quantitative, technical, business, education).

Choose, create, and use models for bivariate data sets.				
In the college-level course, students will:	**Therefore, they need the ability to:**	**These skills should be:**		
		Taught in support course	**Reinforced in college level**	**Taught in college level**
Create a graphical display.				
Analyze data to determine appropriate model.				
Create the model.				
Use model for prediction.				

What background skills would prepare students to engage successfully in activities related to this SLO?

Activity: Plan for action

Create a plan for defining the content of support courses for underprepared students. Plan for how you will:

- Move forward to define a comprehensive set of SLOS for the support course at your institution.

Planning Co-requisite Content

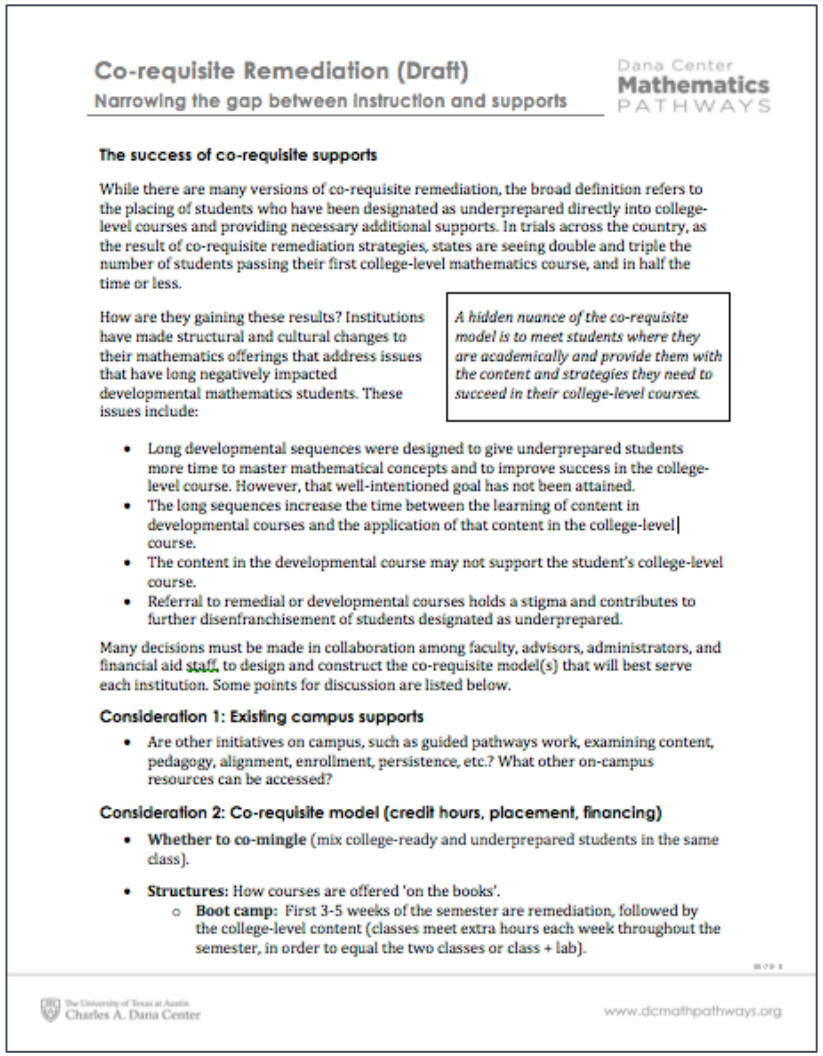
Planning Co-requisite Content

Introduction to Statistics and Co-requisite Support Course Sample Timeline
Adapted from and with thanks to Roane State Community College

Day	Co-requisite Notebook Topics	On-line Lab	*Essentials of Statistics* Triola 5th ed.		MyLabsPlus Assignment
1	Orientation, study habits, time mgmt.; converting between fractions, decimals, percentages; finding a percentage of a number	1	1.1 – 1.2	Orientation; introduction to statistical terms and statistical thinking	1
2	Rounding; estimating; calculating means,	2	1.3 – 1.4	Types of data; collecting sample data	2
3	Decimals, ratios, percent, conversions	3	2.2 – 2.3	Frequency distributions; histograms	3
4	Applications of percent, squares, square roots; order of operations	4	2.4	Graphs that enlighten and graphs that deceive	4
5	Operations on real numbers	5	3.2	Measures of center	5
6	Review of types of data, sampling methods, types of graphs	6	3.3 – 3.4	Measures of variation; measures of relative standing and boxplots	6
7	Review of measures of center and variation	7	Practice Test 1		
8	Comprehensive review of chapters 1 – 3 & basic skills	8	**Test 1**		

Implementing Co-requisite Supports

Implementing Co-requisite Supports



Co-requisite Remediation (Draft)

Narrowing the gap between instruction and supports

Dana Center Mathematics PATHWAYS

The success of co-requisite supports

While there are many versions of co-requisite remediation, the broad definition refers to the placing of students who have been designated as underprepared directly into college-level courses and providing necessary additional supports. In trials across the country, as the result of co-requisite remediation strategies, states are seeing double and triple the number of students passing their first college-level mathematics course, and in half the time or less.

How are they gaining these results? Institutions have made structural and cultural changes to their mathematics offerings that address issues that have long negatively impacted developmental mathematics students. These issues include:

A hidden nuance of the co-requisite model is to meet students where they are academically and provide them with the content and strategies they need to succeed in their college-level courses.

- Long developmental sequences were designed to give underprepared students more time to master mathematical concepts and to improve success in the college-level course. However, that well-intentioned goal has not been attained.
- The long sequences increase the time between the learning of content in developmental courses and the application of that content in the college-level course.
- The content in the developmental course may not support the student's college-level course.
- Referral to remedial or developmental courses holds a stigma and contributes to further disenfranchisement of students designated as underprepared.

Many decisions must be made in collaboration among faculty, advisors, administrators, and financial aid staff, to design and construct the co-requisite model(s) that will best serve each institution. Some points for discussion are listed below.

Consideration 1: Existing campus supports

- Are other initiatives on campus, such as guided pathways work, examining content, pedagogy, alignment, enrollment, persistence, etc.? What other on-campus resources can be accessed?

Consideration 2: Co-requisite model (credit hours, placement, financing)

- **Whether to co-mingle** (mix college-ready and underprepared students in the same class).
- **Structures:** How courses are offered 'on the books'.
  - **Boot camp:** First 3-5 weeks of the semester are remediation, followed by the college-level content (classes meet extra hours each week throughout the semester, in order to equal the two classes or class + lab).

The University of Texas at Austin Charles A. Dana Center

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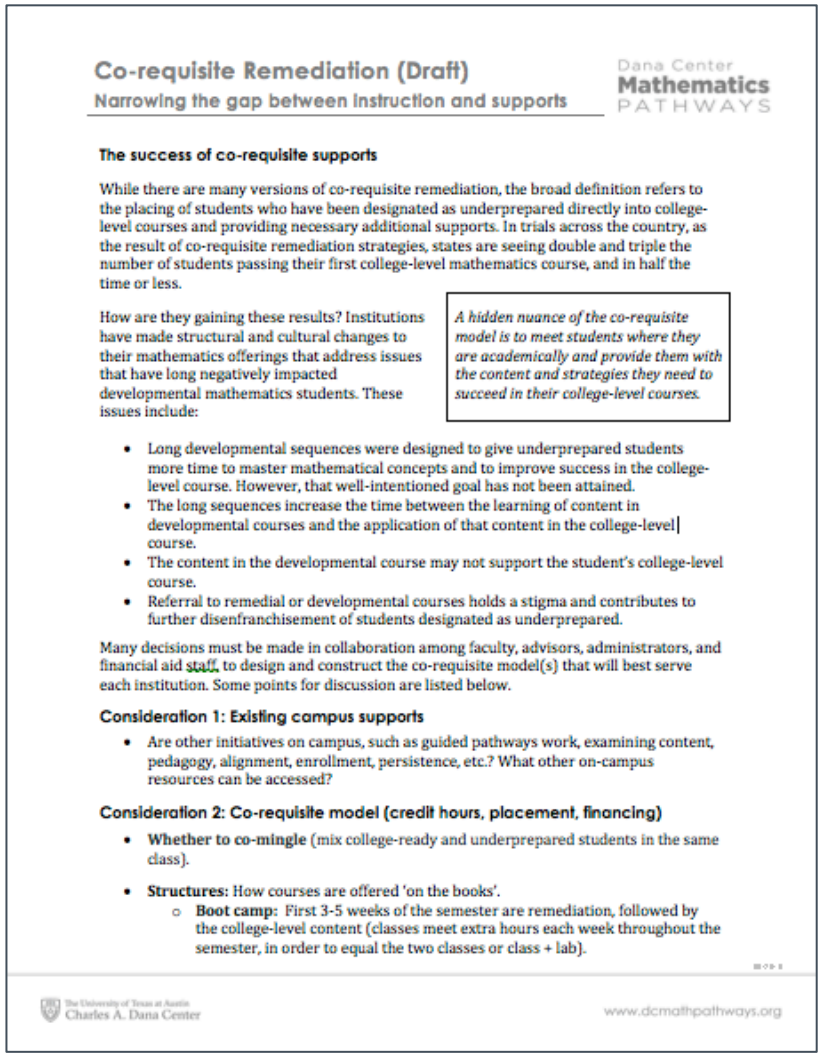
- **Consideration 4:** Cultural shifts

Implementing Co-requisite Supports



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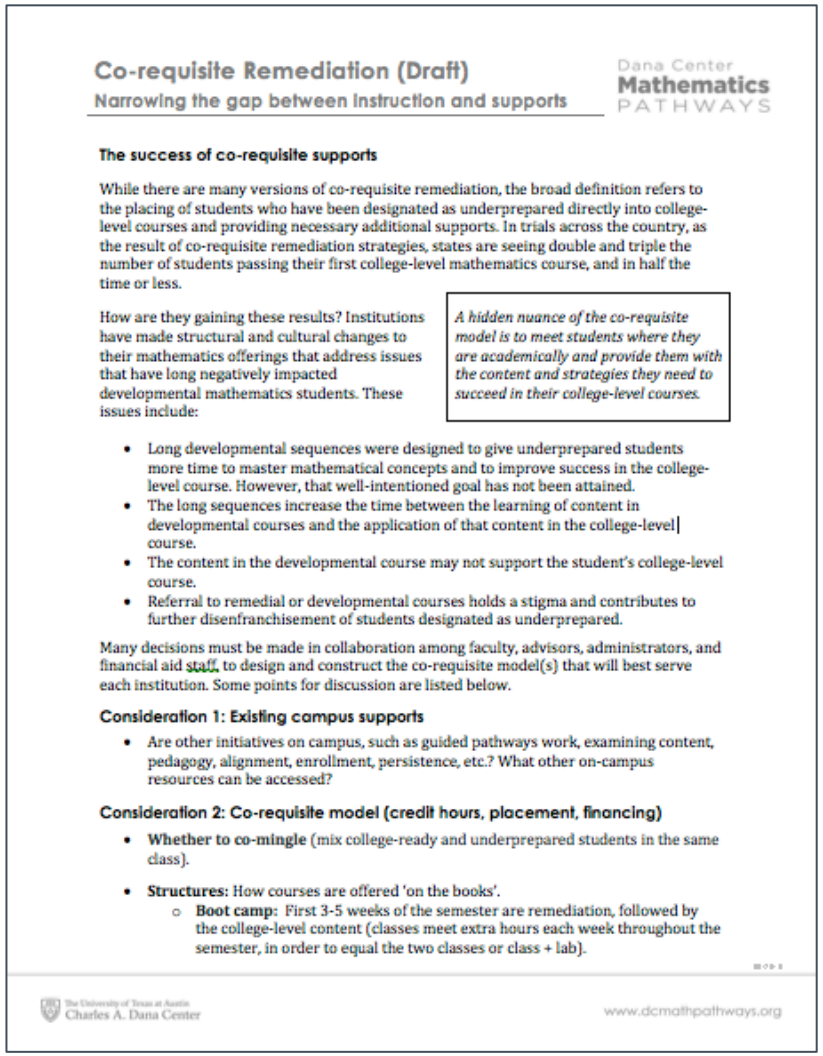

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- **Consideration 1:** Existing campus supports
- **Consideration 2:** Co-requisite model
  - Placement
  - Structure
- **Consideration 3:** Co-requisite content
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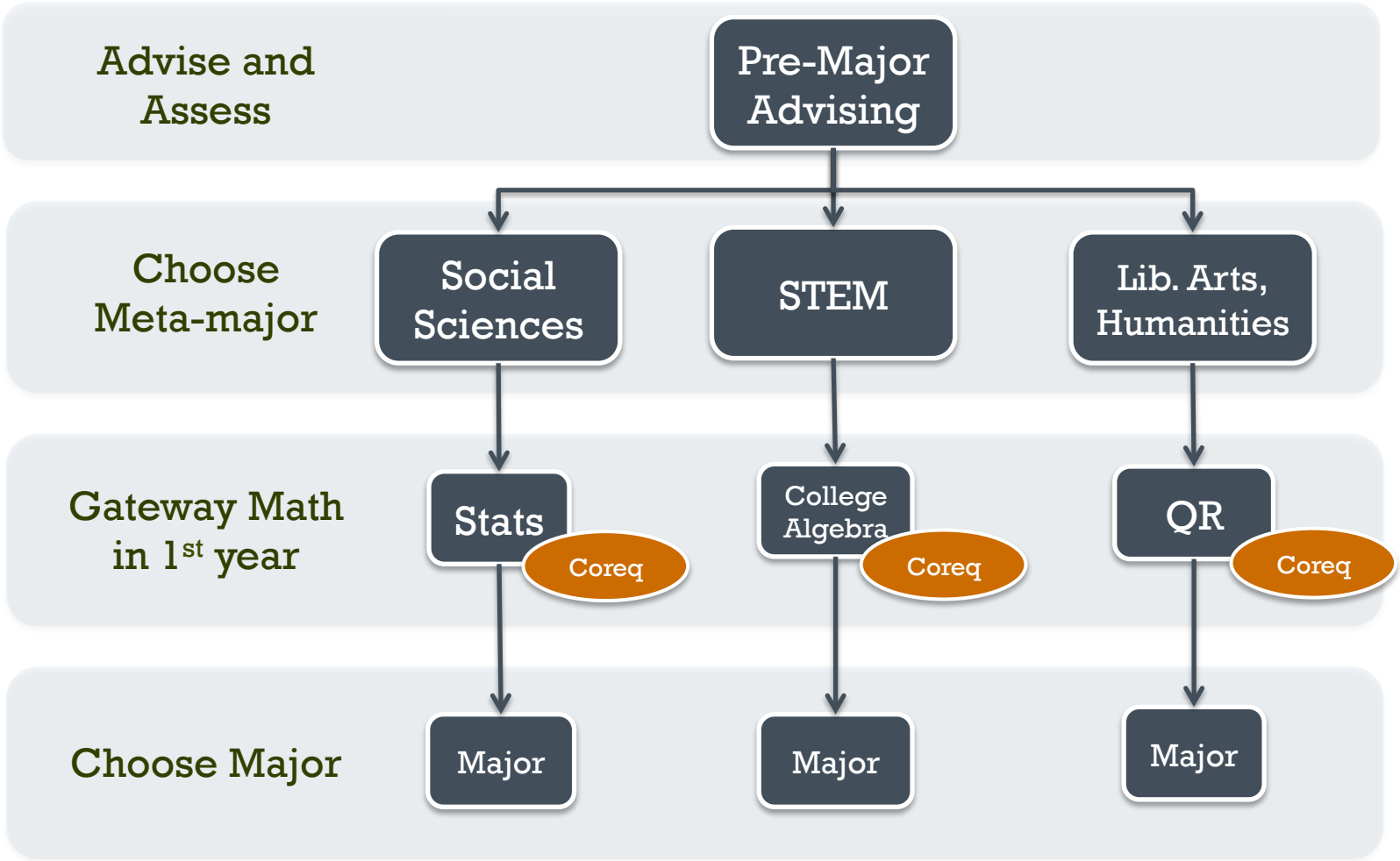
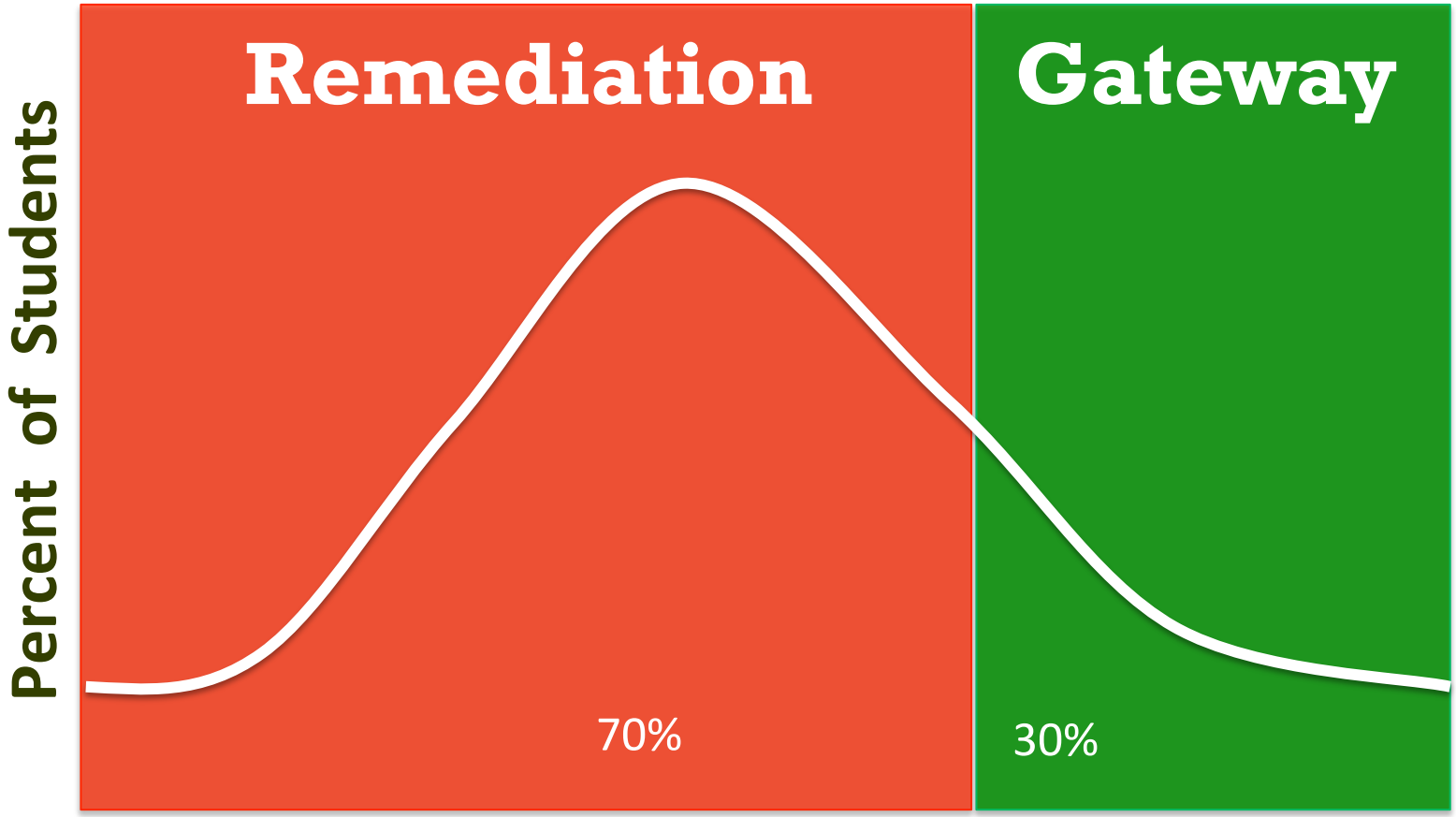
Mathematics Pathways with Co-requisites



Adapted from Complete College America 2016

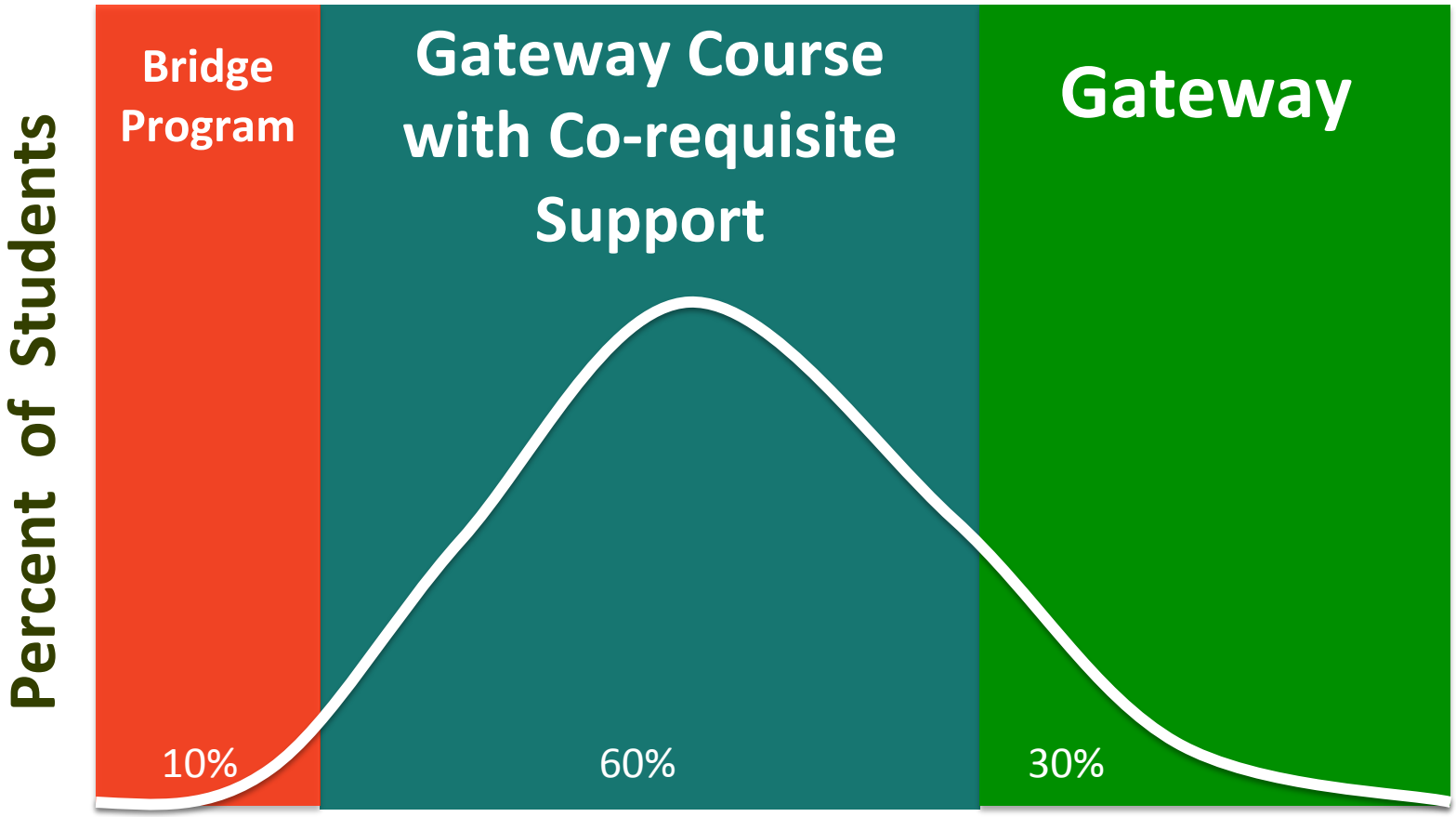
End Use of Traditional Placement



Student Placement Data

- Complete College America 2014

With Co-requisite, Most in College-Level



Student Placement Data

- Complete College America 2014

Activity: Structure Pros and Cons

Discussion Template

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Cohorting	
Pros	**Cons**

Co-mingling	
Pros	**Cons**

Discussion starters:

- How might this structure be perceived by other mathematics department faculty?
- How might this structure impact students' sense of belonging and connection?
- How might this structure be impacted by your institution's context?
- What other issues or questions does this structure raise?

Contact information

- General information about the Dana Center
  www.utdanacenter.org
- Dana Center Mathematics Pathways Resource Site
  www.dcmathpathways.org
- To receive monthly updates about the DCMP, contact us at
  dcmathpathways@austin.utexas.edu
- Connie Richardson
  cjrichardson@austin.utexas.edu
- Jeff Shaver
  jmshaver@austin.utexas.edu

About the Dana Center

The **Charles A. Dana Center** at The University of Texas at Austin works with our nation's education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace.

Our work, based on research and two decades of experience, focuses on K–16 mathematics and science education with an emphasis on strategies for improving student engagement, motivation, persistence, and achievement.

We develop innovative curricula, tools, protocols, and instructional supports and deliver powerful instructional and leadership development.

2016

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