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## About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We work with our nation’s education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at [www.utdanacenter.org](http://www.utdanacenter.org).

## About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—*Statistical Reasoning*, *Quantitative Reasoning*, and *Reasoning with Functions I* and *Reasoning with Functions II* (a two-course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point—a developmental math course that helps students develop foundational skills and conceptual understanding in the context of college-level course material.

In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies—and tenacity—necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.

In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

### Acknowledgments

The development of this course began with the formation of the DCMP **Curricular Design Team**, who set the design standards for the curricular materials of individual DCMP courses would be designed. The team members are:

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The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The **Usability Advisory Group** members are:

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## Contents

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
-	-	Curriculum Overview	xv	viii	-	-
-	-	Suggestions for Prep Week	xxxix	-	-	-
<b>Lessons 1-7: Complex Numerical Summaries; Graphical Displays</b>						
1.A		Data for Life <i>Collect data that will be referred to throughout the semester; supplemental spreadsheet provided</i>	1	1	1	-
1.B		Our Learning Community <i>Student success focus Establish a sense of shared responsibility; provide key information about course content and policies</i>	4	5	4	-
1.C	1.C	Instant Runoff Voting schemes	6	7	13	1.C
1.D	1.D	Borda Count Voting schemes	11	11	18	1.D
2.A	2.A	Graphical Displays <i>Analysis and communication; dotplots, histograms, boxplots; mean; median</i>	14	13	22	2.A
2.B	2.B	Forming Effective Study Groups <i>Student success focus Taking responsibility for own learning and supporting learning of others; setting norms</i>	17	15	26	-
2.C	-	Mini-Project: Graphical Displays <i>Write formal, contextual analysis on compared data; research-related data; sample rubric provided</i>	20	17	32	-
3.A	3.A	Who Is in the Population? <i>Populations; sampling</i>	23	19	38	3.A
3.B	3.B	How Much Water Do I Drink? <i>Analyzing class data; Central Limit Theorem</i>	27	21	43	3.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
3.C	3.C	How Much Water Does Our Class Drink? (Optional) <i>Sample standard deviation</i>	30	23	47	3.C
4.A	4.A	What Are the Risks? <i>Theoretical probability of two or more independent events</i>	35	27	50	4.A
4.B	4.B	Calculating Risk <i>Conditional probability of two or more dependent events</i>	39	31	54	4.B
5.A	5.A	Cost of Living Comparisons <i>Conversion to create equivalent units; supplemental spreadsheet</i>	44	35	58	5.A
5.B	5.B	Index Numbers <i>Using indices such as Consumer Price Index; supplemental spreadsheet</i>	48	39	63	5.B
5.C	5.C	Polls, Polls, Polls! <i>Weighted averages</i>	52	43	67	5.C
5.D	5.D	Average Income <i>Weighted averages and expected value; supplemental spreadsheet</i>	56	47	72	5.D
6.A	6.A	How Can We Smooth the Data? (Optional) <i>Simple and weighted moving averages; supplemental spreadsheet</i>	59	49	76	6.A
6.B	-	Mini-Project: Income Disparities (Optional) <i>Written analysis of graphical display of weighted moving average</i>	63	51	81	-
7.A	7.A	U.S. Budget Priorities <i>Part-part vs. part-whole ratios</i>	68	53	86	7.A
7.B	7.B	Understanding U.S. Budget Priorities <i>Decimals, percentages, and part-whole ratios</i>	72	57	90	7.B
7.C	7.C	Changes to U.S. Budget Priorities <i>Absolute and relative change</i>	78	61	95	7.C
7.D	7.D	Percent of Total U.S. Budget <i>Dotplots used to introduce symmetry and skewness</i>	82	63	99	7.D

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
7.E	7.E	What's My Credit Score? <i>Application of ratios; Practice assignment can be mini-project. Collect data for Lesson 8, Part D; schedule lab for 8.D and 10.A.</i>	87	65	102	7.E
7.F	7.F	U.S. Incarceration Rates <i>Applications of ratios; comparison</i>	91	69	107	7.F

### Lessons 8-12: Mathematical Modeling

8.A	8.A	More Water, Please! <i>Introduction to mathematical modeling</i>	94	71	112	8.A
8.B	8.B	What's My Car Worth? <i>Distinguishing proportionality and linearity</i>	99	75	118	8.B
8.C	8.C	How Money Makes Money <i>Non-linear models</i>	103	79	123	8.C
8.D	8.D	Have My Choices Affected My Learning? <i>Regression using student data. Computer lab day, if possible.</i>	109	83	129	8.D
8.E	8.E	Mini-Project: Progressive and Flat Income Tax Systems (Optional) <i>Informal piecewise linear function</i>	113	87	135	-
8.F	8.F	Mini-Project: Estimating the Number of People in a Crowd (Optional) <i>Using proportionality to estimate</i>	122	93	140	-
9.A	9.A	Depreciation <i>Modeling, interpolation, and extrapolation</i>	129	97	146	9.A
9.B	9.B	Appreciating Depreciation <i>Linear interpolation via similar triangles</i>	135	103	153	9.B
9.C	9.C	How Much Should I Be Paid? <i>Correlation</i>	140	107	159	9.C
9.D	9.D	Why Are You Wearing the Same Old Socks? <i>Correlation vs. causation; strength</i>	148	113	164	9.D
10.A	10.A	Fibonacci's Rabbits <i>Exponential growth; limitations. Computer lab day, if possible.</i>	155	117	168	10.A

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
10.B	10.B	Is It Getting Crowded? <i>Exponential growth; limitations</i>	158	121	173	10.B
<p>You may wish to consider various configurations with the upcoming modeling lessons. For example, you may wish to consider having different groups complete and present the various logistic lessons or having some groups do logistic models while other groups do the periodic models. You may also choose to omit either logistic or periodic models.</p>						
11.A	11.A	Population Growth (Optional) <i>Logistic models</i>	163	125	177	11.A
11.B	11.B	Oh, Deer! (Optional) <i>Time series model of logistic growth</i>	169	129	182	11.B
11.C	11.C	Can You Hear Me Now? (Optional) <i>Logistic models. Spreadsheet demonstration or computer lab day, if possible.</i>	173	133	187	11.C
11.D	11.D	Hares and Lynxes (Optional) <i>Predator-prey</i>	178	137	191	11.D
11.E	11.E	Reindeer and Lichens (Optional) <i>Effects of parameter choices on model predictions</i>	183	141	198	11.E
12.A	12.A	How Long Is the Longest Day? (Optional) <i>Cyclical data</i>	188	145	202	12.A
12.B	12.B	What's My Sine? (Optional) <i>Periodic functions</i>	196	151	206	12.B
12.C	12.C	SIR Disease (Optional) <i>Effect of parameters on a model (epidemics)</i>	201	155	211	12.C
12.D	-	SIR (Continued) (Optional) <i>Create a time-series model using a spreadsheet; Practice assignment could be a mini-project.</i>	206	159	216	-
<b>Lessons 13-15: Statistical Studies</b>						
13.A	13.A	Mind the Gap in Income Inequality <i>Introductory vocabulary for statistical studies</i>	209	161	224	13.A



Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
13.B	13.B	When in Rome . . . <i>Observational and experimental studies and their conclusions</i>	215	165	229	13.B
13.C	13.C	A Lesson Worth Weighting For <i>Sampling processes</i>	219	169	234	13.C
13.D	13.D	Weight . . . There's More! <i>Evaluate and design sampling processes</i>	225	175	241	13.D
14.A	14.A	Blood Pressure and Bias <i>Sampling and non-sampling error</i>	231	181	247	14.A
14.B	14.B	Taking Aim at Bias <i>Types of bias</i>	235	185	251	14.B
14.C	14.C	Conclusions in Observational Studies <i>Minimizing bias; appropriate conclusions</i>	240	189	255	14.C
15.A	15.A	The Video Game Diet <i>Designing experimental studies; cause and effect</i>	244	193	261	15.A
15.B	15.B	All Things in Moderation <i>Confounding variables</i>	248	197	266	15.B
15.C	15.C	The Power of the Pill <i>Blinding; placebo effect; placebos</i>	253	201	271	15.C
15.D	15.D	Designing an Experiment <i>Double blinding; blocking</i>	257	205	276	15.D
15.E	15.E	In Conclusion <i>Culminating lesson on conclusions from statistical studies</i>	262	209	284	15.E

You may wish to consider various configurations with the upcoming lessons on analyzing and writing about graphical displays. For example, you may wish to consider having different groups complete Lesson 16, Parts B, D, E, and F, and present to the class.

### Lessons 16-18: Complex Quantitative Information and Graphical Displays

16.A	16.A	Education Pays <i>Analyzing stacked column graphs</i>	266	213	289	16.A
16.B	16.B	Looking for Links <i>Analyzing comparative stacked columns graphs</i>	271	217	295	16.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
16.C	16.C	It's About Time! <i>Building stacked columns graphs from class data</i>	275	221	301	16.C
16.D	16.D	Connecting the Dots <i>Analyzing motion bubble charts</i>	280	225	207	16.D
16.E	16.E	Big Data (GIS) <i>Analysis problems associated with large, volatile data</i>	285	229	313	16.E
16.F	16.F	Big Brother – They're Watching! <i>Conclusions from heat maps</i>	292	237	319	16.F
17.A	17.A	Decisions, Decisions <i>Decision making based on multiple pieces of quantitative information</i>	296	241	324	17.A
17.B	17.B	The Write Approach to Data <i>Improving written analyses of graphical displays</i>	302	245	330	17.B
17.C	17.C	Numbers Never Lie <i>Misleading and erroneous graphical displays</i>	309	249	344	17.C
17.D	17.D	Can You Feel the Heat? <i>Using data to understand complex issues</i>	316	255	339	17.D
18.A	18.A	Mini-Project: Tornado Climatology <i>Choosing appropriate ways to represent data</i>	321	259	346	-
18.B	18.B	The Making of a Model <i>Various ways to present mathematical models</i>	325	263	352	18.B
18.C	18.C	What a Wonderful World! <i>Using multiple representations to choose a model</i>	329	267	356	18.C
18.D	18.D	Mathematical Models <i>Limitations of models</i>	335	271	361	18.D

Student Resources	
Overview	1
5-Number Summary and Boxplots	3
Algebraic Terminology	5
Coordinate Plane	6
Dimensional Analysis	8
Equivalent Fractions	11
Four Representations of Relationships	13
Fractions, Decimals, Percentages	15
Length, Area, and Volume	17
Mean, Mode, Median	21
Multiplying and Dividing Fractions	25
Number-Word Combinations	28
Order of Operations	29
Probability, Chance, Likelihood, and Odds	30
Properties	32
Ratios and Fractions	35
Rounding and Estimation	36
Scientific Notation	37
Slope	38
Understanding Visual Displays of Information	40
Writing Principles	42