

**The Dana Center Mathematics Pathways © 2016 the Charles A. Dana Center at The University of Texas at Austin, with support from the Texas Association of Community Colleges**

All intellectual property rights are owned by the Charles A. Dana Center or are used under license from the Carnegie Foundation for the Advancement of Teaching. The Texas Association of Community Colleges does not have rights to create derivatives.

**Licensing for State of Texas Community Colleges**

Unless otherwise indicated, the materials in this resource are the copyrighted property of the Charles A. Dana Center at The University of Texas at Austin (the University) with support from the Texas Association of Community Colleges (TACC). No part of this resource shall be reproduced, stored in a retrieval system, or transmitted by any means—electronically, mechanically, or via photocopying, recording, or otherwise, including via methods yet to be invented—without express written permission from the University, except under the following conditions:

- a) *Faculty and administrators* may reproduce and use one printed copy of the material for their personal use without obtaining further permission from the University, so long as all original credits, including copyright information, are retained.
- b) *Faculty may reproduce multiple copies of pages for student use in the classroom*, so long as all original credits, including copyright information, are retained.
- c) *Organizations or individuals other than those listed above* must obtain prior written permission from the University for the use of these materials, the terms of which may be set forth in a copyright license agreement, and which may include the payment of a licensing fee, or royalties, or both.

**General Information About the Dana Center's Copyright**

We use all funds generated through use of our materials to further our nonprofit mission. Please send your permission requests or questions to us at this address:

Charles A. Dana Center  
The University of Texas at Austin  
1616 Guadalupe Street, Suite 3.206  
Austin, TX 78701-1222

Fax: 512-232-1855  
danaweb@austin.utexas.edu  
[www.dcmathpathways.org](http://www.dcmathpathways.org)

Any opinions, findings, conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of The University of Texas at Austin. The Charles A. Dana Center and The University of Texas at Austin, as well as the authors and editors, assume no liability for any loss or damage resulting from the use of this resource. We have made extensive efforts to ensure the accuracy of the information in this resource, to provide proper acknowledgement of original sources, and to otherwise comply with copyright law. If you find an error or you believe we have failed to provide proper acknowledgment, please contact us at danaweb@austin.utexas.edu.

Reproduced by Pearson from electronic files supplied by the author.

ISBN-13: 978-0-13-446748-1

ISBN-10: 0-13-446748-5

1 2 3 4 5 6—UP—19 18 17 16 15

We welcome your comments and suggestions for improvements. Please contact us at danaweb@austin.utexas.edu or at the mailing address above.

## About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales math and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved.

We work with our nation’s education systems to ensure that every student leaves school prepared for success in postsecondary education and the contemporary workplace—and for active participation in our modern democracy. We are committed to ensuring that the accident of where a student attends school does not limit the academic opportunities he or she can pursue. Thus, we advocate for high academic standards, and we collaborate with local partners to build the capacity of education systems to ensure that all students can master the content described in these standards.

Our portfolio of initiatives, grounded in research and two decades of experience, centers on mathematics and science education from prekindergarten through the early years of college. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help educators and education organizations adapt promising research to meet their local needs and develop innovative resources and systems that we implement through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

We have significant experience and expertise in the following:

- Developing and implementing standards and building the capacity of schools, districts, and systems
- Supporting education leadership, instructional coaching, and teaching
- Designing and developing instructional materials, assessments, curricula, and programs for bridging critical transitions
- Convening networks focused on policy, research, and practice

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see our homepage at [www.utdanacenter.org](http://www.utdanacenter.org).

## About the Dana Center Mathematics Pathways

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to improving student success and completion through implementation of processes, strategies, and structures based on four fundamental principles:

1. Multiple pathways with relevant and challenging mathematics content aligned to specific fields of study
2. Acceleration that allows students to complete a college-level math course more quickly than in the traditional developmental math sequence
3. Intentional use of strategies to help students develop skills as learners
4. Curriculum design and pedagogy based on proven practice

The Dana Center has developed curricular materials for three accelerated pathways—*Statistical Reasoning*, *Quantitative Reasoning*, and *Reasoning with Functions I and II* (a two- course preparation for Calculus). The pathways are designed for students who have completed arithmetic or who are placed at a beginning algebra level. All three pathways have a common starting point—a developmental math course that helps students develop foundational skills and conceptual understanding in the context of college-level course material.

In the first term, we recommend that students also enroll in a learning frameworks course to help them acquire the strategies—and tenacity—necessary to succeed in college. These strategies include setting academic and career goals that will help them select the appropriate mathematics pathway.

In addition to the curricular materials, the Dana Center has developed tools and services to support project implementation. These tools and services include an implementation guide, data templates and planning tools for colleges, and training materials for faculty and staff.

## Acknowledgments

The development of this course began with the formation of the DCMP **Curricular Design Team**, who set the design standards for the curricular materials of individual DCMP courses. The team members are:

Richelle (Rikki) Blair, Lakeland Community College (Ohio) Rob Farinelli, College of Southern Maryland (Maryland) Amy Getz, Charles A. Dana Center (Texas) Roxy Peck, California Polytechnic State University (California)	Sharon Sledge, San Jacinto College (Texas) Paula Wilhite, North Texas Community College (Texas) Linda Zientek, Sam Houston State University (Texas)
--	---

The Dana Center then convened faculty from each of the DCMP codevelopment partner institutions to provide input on key usability features of the instructor supports in curricular materials and pertinent professional development needs. Special emphasis was placed on faculty who need the most support, such as new faculty and adjunct faculty. The **Usability Advisory Group** members are:

Ioana Agut, Brazosport College (Texas) Eddie Bishop, Northwest Vista College (Texas) Alma Brannan, Midland College (Texas) Ivette Chuca, El Paso Community College (Texas) Tom Connolly, Charles A. Dana Center (Texas) Alison Garza, Temple College (Texas) Colleen Hosking, Austin Community College (Texas)	Juan Ibarra, South Texas College (Texas) Keturah Johnson, Lone Star College (Texas) Julie Lewis, Kilgore College (Texas) Joey Offer, Austin Community College (Texas) Connie Richardson, Charles A. Dana Center (Texas) Paula Talley, Temple College (Texas) Paige Wood, Kilgore College (Texas)
--	--

Some of the content for this course is derived from the Quantway™ course, which was developed under a November 30, 2010, agreement by a team of faculty authors and reviewers contracted and managed by the Charles A. Dana Center at The University of Texas at Austin under sponsorship of the Carnegie Foundation for the Advancement of Teaching. Quantway™ is copyright © 2011 by the Carnegie Foundation for the Advancement of Teaching and the Charles A. Dana Center at The University of Texas at Austin. Statway™ and Quantway™ are trademarks of the Carnegie Foundation for the Advancement of Teaching.

Development of the *Foundations for Mathematical Reasoning* course was made possible by a grant from the Kresge Foundation. Additional funding and support for the Dana Center Mathematics Pathways was provided by Carnegie Corporation of New York, Greater Texas Foundation, Houston Endowment, Texas legislative appropriations request, and TG.

Any opinions, findings, conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of these funders or The University of Texas at Austin. This publication was also supported through a collaboration between the Charles A. Dana Center, Texas Association of Community Colleges, and Pearson Education, Inc.

### *Project Lead and Authors*

- April Andreas, associate professor, engineering, McLennan Community College (Texas)
- Pauline Chow, senior professor, mathematics, Harrisburg Area Community College (Pennsylvania)
- Christina Hoffmaster, instructor, Vernon College (Texas)
- Connie J. Richardson, advisory lead, Charles A. Dana Center
- Francisco Savina, curriculum lead and lead author, Charles A. Dana Center
- Randell Simpson, resident mathematics instructor, Temple College (Texas)

*Charles A. Dana Center Project Staff*

Adam Castillo, graduate research assistant  
Heather Cook, project manager  
Ophella C. Dano, lead production editor  
Rachel Jenkins, consulting editor  
Phil Swann, senior designer  
Sarah Wenzel, administrative associate  
Amy Winters, lead editor (freelance)

*Math Faculty Reviewers from the DCMF Codevelopment Teams*

Alamo Colleges–Northwest Vista College, San Antonio, Texas  
Austin Community College, Austin, Texas  
Brazosport College, Lake Jackson, Texas  
El Paso Community College, El Paso, Texas  
Kilgore College, Kilgore, Texas  
Lone Star College–Kingwood, Kingwood, Texas  
Midland College, Midland, Texas  
South Texas College, McAllen, Texas  
Temple College, Temple, Texas

**Authors and Reviewers Contracted by the Dana Center**

<b>Quantway™ version 1.0 (2011)</b>	<b>Foundations version 1.0 (2013)</b>
Stuart Boersma, professor of mathematics, Central Washington University (Washington)	Eileen Faulkenberry, associate professor of mathematics, Tarleton State University (Texas)
Mary Crawford-Mohat, associate professor in mathematics, Onondaga Community College (New York)	Christina Hoffmaster, Vernon College (Texas)
Margaret (Peg) Crider, professor in mathematics, retired, Lone Star College (Texas)	Tom Faulkenberry, assistant professor of mathematics, Tarleton State University (Texas)
Caren Diefenderfer, professor of mathematics, Hollins University (Virginia)	Liz Scott, San Augustine Independent School District (Texas)
Amy Getz, manager of community college services, Charles A. Dana Center, University of Texas at Austin (Texas)	Jack Rotman, Lansing Community College (Michigan)
Michael Goodroe, lecturer of mathematics and learning support liaison, Gainesville State College (Georgia)	Jeff Morford, Henry Ford Community College (Michigan)
Cinnamon Hillyard, assistant professor in mathematics, University of Washington Bothell (Washington)	Constance Elko, Austin Community College (Texas)
Robert Kimball, professor in mathematics, retired, Wake Technical Community College (North Carolina)	
Deann Leoni, professor of mathematics, Edmonds Community College (Washington)	
Michael Lundin, professor of mathematics, Central Washington University (Washington)	
Bernard L. Madison, professor in mathematical sciences, University of Arkansas (Arkansas)	
Jeffrey Morford, professor of mathematics, Henry Ford Community College (Michigan)	
Jane Muhich, managing director for community college program development, and director of productive persistence, Carnegie Foundation for the Advancement of Teaching (California)	
Julie Phelps, professor of mathematics, Valencia College (Florida)	

**Pearson Education, Inc. Staff**

Vice President, Editorial Jason Jordan  
 Strategic Account Manager Tanja Eise  
 Editor in Chief Michael Hirsch  
 Senior Acquisitions Editor Dawn Giovannello  
 Editorial Assistant Megan Tripp  
 Digital Instructional Designer Tacha Gennarino  
 Manager, Instructional Design Sara Finnigan  
 Senior Project Manager Dana Toney  
 Director of Course Production, MyMathLab Ruth Berry  
 MathXL Content Developer Kristina Evans

Project Manager Kathleen A. Manley  
 Project Management Team Lead Christina Lepre  
 Product Marketing Manager Alicia Frankel  
 Senior Author Support/Technology Specialist Joe Vetere  
 Rights and Permissions Project Manager Gina Cheselka  
 Procurement Specialist Carol Melville  
 Associate Director of Design Andrea Nix  
 Program Design Lead Beth Paquin  
 Composition Dana Bettez

## Contents

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
-	-	Curriculum Overview	ix	xiii	-	-
-	-	Prep Week <i>Ideas for your syllabus</i>	xxvii	-	-	-
1.A	-	How big is a billion? <i>Quantitative reasoning, large numbers</i>	1	1	1	1.A
1.B	-	Building a learning community <i>Student success focus</i>	-	-	6	1.B
1.C	-	How big is a billion? (continued) <i>Quantitative reasoning, large numbers</i>	3	3	13	1.C
1.D	-	Building a learning community (continued) <i>Student success focus</i>	-	-	18	-
2.A	2.A	Doubling population <i>Large numbers, doubling, rates</i> <i>Introduction to note taking</i>	5	5	29	2.A
2.B	2.B	Scientific notation <i>Representing numbers in scientific notation,</i> <i>converting back to standard notation</i>	8	7	36	2.B
2.C	2.C	Ratios in water use <i>Large numbers, ratios, scientific notation</i>	11	9	42	2.C
2.D	2.D	Analyzing water footprints <i>Scientific notation, ratios</i>	14	11	47	2.D
3.A	3.A	Large numbers in the media <i>Misinformation, strategies for testing information</i>	17	13	52	3.A
3.B	3.B	Seeking help <i>Student success focus</i> <i>Campuses resources, seeking and offering help</i> <i>Note: If this course is paired with the Dana Center</i> <i>Mathematics Pathways' learning frameworks</i> <i>course, this lesson should be inserted after the</i> <i>frameworks class has completed the Scavenger</i> <i>Hunt activity.</i>	21	16	56	-

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
3.C	3.C	Estimating sale prices <i>Estimation, benchmark percentages</i>	23	17	61	3.C
3.D	3.D	Calculating sale prices <i>Value of estimation and calculation, calculating percentages</i>	25	19	66	3.D
3.E	3.E	Developing self-regulation <i>Student Success focus, self assessment</i>	27	21	70	3.E
4.A	4.A	Budgeting operations <i>Use of order of operations, properties, pie charts</i>	29	23	75	4.A
4.B	4.B	Budgeting with spreadsheets <i>Algebraic reasoning through the use of spreadsheet formulas</i>	32	25	79	4.B
4.C	4.C	Graph analysis <i>Introduction to visual displays, misleading scale, relative change over time</i>	35	27	83	4.C
4.D	4.D	Using graphs to understand change <i>Relative size</i>	38	29	89	4.D
5.A	5.A	Displaying table data <i>Stem-and-leaf plots, back-to-back comparison</i>	41	32	94	5.A
5.B	5.B	Relative frequency tables <i>Construct and analyze frequency, relative frequency, cumulative frequency</i>	44	34	99	5.B
5.C	5.C	Displaying data: Histograms <i>Convert frequency tables from 5.B into histograms</i> <i>Note: Optional data display project</i>	48	37	104	5.C
5.D	5.D	Shapes of distributions <i>Using dotplots to describe the shapes of distributions</i>	52	39	110	5.D
6.A	6.A	Measures of central tendency <i>Mean, median, mode, conclusions from statistical summaries, create data sets to meet criterion</i>	56	41	114	6.A
6.B	6.B	Brain power <i>Student success focus</i> <i>How the brain learns</i>	60	43	118	6.B
6.C	6.C	Making decisions with data <i>Use statistical summaries to make decisions</i>	62	45	122	6.C

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
6.D	6.D	Boxplots <i>Analyze a data set via 5-number Summary</i>	64	46	126	6.D
7.A	7.A	The credit crunch <i>Reading strategies to understand financial information</i>	67	48	131	7.A
7.B	7.B	More credit crunch <i>Estimate and calculate credit card Interest</i>	69	49	137	7.B
7.C	7.C	A taxing situation <i>Understand and complete tax forms</i>	72	51	141	7.C
7.D	7.D	A taxing situation (continued) <i>Convert the tax instructions into mathematical expressions</i>	75	53	147	7.D
8.A	8.A	What is the risk? <i>Absolute and relative measures of risk, comparing fraction and decimal forms</i>	79	55	151	8.A
8.B	8.B	An apple a day <i>Evaluate measures of risk</i>	82	57	155	8.B
8.C	8.C	Reducing the risk <i>Percentages, risk reduction</i>	86	60	160	8.C
8.D	8.D	Is reducing the risk worth it? <i>Absolute and relative change, ambiguity in talking about change in a quantity</i>	90	62	164	8.D
9.A	9.A	Comparing categorical data <i>Two-way tables, importance of base value</i>	92	64	168	9.A
9.B	9.B	Interpreting percentages <i>Analysis of abstract information</i>	94	66	173	9.B
9.C	9.C	Do you trust the test? <i>Two-way tables, accuracy in test results</i>	98	69	177	9.C
9.D	9.D	Do you trust the test? (continued) <i>Two-way tables, false-positive and false-negative test results</i>	101	71	190	9.D
10.A	10.A	Population density <i>Ratios, proportional reasoning</i>	104	73	195	10.A
10.B	10.B	Density proportions <i>Scaling, dimensional analysis</i>	106	74	201	10.B



Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
10.C	10.C	State population densities (optional) <i>Estimation strategies</i> <i>Note: Optional spreadsheet use</i>	109	75	206	10.C
10.D	10.D	Apportionment <i>Effect of relative change on Representation</i>	114	78	213	10.D
11.A	11.A	Formulating a plan <i>Variables, evaluate expressions (geometric formulas)</i>	117	81	218	11.A
11.B	11.B	The costs of geometry <i>Building on work with formulas</i>	119	82	222	11.B
11.C	11.C	Modifying and combining formulas <i>Semicircle area, volume</i>	121	84	225	11.C
12.A	12.A	Texting distance <i>Dimensional analysis</i>	123	85	229	12.A
12.B	12.B	The cost of driving <i>Unit rates to compare two options</i>	125	86	234	12.B
12.C	12.C	The true cost of driving <i>Multiple pieces of information, multiple step work</i>	127	88	241	12.C
12.D	12.D	Can the true cost vary? <i>Concrete to abstract approach to a system</i>	130	90	246	12.D
13.A	13.A	Algebra reaction (optional) <i>More complex, unfamiliar formulas</i>	132	91	252	13.A
13.B	13.B	Breaking down a formula <i>Reading to understand/apply a complex formula</i>	134	93	257	13.B
13.C	13.C	Analyzing change in variables <i>Analyze effect of changing values of one variable while other variables remain fixed</i>	136	95	263	13.C
13.D	13.D	Analyzing change in variables (continued) <i>Analyze effect of changing values of one variable while other variables Remain fixed</i>	139	96	268	13.D
14.A	14.A	Body mass index <i>Evaluate and record sequence of steps (multiplication and division only)</i>	141	97	273	14.A
14.B	14.B	Target weight <i>Given target output, undo steps to find input value</i>	143	98	277	14.B

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
14.C	14.C	Blood alcohol content <i>Evaluate and record sequence of steps (multiplication/division and addition/subtraction)</i>	145	100	281	14.C
14.D	14.D	Balancing blood alcohol <i>Given target output, undo steps to find input value</i>	147	102	285	14.D
15.A	15.A	Proportional reasoning in art <i>Determine whether proportions are equivalent</i>	149	104	291	15.A
15.B	15.B	Proportion solutions <i>Solve algebraic proportions</i>	151	106	303	15.B
15.C	15.C	Solving equations <i>Additional practice with solving</i>	153	107	307	15.C
15.D	15.D	More work with equations (optional) <i>Equations chosen by instructor</i>	156	109	311	15.D
15.E	15.E	Proportional viewing (optional) <i>Equations chosen by instructor</i>	157	110	316	15.E
16.A	16.A	Describing rates <i>Slope as a unit rate (<math>y</math>-intercept = 0)</i>	159	112	320	16.A
16.B	16.B	Comparing rates <i>Compare/contrast slopes (<math>y</math>-intercept = 0)</i>	163	114	324	16.B
16.C	16.C	Interpreting change <i>Calculating slope from points, (<math>y</math>-intercept <math>\neq 0</math>)</i>	166	116	328	16.C
16.D	16.D	Where do we start? <i>Calculating <math>y</math>-intercept by backing out the effect of the variable term</i>	168	117	332	16.D
16.E	16.E	Predicting costs <i>Formalizing the calculation of <math>y</math>-intercept</i>	172	119	336	16.E
17.A	17.A	Expressing linear relationships <i>Graphs, tables, algebraic, and verbal representations Intersecting lines</i>	176	121	342	17.A
17.B	17.B	Making the call <i>Using multiple representations to make decisions</i>	180	124	346	17.B
17.C	17.C	Close enough <i>Scatterplots and trend lines</i>	184	126	350	17.C
17.D	17.D	Predicting budget increases (optional) <i>Using a trend line to interpolate and extrapolate</i>	189	130	356	17.D

Lesson	Preview Assignment	Lesson Title and Description	In-Class Activities with Answers	In-Class Activities (Student)	Lesson Planning Suggestions	Practice Assignment
18.A	18.A	Pricing Your product <i>Developing formulas for product markups and discounts</i>	192	132	360	18.A
18.B	18.B	Backing out the sales tax <i>Determining the original amount</i> <i>Note: Optional mini-project available</i>	195	134	366	18.B
18.C	18.C	Compound interest makes cents <i>Develop exponential formula for annual interest</i>	198	136	372	18.C
18.D	18.D	Long-term growth <i>Continue work with annual compounding</i>	200	138	376	18.D
19.A	19.A	More compounding <i>Compounding monthly then abstract to general form</i>	202	140	381	19.A
19.B	19.B	Depreciation <i>Exponential decay</i> <i>Note: Optional project available</i>	204	142	386	19.B
19.C	19.C	Payday loans <i>Effect of extremely high interest</i>	206	143	393	19.C
19.D	19.D	Neither a borrower ... <i>Linear loan model</i> <i>Note: This lesson can be combined with 19.C by having different groups complete each lesson.</i>	208	144	399	19.D
19.E	19.E	Credit card repayment (optional) <i>Effect of making minimum payments</i>	210	145	404	19.E

Student resources	
Overview	1
5-Number Summary and Boxplots	3
Algebraic Terminology	5
Coordinate Plane	6
Dimensional Analysis	8
Equivalent Fractions	11
Four Representations of Relationships	13
Fractions, Decimals, Percentages	15
Length, Area, and Volume	17
Mean, Mode, Median	21
Multiplying and Dividing Fractions	25
Number-Word Combinations	28
Order of Operations	29
Probability, Chance, Likelihood, and Odds	30
Properties	32
Ratios and Fractions	35
Review Yourself for Exam 1	36
Review Yourself for Exam 2	38
Review Yourself for Exam 3	42
Review Yourself for Exam 4	46
Review Yourself for the Final Exam	49
Rounding and Estimation	56
Scientific Notation	57
Slope	58
Understanding Visual Displays of Information	60
Writing Principles	62