What Is Rigor in Mathematics Really?

Whenever major developments occur in any discipline, unintended consequences arise that need to be addressed. When the focus for entry-level mathematics shifted from access to success over a decade ago, it catalyzed the advocacy and implementation of accelerated multiple mathematics pathways alongside the algebraic-intensive pathway. Subsequent increases in success have been exciting, showing upwards of three times the success rates for students in one third of the time for some programs. With these startling increases came a widespread concern about maintaining rigor within the discipline.

In response to this concern, the Charles A. Dana Center engaged in a study of the meaning and intention of rigor in mathematics education. This paper first explores the meaning of rigor in mathematics education through the synthesis of interviews with leading mathematicians and educators, and presents a review of the literature in higher education and K–12. It concludes by offering recommendations for a shared definition of rigor and its implications for curriculum and instruction.

THE INCREASING IMPORTANCE OF MATHEMATICS

In the past 20 years, mathematics has become increasingly important to a growing number of fields of study and their related professions. In 1998, the National Science Foundation released its *Report of the Senior Assessment Panel for the International Assessment of U.S. Mathematical Sciences*, which listed 11 fields of study that interfaced with mathematics including physics, chemistry, economics, and manufacturing. The National Research Council's 2013 report, *The Mathematical Sciences in 2025*, expanded this number to 21 fields and predicted continued growth. New fields of study that were added to the list included entertainment, social networks, ecology, computer science, information processing, marketing, and defense, demonstrating the growth in jobs for "workers with mathematical science skills at all degree levels, regardless of their field of training" (National Research Council, 2013, p. 69).

As a result, numerous calls have been made for a change in mathematics curricula (Saxe & Braddy, 2015).[1] *The Mathematical Sciences in 2025* noted that mathematics departments "have not kept pace with the large and rapid changes in how the mathematical sciences are used in science, engineering, medicine, finance, social science, and society at large" (National Research Council, 2013, p. 10). The report suggested that "[n]ew educational pathways for training in the mathematical sciences need to be created—for students in mathematical sciences departments, for those pursuing degrees in science, medicine, engineering, business, and social science, and for those already in the workforce needing additional quantitative skills" (p. 11).

As the field of mathematics was wrestling with economic demands, it was also gaining awareness of the need to improve student success rates. A growing body of research demonstrated that traditional developmental sequences and college gateway courses in mathematics proved to be barriers to student success. For example, in the watershed national study conducted with Achieving the Dream institutions, an estimated 60 percent of incoming two-year college students were placed into at least one development math course each year but only 20 percent of these students completed a college-level math course (Bailey, Jeong, & Cho, 2010). In addition, approximately half of all students who enrolled in traditional college algebra failed the course (Saxe & Braddy, 2015). Solutions to improve these problems centered on developing additional pathways and restructuring course sequences to improve course and degree completion. Among these solutions was the development of quantitative reasoning and statistical mathematics pathways, increasing the relevance of mathematics for many students in their chosen programs of study and careers.

> **"Mathematics has become increasingly important to a growing number of fields of study and their related professions."**

MATHEMATICS PATHWAYS

To address these economic and student success imperatives, the Charles A. Dana Center has been working with states across the country to change the structure of entry-level mathematics to include small numbers of multiple mathematics pathways that align with college students' programs of study and to ensure that underprepared students are successful in completing their first college-level math course within their first year of college (Dana Center Mathematics Pathways, n.d.). As of early 2018, more than 15 states are implementing mathematics pathways

on a broad scale. Along with several other organizations, the Dana Center is also in the early stages of redesigning high school mathematics to better align with evolving definitions of college readiness. Several national initiatives are focused on the development of or support for mathematics pathways including the Carnegie Foundation for Advancement in Teaching's Quantway™ and Statway™, California Acceleration Project (CAP), Advancing Math Pathways for Student Success (AMPSS), and Transforming Postsecondary Education in Mathematics (TPSE).

These state and national projects share similar goals: 1) to significantly reduce the high failure rate in developmental and gateway math courses; 2) to offer courses directly relevant to students' broad educational programs; and 3) to provide a range of mathematics pathways so that all students have access to and success in building the mathematical skills essential for their lives and chosen professions.

As these efforts are being scaled widely in schools and mathematics departments across the country, the Dana Center has sought to learn from K–12 and higher education faculty about their experiences in teaching students to use these new models. While there are many successes to celebrate, we have also learned about challenges. Among the things we have learned, one concern stands out: that changing mathematics curricula away from an emphasis on algebra may lead to less rigorous courses. At first, the remedy to this concern seems obvious: to ensure that the elements of rigor are incorporated into all courses. However, as we explored this issue in more depth, we realized that there was no consensus on a definition of rigor to apply in addressing the effectiveness of mathematics education.

DIFFERENT VIEWS ABOUT RIGOR

Rigor is understood and used differently by both the K–12 and higher education sectors. Four primary views of rigor have emerged from our analysis. The first view of rigor in mathematics is that it is the same as "mathematical rigor," which is the use of logical deductions from stated hypotheses to prove theorems. The second view of rigor focuses on adhering to traditionally prescribed content. Mathematics departments often determine course content by committee and, to be inclusive of its members, agree to a long list of topics and concepts that may undermine flexible adaptation to student needs. The third view finds rigor to be synonymous with increased difficulty and more challenging content. Rigor is associated with "advanced courses," comprehensive presentation and lectures on all possible topics and techniques, high-stakes summative tests, low pass rates and/or low grades. A final perception is that rigor and college algebra may be used interchangeably. In this view, college algebra stands as a proxy for rigor in mathematics in part because skills developed in the course open many other mathematical options to students and because mastery of algebraic skills signifies readiness to proceed with a student's mathematical education.

Until the advent of multiple mathematics pathways, virtually all students were required to take college algebra regardless of how well it aligned to their career aspirations and goals. *A Common Vision* (Saxe & Braddy, 2015) described the issue in the following way:

> Current college algebra courses serve two distinct student populations: 1) the overwhelming majority for whom it is a terminal course in mathematics, and 2) the relatively small minority for whom it is a gateway to further mathematics. Neither group is well-served by the traditional version of the college algebra course. There is a mismatch between a curriculum designed to prepare students for calculus and the reality that only a small proportion of these students subsequently enroll in calculus. (p. 13)

From the Dana Center's point of view, using college algebra as a proxy for rigor unnecessarily limits the conversation about rigor to only one area of mathematics. When college algebra is used as a proxy for rigor, mathematical modeling, geometric and numeric methods, statistics, and quantitative reasoning often are labeled as not rigorous. Yet we know that arithmetic, quantitative reasoning, algebra, and statistics can all be rigorous courses. The ability to perform algebraic manipulations, a primary objective of college algebra, in and of itself does not provide clear information about what rigor is. Consequently, understanding college algebra is not the equivalent of rigor in all mathematical fields.

We believe the lack of clarity and consensus about the term rigor matters for two reasons. A shared definition of rigor offers the opportunity to ground the conversation in aligning K–12 and higher education. Such a definition allows both sectors to identify and move towards common expectations, thereby creating a more consistent experience in mathematics for students across the K–16 continuum. Too often, students experience sudden shifts in expectations at each level of transition: when they leave elementary school, when they leave middle school, and when they leave high school. Multiple understandings and uses of rigor undermine the mathematical learning experience available to students. Not teaching in a rigorous way may disproportionately affect some students more than others, particularly those from historically disadvantaged and traditionally underserved groups.

To better understand the meaning of rigor, we conducted a literature review and interviewed mathematicians at national conferences including the American Mathematical Association of Two-Year Colleges (AMATYC) in November 2016 and November 2017, the National Association of Developmental Educators (NADE) in March 2017, and the Michigan Math Summit in June 2017. In addition, we conducted a series of focused interviews with the following individuals:

David Bressoud, DeWitt-Wallace Professor, Macalester College, Lead Principal Investigator of MAA National Study of College Calculus, and Director of the CBMS

Diane Briars, Mathematics Education Consultant and Past President, National Council of Teachers of Mathematics (NCTM)

Mark Green, Professor Emeritus and Distinguished Research Professor, University of California, Los Angeles

Matt Larson, President, National Council of Teachers of Mathematics (NCTM)

Bill McCallum, Founder, Illustrative Mathematics and University Distinguished Professor of Mathematics, University of Arizona

Roxy Peck, Professor Emerita, California Polytechnic State University–San Luis Obispo

Myra Snell, Professor of Mathematics, Los Medanos College and Co-Founder of the California Acceleration Project (CAP)

Doug Sovde, Director of K–12 Education Strategy, Policy and Services, The Charles A. Dana Center

Uri Treisman, Professor of Mathematics, The University of Texas at Austin, and Executive Director of The Charles A. Dana Center

While there are gaps in the literature about rigor in mathematics, especially from the higher education perspective, there was remarkable consistency among interviewees about what constitutes rigor. Notably, the definition that emerged was not dependent on sector (K–12 or higher education), nor was it defined based on course content or course level. Rather, the definition centered on whether students are able to engage with and use their mathematical knowledge. Drawing on insights and observations from these sources, this paper offers a shared definition of rigor in mathematics for K–12 and higher education, explores the importance of rigor for all students, places rigor in the context of an effective mathematics course, and provides recommendations for how to implement this definition across sectors.

RIGOR IN MATHEMATICS: A SHARED DEFINITION FOR K–12 AND HIGHER EDUCATION

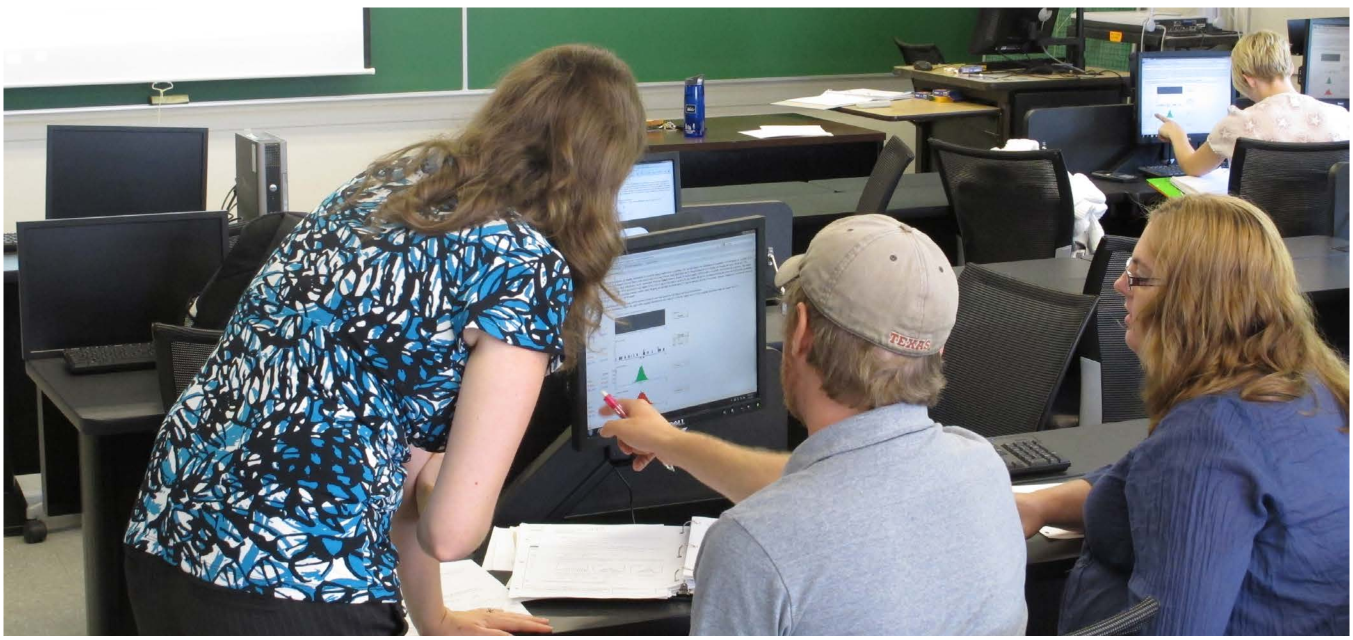
As we investigated the research on rigor in mathematics, we sought a definition that would serve the mathematics community from kindergarten through graduate school. Unlike our interviews with mathematicians, our literature review revealed a wide variety in definitions. For example, rigor has been defined as “learning in which scholars demonstrate a thorough in-depth mastery of challenging tasks to develop cognitive skills through reflective thought, analysis, problem-solving, evaluation, or creativity” (Houston Independent School District, 2008). Other research has

> **“Rigor in mathematics is a set of skills that centers on the communication and use of mathematical language.”**

divided rigor into two categories, offering one definition for content and another for instruction (Hull, Balka, & Harbin Miles, 2013a, 2013b, 2013c). Rigor in mathematics has also been defined as three developmental stages: pre-rigor, rigor, and post-rigor (Tao, n.d.). Finally, rigor has been defined as a “deep, authentic command of mathematical concepts” and includes procedural fluency and skills, conceptual understanding, and application (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010a). For many in both K–12 and higher education, these definitions outline the essential qualities of good instruction, which are important for learning mathematics but are not necessarily rigor.

The literature also described content and process standards in K–12 and higher education. These sources provided valuable insight and informed our definition of rigor. Here, we offer some examples of standards that were helpful to our thinking. Early K–12 expectations included emphasis on problem solving, reasoning, communication, and building connections (NCTM, 2000). Later, standards evolved to include reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling, attention to precision, and identifying and using structure (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Similarly, higher education standards focused on assessing the correctness of solutions, stating problems carefully, articulating assumptions, understanding the importance of precise definition, and reasoning logically (Saxe & Braddy, 2015). Mathematical standards for the first two years of college highlighted utilizing structure, making sense of and solving problems, communicating mathematically, and doing so with precision (AMATYC, 2017).

Synthesizing the literature and information from our interviews, we conclude that rigor in mathematics is a set of skills that centers on the communication and use of mathematical language. Specifically, students must be able to communicate their ideas and reasoning with *clarity* and *precision* by using the appropriate mathematical symbols and terminology. Here, clarity means logical thinking, inference, and deduction. Precision means detailed and careful use of mathematical language. Clarity and precision are necessary so that students can demonstrate to themselves and others that their mathematical approach works and is appropriate in the context for which it has been used. For a given problem, students who have been exposed to rigor in their coursework are able to ask and answer questions such as *What procedure is appropriate? Will that procedure work? How do we know?* It is important to note that this definition of rigor does not preclude or eliminate problem-solving or the creativity that goes with it. On the contrary, rigor frames those processes through a lens that unifies mathematics across the education continuum.



This definition recognizes that mathematics is a technical language separate from other languages. As a result, mathematical language needs to be taught intentionally because mathematicians are using language in a specialized way. For students to be successful in mathematics, they must learn to communicate fluently in that language and demonstrate the ability to use that language in the performance of mathematics.

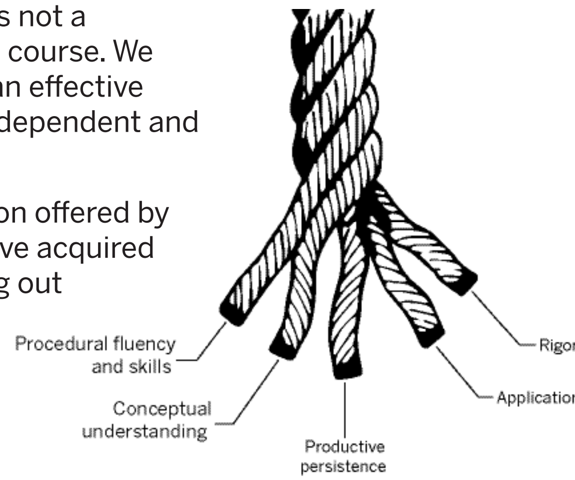
Mathematical language is learned by doing mathematics in much the same way that language is learned: by immersion in the culture in which the language is used and central to everyday life. Rigor is more than teaching mathematical vocabulary. Rather, it is helping students develop the habits of mind that allow them to communicate and think with the precision and clarity that mathematics requires. In a course where rigor is taught and integrated into coursework, students should understand the underlying logic of what they are learning from a mathematical point of view. Coursework reflects good mathematical practice by designing classroom activities and assignments that model rigor. Instructors can do this in a variety of different ways, including:

- Demonstrating that premises of the course are solidly based.
- Emphasizing proper notation in their explanations.
- Supporting students as they develop and use precise mathematical language.
- Giving students feedback about the clarity of their reasoning.
- Encouraging alternative approaches.
- Asking students about the reasonableness of their answers.
- Including new situations where students need to extend their understanding.

Finally, the definition of rigor in mathematics offered here is not dependent on sector (K–12 or higher education) or course content or level. Elementary students learning arithmetic can learn how to express and should be expected to express clarity and precision in their ideas, just as college students learning statistics, for example, are expected to do.

UNDERSTANDING RIGOR IN THE CONTEXT OF AN EFFECTIVE MATHEMATICS COURSE

While rigor is important for mastery of mathematics, it is not a replacement for other elements essential to an effective course. We propose that one way to understand the role of rigor in an effective mathematics course is to imagine a rope[2] with five interdependent and intertwined strands. The strands of the rope include:



Procedural fluency and skills – Using the definition offered by the National Research Council (2001), students have acquired procedural fluency when they have "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (p. 116). In arithmetic, for example, this means students are able to add, subtract, multiply, and divide numbers accurately and with confidence.

Conceptual understanding –Students demonstrate "comprehension of mathematical concepts, operations, and relations" (National Research Council, 2001, p. 116). Two examples illustrate this idea. In algebra, students show an understanding of when to use the quadratic equation, how to solve it, and how to interpret and use the results. In statistics, students show an understanding of when to use a particular inference test, state the assumptions, and demonstrate how to interpret and use the results.

Productive persistence –Students use tenacity in solving problems and employ a variety of effective learning strategies to successfully engage with coursework (Carnegie Foundation for the Advancement of Teaching, n.d.; Sylva & Whyte, 2013). For example, they are engaged in finding effective resources, motivated to wrestle with a problem until they can solve it, and interested in evaluating their work to find errors.

Application – Students correctly apply mathematical knowledge in new situations (National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010a). For example, application of algebra in calculus such as differentiating exponentials.

Rigor – Students use mathematical language to communicate effectively and to describe their work with clarity and precision. Students demonstrate that what they have done works, when it works, and why the procedure they selected is appropriate. The student can answer the question, "How do we know?" In calculus, for example, students should associate derivatives not with the rote procedure (e.g., $dx^n/dx = nx^{n-1}$) but with the concept of instantaneous rate of change. Similarly, in the statistics example above, students should associate the inference test not with the calculation of the t-score but with the concept of population, sample characteristics, and other questions that are generated from the calculation.

RIGOR AS A MATTER OF EQUITY AND OPPORTUNITY

The increasing demands of a data-driven society require more mathematical skills and deeper understanding of when and how to utilize those skills. People need to apply those skills and knowledge both in their personal and work lives. For example, informed voters should understand polls and make judgments about candidates using polling data. Retirement planning is no longer a matter of relying on an employer to provide a pension, but likely requires careful management of investments. Jobs in every sector—from lawyers to teachers to journalists and many others—require knowledge of mathematical skills to understand the implications of large data sets. Other professions, such as nursing, require the use of arithmetic as well as algebra to calculate medicine dosages and IV drip rates, determine drug titration, and convert between the customary and metric systems.

In each of these cases, it is important for people to be able to think deeply about the mathematical calculations they are asked either to assess or to make and not simply to rely on automaticity of algorithms. In short, rigor is important not only for the mathematical benefits to students, but for the mathematical and quantitative reasoning challenges that are embedded in the daily lives of productive and engaged members of a data-driven society.

While rigor matters for every student, it is particularly important for students from historically disadvantaged and underserved groups. These students are more likely to encounter mathematics courses and instruction that are focused on procedural fluency and adherence to rules rather than the skills of reasoning, logical thinking, argumentation, and precision that lead to deeper engagement in mathematics. Such skills are not only essential to be successful, but they are also important for the management of daily life and citizenship as noted above.

During the past two years, our conversations with mathematics faculty have revealed wide divergence in both what rigor means and whether it is important. For many faculty, the importance of teaching all students rigor is clear. Other faculty express interest in rigor but want a better understanding of what rigor is and how to incorporate it into their teaching. However, some faculty, especially those teaching lower division courses, openly express that they do not see the need to teach rigor to their students. After all, they argue, their students are not going to be mathematics majors and need only basic skills. Yet the opportunity to learn rigor may help students not only determine if they will pursue mathematics beyond minimum requirements and gateway courses, but inform their citizenship and advance their careers.

Students respond to the opportunities faculty provide and make intentional choices based on their classroom experiences. If a course is not rigorous (i.e., if students are not taught the language of mathematics and how to use it), they will not understand where an idea, theorem, or solution came from or the truth of it. If students do not learn the truth of a given idea, they will not believe, use, or enjoy the concepts being taught. Simply, students will not recognize the beauty and power of mathematics. If students do not enjoy mathematics and understand its relevance in their lives, it is unlikely that they will pursue mathematics or the growing number of fields with which mathematics interfaces as a career.

Ultimately, students come to a crossroads. Will they pursue mathematics when they do not understand and do not enjoy the work? At this moment, and perhaps similar moments thereafter, is where the decision not to continue with mathematics is made—not because students are not interested but because they have not been given the opportunity to learn the language of mathematics and the insight it brings.

RECOMMENDATIONS FOR MOVING FORWARD WITH A SHARED DEFINITION OF RIGOR

The definition of rigor offered here has implications for curriculum, coursework, and instruction from kindergarten through graduate school. The Charles A. Dana Center offers five recommendations below to encourage further exploration with and to help move this shared definition of rigor forward. We understand that this is the beginning of the conversation and we look forward to the opportunity to work with others to operationalize this definition in the future.

Recommendation 1: Provide faculty, administrators, and system leaders with professional development opportunities. While there is wide variation in professional development for K–12 and higher education faculty, it is important that mathematics faculty and system leaders have the opportunity to consider the shared definition offered here and its implications for all students. Rigor in mathematics is not simply a matter of revised curriculum, although this is important as noted in Recommendation 2 below. It is the instruction and the modeling of rigor that bring the definition and the opportunities it offers to life. A mathematics course may appear to be rigorous based on the textbook table of contents or the course syllabus but the ultimate test of instruction is whether students leave the course with rigor and its attendant skills embedded in their thinking.

Recommendation 2: Consider revisions to curricula that embrace rigor in mathematics. In partnership with instruction that models rigor, faculty need curricula that provide meaningful opportunities for students to learn mathematical language and develop strong skills in how to use it. Revisions to curricula should expect students to reason with clarity and precision. These revisions might include additional classroom time devoted to reasoning and argumentation skills. As with all mathematical skills, revisions should focus on skill development and practice both inside and outside of the classroom.

Recommendation 3: Consider ways to utilize technology in the integration of rigor in mathematics. As faculty and curriculum designers factor rigor in mathematics into their coursework and content, they may want to consider what would be appropriate uses of technology in the process of doing mathematics. Students who have developed a deep awareness of language and how to use it may the benefit from technological tools that help illustrate mathematical ideas and concepts.

Recommendation 4: Encourage faculty and system leaders to engage in conversations with colleagues across the educational continuum. Rigor in mathematics is premised on understanding the nature of mathematical language and how to use it. Rigor is also subject to a wide variety of interpretations. To help bring cohesion to the mathematics community and related fields, faculty and system leaders may want to consider a series of focused conversations exploring a shared definition of rigor in mathematics. In particular, these discussions may benefit from including faculty members from disciplines across an institution.

Recommendation 5: Encourage faculty and system leaders to examine and revise their understanding of rigor as it relates to course requirements for graduation, admissions, and placement. "Algebra II" has served as a symbolic proxy for rigor at both state-level high school graduation and higher education admissions requirements. Substantively, this course has served two main purposes for students: 1) to complete their learning of fundamental algebraic concepts begun in Algebra I and Geometry courses; and 2) to prepare them for success in the Calculus pathway. As multiple mathematics course pathways become more widely available to students in postsecondary settings, the legacy Algebra II course ought not be considered as the sole gateway to college. These mathematics pathways offer relevant and equally challenging high school course options that include algebra and that embody rigor. Employing the definition of rigor offered here, Quantitative Reasoning and Statistics ought to be considered as equivalent alternatives to Algebra II.

CONTINUING THE CONVERSATION

For the Dana Center, this paper is the beginning of a larger conversation about the importance of rigor in mathematics. We welcome your feedback and hope you will join us in this dialogue. For more information contact Connie Richardson, connie.richardson@austin.utexas.edu.

ENDNOTES

[1]This report, published by Mathematical Association of America, is based on seven curricular guides from five professional associations (American Mathematical Association of Two-Year Colleges, American Mathematical Society, American Statistical Society, Mathematical Association of America, and Society for Industrial and Applied Mathematics) that "call for multiple pathways into and through mathematical sciences curricula" (Saxe & Braddy, 2015, p. 13).

[2]The rope metaphor is adapted from the National Research Council's report *Adding It Up: Helping Children Learn Mathematics* (2001).

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The University of Texas at Austin
Charles A. Dana Center

About the Charles A. Dana Center at The University of Texas at Austin

The Dana Center develops and scales education innovations to support educators, administrators, and policymakers in creating seamless transitions throughout the K–14 system for all students, especially those who have historically been underserved. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

We help local systems adapt promising research to meet their needs, and we develop innovative resources and tools that are implemented through multiple channels, from the highly local and personal to the regional and national. We provide long-term technical assistance, collaborate with partners at all levels of the education system, and advise community colleges and states.

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations. We have worked with states and education systems throughout Texas and across the country. For more information about our programs and resources, see **www.utdanacenter.org**.

About the Dana Center Mathematics Pathways (DCMP)

The Dana Center Mathematics Pathways (DCMP) is a systemic approach to dramatically increasing the number of students who complete math coursework aligned with their chosen program of study and who successfully achieve their postsecondary goals. The DCMP was initially launched as the New Mathways Project (NMP) in 2012 through a joint enterprise with the Texas Association of Community Colleges. For more information about the DCMP, see **www.dcmathpathways.org**.

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