The University of Texas at Austin
Charles A. Dana Center

Re-envisioning the Pathway to Calculus to Broaden Participation in STEM Programs

“. . . STEM-related jobs are among the best our economy offers, as evidenced by their high wages and lower unemployment rates than in other sectors. The increased supply of jobs in these fields will offer an opportunity to reduce income inequality in the United States. This opportunity can be captured only by increasing the number of U.S.-born college graduates with training in STEM fields from all demographic sectors of U.S. society.”

President’s Council of Advisors on Science and Technology, *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics* (2012).

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A Call to Re-envision

Professionals in Science, Technology, Engineering, and Mathematics (STEM) help drive our nation's innovation and competitiveness in a variety of fields and industries. According to the Department of Commerce, STEM occupations are expected to grow at a rate of 1.4 times faster than non-STEM occupations, and the United States will need approximately one million more STEM professionals between 2014 and 2024 (Noonan, 2017). However, the decreasing number of students deemed eligible for college-level mathematics, coupled with poor outcomes in developmental mathematics sequences, is negatively impacting the number of STEM degrees awarded. The insufficient number of students prepared to succeed in a college-level calculus course in their freshman year significantly reduces the pool of students likely to graduate with a STEM degree in four years (Kreysa, 2006). Ultimately, mathematics departments are left to grapple not only with how to address the poor success rates of their developmental STEM students, but also how to increase enrollments in calculus and the number of STEM graduates.

In an effort to support mathematics departments in addressing these challenges, the Dana Center Mathematics Pathways (DCMP) launched the Pathway to Calculus Project in 2014. The two goals of this project were to: 1) *develop a curriculum design process* to support mathematics departments seeking to re-envision pathways to calculus on their own campuses, and 2) *implement the process* to develop a re-envisioned pathway that improves calculus outcomes for **intermediate algebra ready students** to broaden participation in STEM fields.

To meet the first goal, the DCMP sought input from a wide range of researchers and mathematics faculty, and convened two teams. The **content design team** and the **structure design team** set out to do the following:

- Identify difficult concepts in calculus to aid math departments in designing appropriate pre-requisite courses that adequately prepare students for calculus.
- Develop a set of overarching content principles to guide math departments in selecting appropriate mathematical content for pre-requisite courses that lead to calculus.
- Develop a set of essential structural elements to assist math departments in making decisions about the types of pedagogy and classroom structures that support student preparation for calculus.


Dr. David Bressoud

The DCMP then recruited a team of experienced authors to develop a re-envisioned pathway to calculus based on the vision put forth by the content and structure teams. These authors used a deliberate review process to develop the **DCMP's Pathway to Calculus**, which comprises two college-level courses designed to prepare students for calculus. Faculty from a variety of institutions began teaching these courses in 2016, providing feedback to DCMP as part of a continuous improvement process.

This paper documents the recommendations of the content design team and structure design team, and describes the development of DCMP's Pathway to Calculus.

Developing a Curriculum Design Process

A Backmapping Approach

The word *precalculus* describes the mathematical knowledge and skills learned prior to the study of college-level calculus. The publication of college textbooks , therefore, encompasses an extensive range of topics that may or may not be essential for success in calculus. When designing courses around these textbooks, instructors must often choose which topics to cover and which to leave out. Generally, students are introduced to functions and relations, learn properties about families of functions and identify features of their graphs, practice numerous algebraic manipulations, use theorems to locate rational roots of polynomials, and prove trigonometric identities. Often, however, the end result is an experience that emphasizes algebraic manipulations while losing sight of some of the most important conceptual features.

In order to identify the most appropriate content for inclusion in a re-envisioned pathway to calculus, the DCMP invited prominent researchers in mathematics education to join the content design team. The primary goal of the content design team was to examine the large amount of curricular material usually associated with precalculus in light of current research in mathematics education, and to decide which skills, concepts, and procedures best prepare students for success in calculus.

In 2014, the team convened in spring and summer through a series of phone conferences and face-to-face meetings. Drawing from the team's individual experiences in the classroom, knowledge of mathematics education research, and understanding of how undergraduate institutions approach calculus, the members were able to identify aspects of calculus that tend to be the most demanding and difficult for students.

Content Design Team

- **David M. Bressoud**, DeWitt Wallace Professor of Mathematics, Macalester College (St. Paul, MN)
- **Stuart Boersma**, Professor of Mathematics, Central Washington University (Ellensburg, WA)
- **Helen Burn**, Professor of Mathematics, Highline Community College (Des Moines, WA)
- **Marilyn P. Carlson**, Professor of Mathematics, Arizona State University (Phoenix, AZ)
- **Eric Hsu**, Professor of Mathematics, San Francisco State University (San Francisco, CA)
- **Michael Oehrtman**, Associate Professor of Mathematical Sciences, University of Northern Colorado (Greeley, CO)
- **Frank Savina**, Course Program Specialist, Charles A. Dana Center (Austin, TX)

Difficult Concepts in Calculus Identified by the Content Design Team

Function as process—including thinking of a function as a complete activity; understanding that functions are processes that can be reversed and composed.

Covariation—including understanding dynamic geometric relationships; communicating change with correct language; understanding the chain rule and how compositions of functions transmit change.

Communicating about change and rates of change—including interpreting graphs of f, f', f''; understanding limiting behavior; identifying rate as a quantity in and of itself; examining change over entire intervals; understanding what it means for negative quantities to increase or decrease.

Limits and approximations—including understanding the definite integral as an accumulator.

Riemann sums—including making connections between the use of technology and summation notation; using and conceptualizing approximation rules.

Exponential and logarithmic growth—including the fact that each mathematical relationship is a function in its own right and also an inverse of the other.

Function notation—including distinguishing input from output and variables from parameters; using different symbols to represent equal processes.

For each difficult concept, the content design team also determined specific content and activities that would best prepare students for those portions of calculus—a process members called "backmapping." As expected, a long list of mathematical content resulted. However, by identifying and applying four well-defined, overarching principles, the content design team selected the most essential content for a re-envisioned pathway to calculus.

Selecting Essential Content

While the team members recognized that many of the content areas were clearly *helpful* in preparing students for calculus, they aimed to identify those content areas that were both immediately *meaningful to students* and *essential for student success* in calculus. Identifying these areas necessitated making difficult decisions to pare down the content; this approach produced the added benefit of creating a more manageable and focused curriculum.

As the team began to develop student learning outcomes, four overarching content principles emerged, delineating what students needed to achieve in order to succeed in calculus and beyond.

Overarching Content Principles

1. ***Deep understanding of the function process:*** A strong conceptual understanding of the process view (rather than the action view) of function gives students a critical mathematical foundation to support future learning in STEM fields. A curriculum that stresses the process view of a function prepares students to analyze function outputs on entire intervals of inputs, reason about inverting functions by reversing processes, and make stronger connections between the graph of a function and the function's relationship to generalized inputs and outputs. In such a curriculum, students also understand that a function is independent of a formula and are able to communicate about functions using multiple representations.

2. ***Proficiency in covariational reasoning:*** The ability to analyze two quantities simultaneously, how those quantities change, and how they covary enables students to better understand the unique and dynamic problem situations addressed by calculus and related disciplines. Courses in a pathway to calculus should provide students many opportunities to explore dynamic function relationships and help students more easily conceptualize the notions of an average rate of change, and the transition between an average rate of change and an instantaneous rate of change.

3. ***Fluency in communication with functions and function notation:*** Students communicate orally and in writing as they analyze function behavior using multiple representations. Courses in a pathway to calculus should engage students at the notational level by having them directly examine the need for function notation and by requiring them to interpret to and from function notation. Such communication skills are essential to developing students' mathematical knowledge and logical reasoning skills.

4. ***Facility with meaningful approaches to algebraic reasoning:*** Students engage with the curriculum content as they develop their algebra and problem-solving skills within authentic and relevant STEM contexts. Students create, explore, and interpret mathematical models and use algebra as a way of extracting additional information from a model or mathematical problem. This approach to algebra gives students an immediate appreciation of the usefulness of algebra and algebraic reasoning.

Just as these content principles shaped the work of the DCMP's content design team, they can also assist curriculum designers and developers in mathematics departments to develop student learning outcomes and, perhaps more important, serve as a lens to guide the curriculum writing process. By designing curricula that support these content principles, mathematics departments can create pathways to calculus that better prepare students to master difficult concepts in calculus by giving them the targeted skills and conceptual understanding to succeed. (See the Resources section for student learning outcomes and course outlines of DCMP's Pathway to Calculus.)

A Research-Based Approach to Making Pedagogical Decisions

To adequately support STEM-intending students in preparing for calculus, the DCMP invited prominent individuals in the mathematics field to join the structure design team. This 14-person team comprised mathematics faculty members from colleges and universities, and individuals affiliated with the Mathematical Association of America (MAA) and the American Mathematical Association of Two-Year Colleges (AMATYC).

The goal of the structure design team was to make important decisions about pedagogy, student supports, and course structures that enable students to succeed in calculus. The team began its work in January 2014 with a conference call to discuss the team's initial assignment: To identify promising programs across the country that were restructuring their developmental algebraically intensive sequences. Team members spent three months researching each of these programs.

In May 2014, the structure design team met for two days to make structure recommendations based on their collective research. The morning agenda for the first day included a review of those promising programs and a discussion around their similarities and differences. In the afternoon, the structure design team was divided into three groups to brainstorm possible course designs. Throughout the day, DCMP staff led several activities that were intentionally designed to allow team members to make their own conclusions, share those thoughts in small groups, discuss potential issues about selected practices, and synthesize their ideas as a team.

At the conclusion of the first day, consensus emerged around four key ideas for structure:

- In-class collaboration
- Instant-feedback environment
- Resources for at-home independent learning
- Delayed feedback through a variety of projects

The second day of the convening began with a combined meeting of the content design and structure design teams. The content design team shared its strong belief that an active learning pedagogy intentionally built around the four overarching content principles would help students succeed in calculus. In addition, the structure design team reviewed the proposed content to help make its decisions regarding the number of courses and credit hours.

Through a series of follow-up virtual meetings, the structure design team refined the four key ideas into a set of three essential structural elements that adequately support student preparation for calculus. These structural elements informed the course content for the DCMP's Pathway to Calculus, and can guide mathematics faculty as they think about the types of structures that help students succeed in calculus.

Structure Design Team

- **John P. (JP) Anderson**, Professor of Mathematics, San Jacinto College, South Campus (Houston, TX)
- **Colleen Berg**, Instructor, Texas Tech University (Lubbock, TX)
- **Caren L. Diefenderfer**, Governor of the MD–DC–VA Section of the Mathematical Association of America and Professor of Mathematics, Hollins University (Roanoke, VA)
- **Suzanne Dorée**, Chair of the Mathematical Association of America's CRAFTY subcommittee and Professor of Mathematics, Augsburg College (Minneapolis, MN)
- **Bekki George**, Instructional Assistant Professor, University of Houston, Main Campus (Houston, TX)
- **Suzie Goss**, Professor of Mathematics, Lone Star College–Kingwood (Kingwood, TX)
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- **Joanne Peeples**, Professor of Mathematics, El Paso Community College (El Paso, TX)
- **Virgil Pierce**, Associate Professor of Mathematics, The University of Texas–Pan American (Edinburg, TX)
- **Jim Roznowski**, AMATYC Past President, Delta College (Emeritus) (University Center, MI)

Essential Structural Elements

Element 1

Utilize a meaningful active-learning environment grounded in authentic STEM contexts.

The in-class component should provide immediately meaningful content within a variety of STEM-specific contexts, and be anchored in active-learning lessons designed to provide frequent opportunities for students to work collaboratively in pairs or small groups to solve problems that range from easy (to build skills and confidence) to more challenging (to facilitate deeper learning and increase students' constructive perseverance).

Both the content and structure design teams felt strongly that these collaborative experiences provide students opportunities to build and reinforce their conceptual understanding, and develop their capacities to learn inside and outside of class in their subsequent STEM courses (e.g., group work in calculus and labs in the sciences). Additionally, an essential function of the in-class component is to allow students to experience—and process—failed attempts in an academically supportive environment. The structure design team felt strongly that social connections created through the in-class learning community help drive and sustain student engagement and motivation.

Element 2

Provide instant feedback outside of class.

The curriculum should include a variety of problems through an online platform that ranges from entry level to more challenging to give students ample opportunities to work independently outside of class. The platform should provide students access to hints, answers, and explanations so they can experience immediate feedback on their understanding and skill mastery; it should also include homework problems in mathematical vocabulary, computations and processes, and correct use of mathematical notation.

Element 3

Require independent projects outside of class.

Give students opportunities to work on class projects regularly, as independent learners, in an environment without instant feedback. During this independent learning time, students work on open-ended problems based on recent in-class activities and contexts that enable them to work on more open-ended problems, practice their written communication skills, and take time to check the reasonableness of their answers.

Implementing the Curriculum Design Process

Developing DCMP's Pathway to Calculus

To adequately accelerate and prepare **intermediate algebra ready students** for the algebraic demands of calculus and other courses needed for STEM programs of study, two courses that included significant algebraic skill building were developed. The need for significant algebraic skill building became obvious after the structure design team considered the content design team's recommendations for course content. Ultimately, both design teams concluded that the first course should be a five-credit, semester-long co-requisite course (since it provides most of the necessary algebraic skill building) and that the second course should be a four-credit, semester-long course (since it requires less of the algebraic skill building) that leads directly into calculus.

The content design team's backmapping approach, combined with an understanding of appropriate structures and pedagogy, helped identify overarching content design principles and clarified content learning outcomes. These, in turn, were utilized to develop detailed course outlines that guided the curriculum development process.

In Fall 2014, curriculum authors used the roadmap laid out in the course outlines and the content and structure teams' recommendations to begin developing the **DCMP's Pathway to Calculus**, a year-long sequence comprising two college-level courses: *Reasoning with Functions I* and *Reasoning with Functions II*.

Over a span of two years, the authors developed a total of 188 lesson parts. Each lesson part has four main components:

1. Preview Assignment: an out-of-class learning experience that prepares students to engage and succeed in the following day's activities
2. In-Class Activity: a fully contextualized 25-minute lesson that leverages active learning and small group work to help students persevere through the problem-solving process
3. Practice Assignment: an out-of-class-learning experience that provides opportunities for students to practice skills and concepts learned in class and extend their learning
4. Instructor Notes: a detailed set of notes for the instructor that identifies learning outcomes for each lesson part, suggests pedagogical approaches to lesson facilitation, and flags places where students may struggle

Intermediate algebra ready students would begin the DCMP's Pathway to Calculus with *Reasoning with Functions I*, while students who would traditionally enter a beginning algebra course would begin the pathway by taking a traditional beginning algebra course or the DCMP's developmental course *Foundations of Mathematical Reasoning*.

Implementation and Data Gathering

To test the efficacy of DCMP's Pathway to Calculus, Dana Center staff selected one community college and one 4-year university to teach *Reasoning with Functions I* (RF1) and *Reasoning with Functions II* (RF2). Palomar College in San Marcos, California, and the University of Cincinnati in Ohio were selected for the study due to their large and diverse student populations.

Palomar College offered DCMP's Pathway to Calculus in Spring 2016, while the University of Cincinnati began teaching the pathway in Spring 2017. A calendar of course offerings follows:

ENROLLMENT IN DCMP'S PATHWAY TO CALCULUS						
Course	**Spring 2016**	**Fall 2016**	**Spring 2017**	**Fall 2017**	**Spring 2018**	**Total Students**
Reasoning with Functions I	2 sections 29 students	2 sections 30 students	1 section 28 students	5 sections 149 students	4 sections 84 students	320
Reasoning with Functions II	None	2 sections 29 students	1 section 21 students	3 sections 76 students	3 sections 43 students	169
					Total:	**489**

Summative Evaluation

The Dana Center's evaluation team is currently collecting and analyzing data from the two institutions. Evaluators will continue to monitor student success and persistence rates throughout this pathway to calculus and, more important, student success rates in calculus. By disaggregating the data by race, ethnicity, and gender, the Dana Center hopes to gauge the efficacy of this re-envisioned pathway to calculus.

A Framework for Re-envisioning a Pathway to Calculus

The curriculum design and development processes outlined in this paper serve as a model for collaboration within institutions of higher education seeking to broaden diverse participation and success in STEM fields. They are designed to launch a process of continuous improvement that enables institutions to update and improve their curricula as part of an ongoing effort to better serve students. Mathematics departments can utilize this model as a framework to design and develop pathways to calculus that prepare their students to succeed in calculus. The overarching content principles and essential structural elements can guide faculty in designing engaging, high-quality courses that lead students to and through calculus. The work of the DCMP Pathway to Calculus Project aims to push the field to address the important need of increasing student success in STEM related fields.

Resources

Learning Outcomes

Reasoning with Functions I: Student learning outcomes and table of contents

https://dcmathpathways.org/resources/dcmp-reasoning-functions-i-overview-and-learning-outcomes

Reasoning with Functions II: Student learning outcomes and table of contents

https://dcmathpathways.org/resources/dcmp-reasoning-functions-ii-overview-and-learning-outcomes

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Suggested Readings

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About the Dana Center

The Charles A. Dana Center develops and scales mathematics and science education innovations to support educators, administrators, and policy makers in creating seamless transitions throughout the K–16 system for all students, especially those who have historically been underserved. We focus in particular on strategies for improving student engagement, motivation, persistence, and achievement.

The Center was founded in 1991 at The University of Texas at Austin. Our staff members have expertise in leadership, literacy, research, program evaluation, mathematics and science education, policy and systemic reform, and services to high-need populations.

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